



ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΙΑΣ



# Démographie spatiale/Spatial Demography

## Session 6: Measures of Spatial Autocorrelation

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# Άδειες Χρήσης

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# Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στα πλαίσια του εκπαιδευτικού έργου του διδάσκοντα.
- Το έργο «**Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Θεσσαλίας**» έχει χρηματοδοτήσει μόνο τη αναδιαμόρφωση του εκπαιδευτικού υλικού.
- Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.



# Weight Matrix (1)

In order to measure spatial autocorrelation, one first needs to define what is meant by two values being close together, namely a distance measure must be determined.

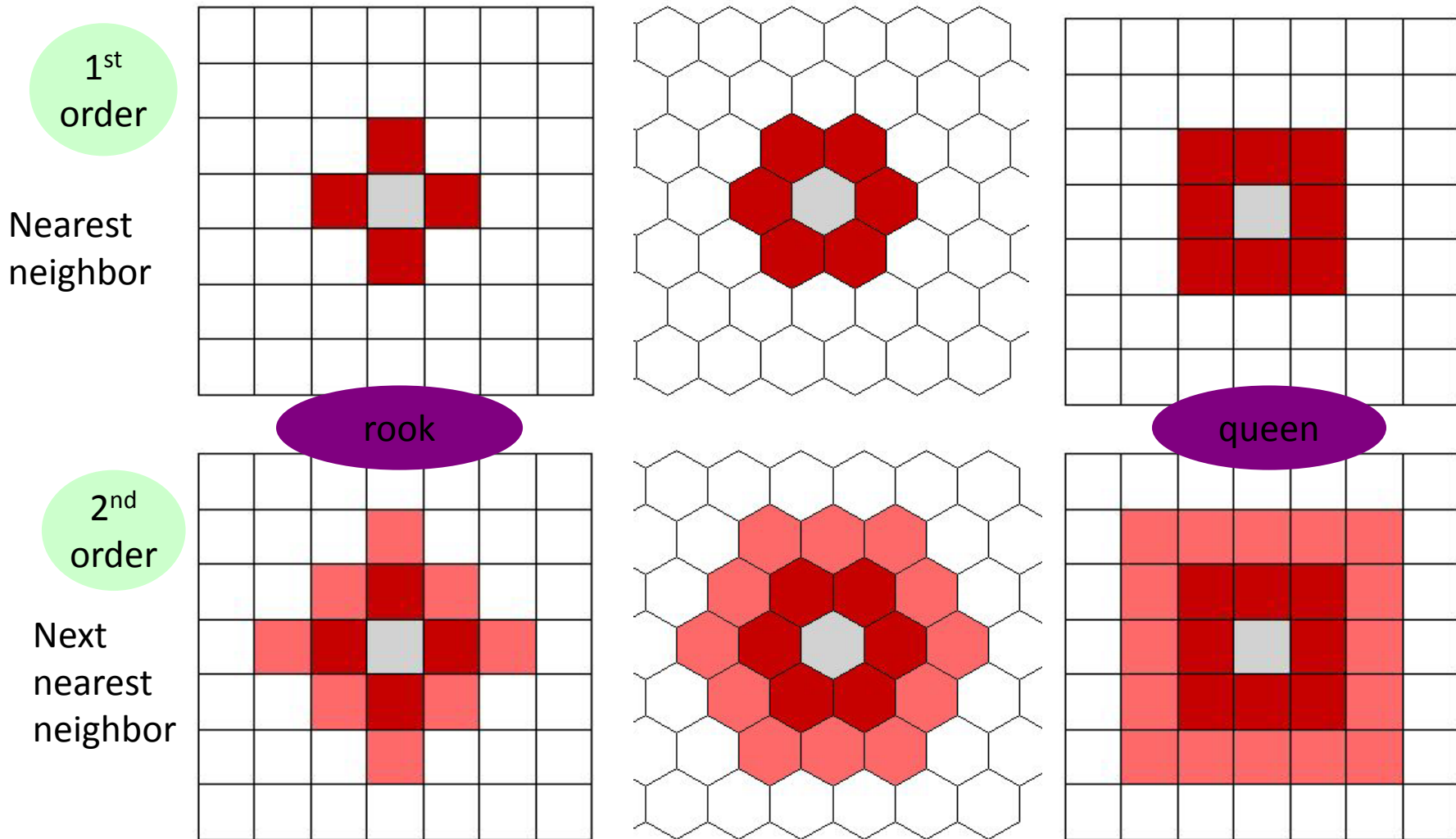
These distances are presented in weight matrix, which defines the relationships between locations of corresponding values. If data are collected at  $n$  locations, then the weight matrix will be  $n \times n$  with zeroes on the diagonal.

Weights can be based on Contiguity [binary (0,1)]

and

on Distance-continuous variable

# Weight Matrix (2)



# Global Moran's I (1)

It is based on cross-products of the deviations from the mean, as:

The Moran's  $I$  statistic for spatial autocorrelation is given as:

$$I = \frac{n \sum_{i=1}^n \sum_{j=1}^n w_{i,j} z_i z_j}{S_0 \sum_{i=1}^n z_i^2} \quad (1)$$

where  $z_i$  is the deviation of an attribute for feature  $i$  from its mean ( $x_i - \bar{X}$ ),  $w_{i,j}$  is the spatial weight between feature  $i$  and  $j$ ,  $n$  is equal to the total number of features, and  $S_0$  is the aggregate of all the spatial weights:

$$S_0 = \sum_{i=1}^n \sum_{j=1}^n w_{i,j} \quad (2)$$

The  $z_I$ -score for the statistic is computed as:

$$z_I = \frac{I - \mathbf{E}[I]}{\sqrt{\mathbf{V}[I]}} \quad (3)$$

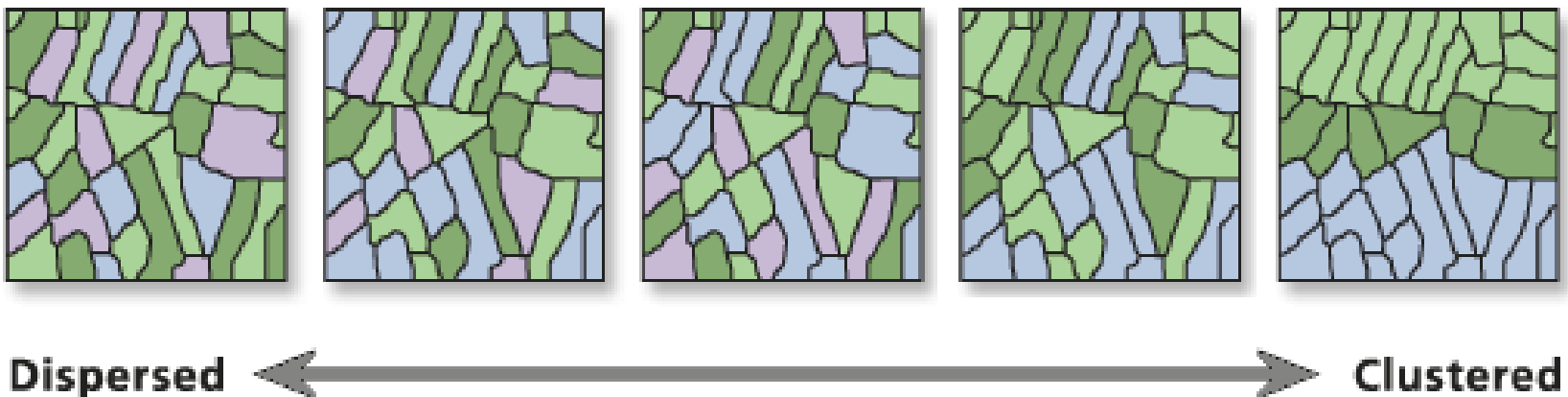
where:

$$\mathbf{E}[I] = -1/(n - 1) \quad (4)$$

$$\mathbf{V}[I] = \mathbf{E}[I^2] - \mathbf{E}[I]^2 \quad (5)$$

# Global Moran's I (2)

- Values from -1 to 1
- In general, a Moran's Index value near +1.0 indicates clustering while an index value near -1.0 indicates dispersion. However, without looking at statistical significance you have no basis for knowing if the observed pattern is just one of many, many possible versions of random.



Source

[http://resources.esri.com/help/9.3/arcgisengine/java/gp\\_toolref/spatial\\_statistics\\_tools/how\\_spatial\\_autocorrelation\\_colon\\_moran\\_s\\_i\\_spatial\\_statistics\\_works.htm](http://resources.esri.com/help/9.3/arcgisengine/java/gp_toolref/spatial_statistics_tools/how_spatial_autocorrelation_colon_moran_s_i_spatial_statistics_works.htm)

# Local indicators of spatial association (LISA) (1)

Anselin (1995) defined LISA as:

The local statistics for each observation gives an indication of the extent of significant spatial clustering of similar values around that observation.

The sum of local statistics for all observation is proportional (or equal) to a corresponding global statistics.



# Local indicators of spatial association (LISA) (2)

The Local Moran's  $I$  statistic of spatial association is given as:

$$I_i = \frac{x_i - \bar{X}}{S_i^2} \sum_{j=1, j \neq i}^n w_{i,j} (x_j - \bar{X}) \quad (1)$$

where  $x_i$  is an attribute for feature  $i$ ,  $\bar{X}$  is the mean of the corresponding attribute,  $w_{i,j}$  is the spatial weight between feature  $i$  and  $j$ , and:

$$S_i^2 = \frac{\sum_{j=1, j \neq i}^n w_{ij} x_j}{n - 1} - \bar{X}^2 \quad (2)$$

with  $n$  equating to the total number of features.

The  $z_{I_i}$ -score for the statistics are computed as:

$$z_{I_i} = \frac{I_i - \mathbf{E}[I_i]}{\sqrt{\mathbf{V}[I_i]}} \quad (3)$$

where:

$$\mathbf{E}[I_i] = - \frac{\sum_{j=1, j \neq i}^n w_{ij} x_j}{n - 1} \quad (4)$$

$$\mathbf{V}[I_i] = \mathbf{E}[I_i^2] - \mathbf{E}[I_i]^2 \quad (5)$$

Source:

[http://resources.esri.com/help/9.3/arcgisengine/java/gp\\_toolref/spatial\\_statistics\\_tools/how\\_cluster\\_and\\_outlier\\_analysis\\_colon\\_anselin\\_local\\_moran\\_s\\_i\\_spatial\\_statistics\\_works.htm](http://resources.esri.com/help/9.3/arcgisengine/java/gp_toolref/spatial_statistics_tools/how_cluster_and_outlier_analysis_colon_anselin_local_moran_s_i_spatial_statistics_works.htm)

# Moran Scatter Plot (1)

“Moran’s I spatial autocorrelation statistic is visualized as the slope in the scatter plot with the spatially lagged variable on the vertical axis and the original variable on the horizontal axis. The variables are standardized to facilitate interpretation and categorization of the type of spatial autocorrelation (cluster or outlier).”

Source: Anselin (2003)

We are going to elaborate further during the lab sessions.....



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# End of Session

