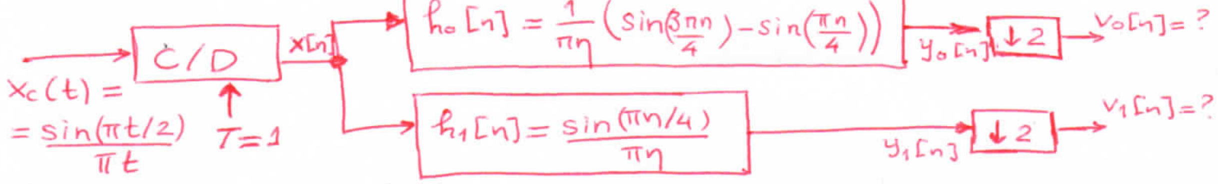
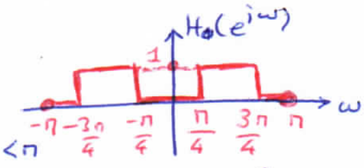
HY 342 | DSP-MIDTERM | 2018-19-A 04/12/2018

1

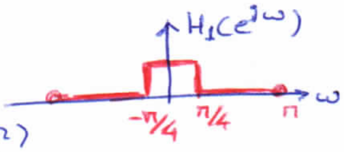


• Δουλεύουμε στο πεδίο της συχνότητας:

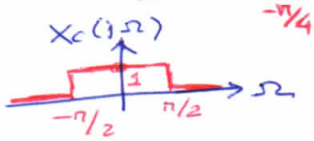
$$h_0[n] = \frac{\sin\left(\frac{3\pi n}{4}\right)}{\pi n} - \frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n} \Rightarrow H_0(e^{j\omega}) = \begin{cases} 1; & \frac{\pi}{4} < |\omega| < \frac{3\pi}{4} \\ 0; & |\omega| < \frac{\pi}{4} \ \& \ \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$



$$h_1[n] = \frac{\sin(\pi n/4)}{\pi n} \Rightarrow H_1(e^{j\omega}) = \begin{cases} 1; & |\omega| < \frac{\pi}{4} \\ 0; & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

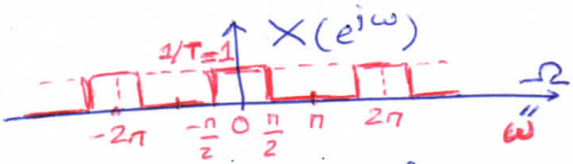


• $x_c(t) = \frac{\sin(\pi t/2)}{\pi t} \Rightarrow X_c(j\Omega) = \begin{cases} 1; & |\Omega| < \pi/2 \\ 0; & \text{αλλού} \end{cases}$



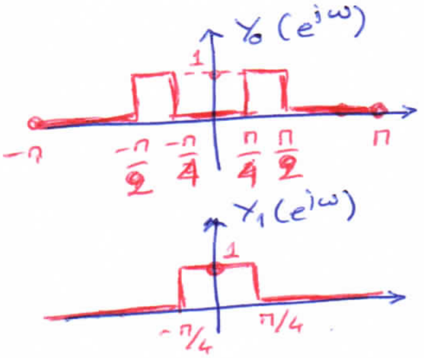
SAMPLING $\Longrightarrow$ (T=1)

$\Omega_s = \frac{2\pi}{T} = 2\pi$

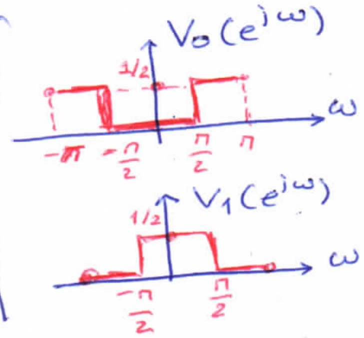


$\left(x[n] = \frac{\sin(\pi n/2)}{\pi n} \right)$

FILTERING $\Longrightarrow$



↓2 $\Longrightarrow$



$$\text{DTFT}^{-1} \Longrightarrow \left\{ v_0[n] = (-1)^n \frac{\sin\left(\frac{\pi n}{2}\right)}{2\pi n} \ ; \ v_1[n] = \frac{\sin\left(\frac{\pi n}{2}\right)}{2\pi n} \right\}$$

2

FIR TYPE (I) $M=4$, $h[n] \in \mathbb{R}$ $\Rightarrow$ $H(z)$, $h[n]$, POLE-ZERO-DIAG. IMPLEMENTATION (EFFICIENT)

ZERO @ $1+j$

$x[n] = (-1)^n \Rightarrow y[n] = 25(-1)^n$

• Γραμμικότητα φάσης $z_0 \Rightarrow 1/z_0$
• Πραγματική $h[n]$ $z_0 \Rightarrow z_0^*$
$\Big\} \Rightarrow z_0, z_0^*, \frac{1}{z_0}, \frac{1}{z_0^*}$ ZEROS δηλ. $1+j$, $1-j$, $\underbrace{\frac{1}{1+j}}_{\frac{1}{2}-\frac{1}{2}j}$, $\underbrace{\frac{1}{1-j}}_{\frac{1}{2}+\frac{1}{2}j}$

Άρα $H(z) = A(1 - z^{-1}(1+j))(1 - z^{-1}(1-j))(1 - \frac{1}{1+j}z^{-1})(1 - \frac{1}{1-j}z^{-1}) =$

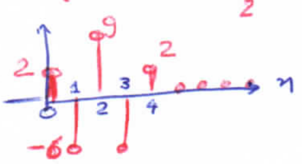
$= A(1 - 2z^{-1} + 2z^{-2})(1 - z^{-1} + \frac{z^{-2}}{2}) = A(1 - 3z^{-1} + \frac{9}{2}z^{-2} - 3z^{-3} + z^{-4}) = \boxed{2 - 6z^{-1} + 9z^{-2} - 6z^{-3} + 2z^{-4}}$

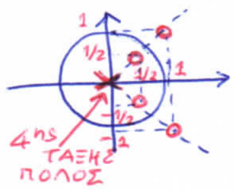
ΠΡΑΞΕΙΣ
$$\begin{cases} 1 - 2z^{-1} + 2z^{-2} \\ \quad - z^{-1} + 2z^{-2} - 2z^{-3} \\ \quad\quad \frac{z^{-2}}{2} - z^{-3} + z^{-4} \end{cases}$$

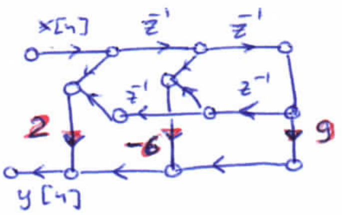
$x[n] = (-1)^n \Rightarrow y[n] = (-1)^n H(-1) = (-1)^n \cdot A \cdot \frac{25}{2} = 25(-1)^n$

$\Rightarrow \boxed{A=2}$

$h[n]$:







3

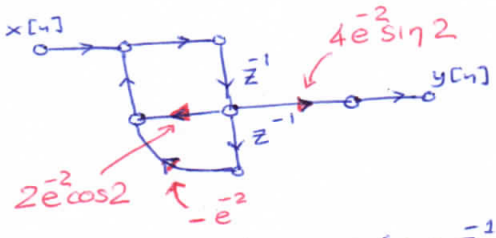
$$H_c(s) = \frac{2}{s^2+2s+2} \xrightarrow{T=2} \begin{cases} \text{IMPULSE INVARIANCE } H(z), \text{ IMPLEMENTATION (DIRECT)} \\ \text{BILINEAR TRANSFORM } H(z), \text{ DIRECT FORM IMPL, POLES ZEROS} \end{cases}$$

- IMPULSE INVARIANCE

$$H_c(s) = \frac{2}{(s+1)^2 + 1^2} \xrightarrow{\mathcal{L}^{-1}} h_c(t) = 2e^{-t}\sin(t).u(t) \xRightarrow{T=2} h[n] = 4e^{-2n}\sin(2n)u[n]$$

$$h[n] = T h_c(nT)$$

$$\xRightarrow{Z} \boxed{H(z) = \frac{[4e^{-2}\sin(2)]\,z^{-1}}{1-[2e^{-2}\cos(2)]z^{-1} + e^{-2}z^{-2}}}$$

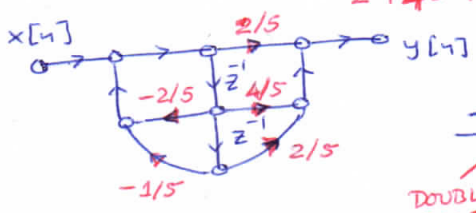


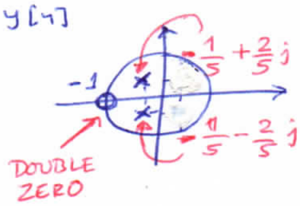
- BILINEAR TRANSFORM

$$H_c(s) = \frac{2}{s^2+2s+2} \underset{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}{\xRightarrow{T=2}} H(z) = \frac{2}{\frac{(1-z^{-1})^2}{(1+z^{-1})^2} + 2\frac{1-z^{-1}}{1+z^{-1}} + 2} = \frac{2(1+z^{-1})^2}{\underbrace{(1-z^{-1})^2 + 2(1-z^{-2}) + 2(1+z^{-1})^2}}$$

$1-2z^{-1}+z^{-2}$
$2 \qquad -2z^{-2}$
$2+4z^{-1}+2z^{-2}$

$$\Rightarrow \boxed{H(z) = \frac{2+4z^{-1}+2z^{-2}}{5+2z^{-1}+z^{-2}}}$$





ΡΙΖΕΣ ΠΑΡΑΝΟΜΑΣΤΗ: $-\frac{1}{5} \pm \frac{2}{5}j$ (POLES)

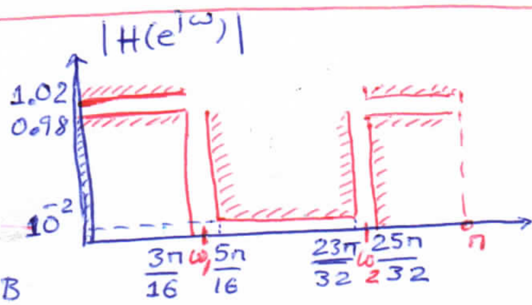
» ΑΡΙΘΜΗΤΗ : $-1, -1$ (ZEROS)

4

BANDSTOP FIR FILTER BY WINDOWING

$0.98 < |H(e^{j\omega})| < 1.02$; $\quad |\omega| \leq \frac{3\pi}{16}$ & $\frac{25\pi}{32} \leq |\omega| \leq \pi$

$|H(e^{j\omega})| < 0.01$; $\quad \frac{5\pi}{16} \leq |\omega| \leq \frac{23\pi}{32}$

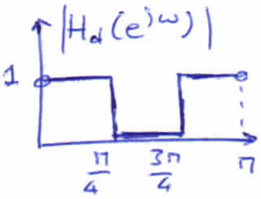


- $\delta_1 = 0.02$, $\delta_2 = 0.01$ $\Rightarrow \delta = \min\{\delta_1, \delta_2\} = 10^{-2} \Rightarrow -20\log_{10}\delta = 40\,dB$
  $\Rightarrow \boxed{\text{HAMMING WINDOW}}$

- $\Delta\omega_1 = \frac{5\pi}{16} - \frac{3\pi}{16} = \frac{2\pi}{16}$
  $\Delta\omega_2 = \frac{25\pi}{32} - \frac{23\pi}{32} = \frac{2\pi}{32} = \frac{\pi}{16}$
  $\Bigg\} \Rightarrow \Delta\omega = \min\{\Delta\omega_1, \Delta\omega_2\} = \frac{\pi}{16} = \frac{8\pi}{M} \Rightarrow M = 8.16 = \boxed{128}$

  TYPE (I) ✓

- $\omega_1 = \left(\frac{3\pi}{16} + \frac{5\pi}{16}\right)/2 = \frac{8\pi}{32} = \frac{\pi}{4}$
  $\omega_2 = \left(\frac{23\pi}{32} + \frac{25\pi}{32}\right)/2 = \frac{48\pi}{32.2} = \frac{3\pi}{4}$



$N_b = 3$

$G_1 = 1,\ G_2 = 0,\ G_3 = 1, (G_4 = 0)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad N_b + 1$

- Άρα:

$$\boxed{h_d[n] = \frac{\sin\left(\frac{\pi}{4}[n-64]\right) - \sin\left(\frac{3\pi}{4}[n-64]\right) + \sin\left(\pi[n-64]\right)}{\pi(n-64)} \cdot \left(0.54 - 0.46\cos\left(\frac{\pi n}{64}\right)\right) \quad \text{για } 0 \leq n \leq 128}$$