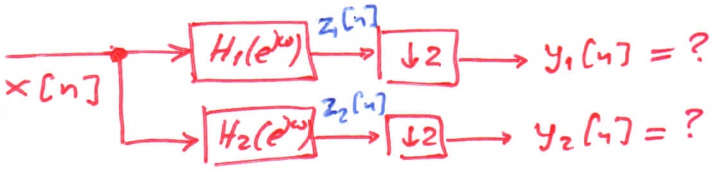
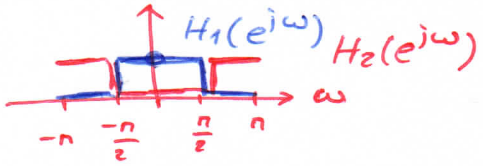
HY 342 — DSP – MIDTERM — 2016–2017–A

(23/11/2016)

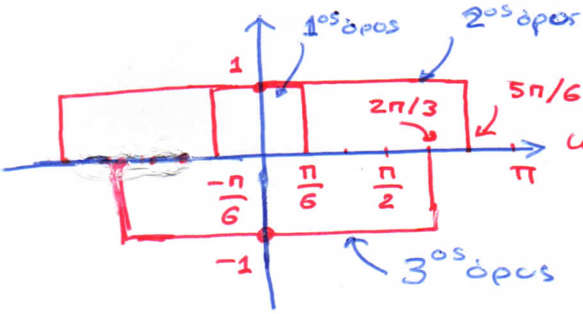
1

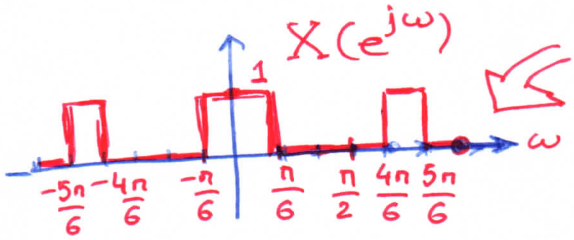




$$x[n] = \frac{1}{\pi n}\left[\sin\frac{\pi n}{6} + \sin\frac{\pi n 5}{6} - \sin\frac{\pi n 2}{3}\right]$$

$x[n] = \cdots \overset{DTFT}{\Rightarrow} X(e^{j\omega})$ δίνεται από το σχήμα:

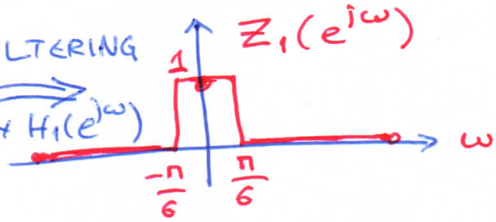




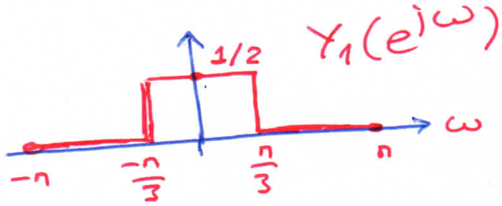
Στη συνέχεια:

$$Y_i(e^{j\omega}) = \frac{1}{2}\left[Z_i(e^{j\frac{\omega}{2}}) + Z_i(e^{j(\frac{\omega}{2}-\pi)})\right]$$

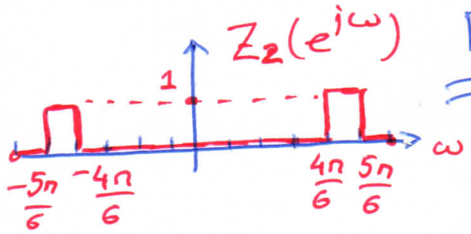
FILTERING BY $H_1(e^{j\omega})$



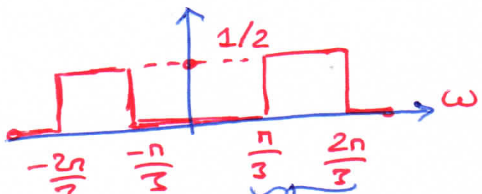
$\boxed{\downarrow 2}$



FILTERING BY $H_2(e^{j\omega})$



$\boxed{\downarrow 2}$



προέρχεται από:

$$\begin{cases} -\frac{5\pi}{6} + \pi = \frac{\pi}{6} (*2) = \frac{\pi}{3} \\ -\frac{4\pi}{6} + \pi = \frac{2\pi}{6} (*2) = \frac{2\pi}{3} \end{cases}$$

Κατά συνέπεια:

$$y_1[n] = \frac{1}{2\pi n}\cdot \sin\frac{\pi n}{3}$$

$$y_2[n] = \frac{1}{2\pi n}\cdot\left(\sin\frac{2\pi n}{3} - \sin\frac{\pi n}{3}\right)$$

2 FIR TYPE II

5 ΜΗΔΕΝΙΚΑ (= M)

$H(\sqrt{3}j) = 0$

ΠΡΑΓΜΑΤΙΚΟΙ ΣΥΝΤΕΛΕΣΤΕΣ

$x[n] = 1 \Rightarrow y[n] = 20$

$\Rightarrow H(z) = ?$ ΚΑΙ ΑΛΛΕΣ ΕΡΩΤΗΣΕΙΣ

- TYPE II $\Rightarrow H(-1) = 0$
- SYMMETRIC FIR: $H(z_0) = 0 \Rightarrow H\left(\frac{1}{z_0}\right) = 0$
- REAL $h[n]$: $H(z_0) = 0 \Rightarrow H(z_0^*) = 0$

$\xRightarrow{z_0 = \sqrt{3}j}$ ΜΗΔΕΝΙΚΑ ΣΤΑ: $\left\{-1, \pm\sqrt{3}j, \pm\frac{1}{\sqrt{3}}j\right\} \Rightarrow$

$$\Rightarrow H(z) = A\cdot(1 + z^{-1})\cdot(1 + \sqrt{3}j\, z^{-1})\cdot(1 - \sqrt{3}j\, z^{-1})\cdot\left(1 + \frac{1}{\sqrt{3}j} z^{-1}\right)\cdot\left(1 - \frac{1}{\sqrt{3}j} z^{-1}\right)$$

$$\Rightarrow H(z) = A\,(1 + z^{-1})(1 + 3z^{-2})\left(1 + \frac{1}{3} z^{-2}\right) \Rightarrow$$

$$\Rightarrow H(z) = A\,(1 + z^{-1})\left(1 + \frac{10}{3} z^{-2} + z^{-4}\right) \Rightarrow$$

$$\Rightarrow H(z) = A\left(1 + z^{-1} + \frac{10}{3} z^{-2} + \frac{10}{3} z^{-3} + z^{-4} + z^{-5}\right) \quad (1)$$

$$x[n] = 1^n \Rightarrow y[n] = H(1)\cdot 1^n = 32 \Rightarrow A\left(4 + \frac{20}{3}\right) = 32 \Rightarrow A = 3 \quad (2)$$

a. Από (1), (2) $\Rightarrow$ $\boxed{H(z) = 3 + 3z^{-1} + 10z^{-2} + 10z^{-3} + 3z^{-4} + 3z^{-5}}$

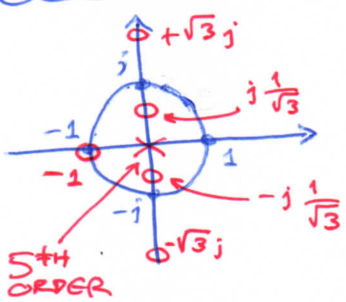
b. Προφανώς $\boxed{\measuredangle H(e^{j\omega}) = -\frac{5\omega}{2}\ (+\pi)}$ κατά διαστήματα

d. $x[n] = (-1)^n \Rightarrow$

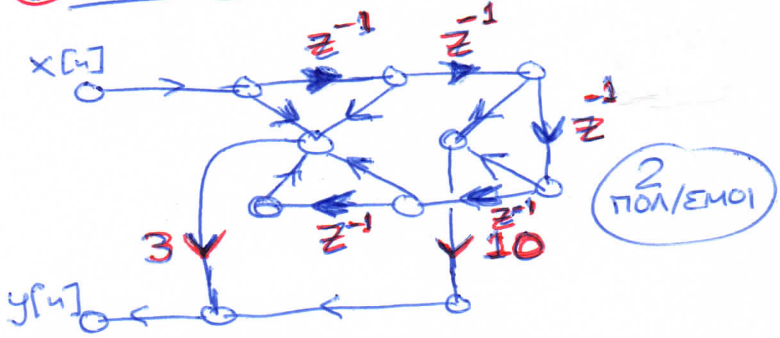
$\Rightarrow y[n] = H(-1)\cdot x[n]$

$\Rightarrow \boxed{y[n] = 0}$

c. ΔΙΑΓΡΑΜΜΑ ΜΗΔΕΝΙΚΩΝ-ΠΟΛΩΝ



e. EFFICIENT IMPLEMENTATION:



3a

$$\left.\begin{array}{c} H_c(s) = \dfrac{1}{(s+1)^3} \\ \text{IMPULSE INV.} \\ T = 2 \end{array}\right\} \Rightarrow H(z) = ?$$

- $H_c(s) = \dfrac{1}{(s+1)^3} \xrightarrow[\mathcal{L}^{-1}(\cdot)]{\text{ευστάθεια}} h_c(t) = \dfrac{t^2}{2} e^{-t} u(t) \xrightarrow[\text{IMPULSE INVARIANCE}]{T=2}$

$(h[n] = T h_c(nT))$

$\Rightarrow h[n] = 4n^2 e^{-2n} u[n] \Rightarrow$

$$\xrightarrow[\mathcal{Z}(\cdot)]{} \boxed{H(z) = \frac{(4e^{-2})z^{-1} + (4e^{-4})z^{-2}}{(1 - e^{-2}z^{-1})^3}}$$

- Γιατί: $n(n-1)a^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \dfrac{2a^2 z}{(z-a)^3} \Rightarrow$

$\quad n^2 a^n u[n] - n a^n u[n]$

$\Rightarrow n^2 a^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \dfrac{2a^2 z}{(z-a)^3} + \dfrac{az}{(z-a)^2} = \dfrac{2a^2 z + az^2 - a^2 z}{(z-a)^3}$

$\Rightarrow n^2 a^n u[n] = \dfrac{az^2 + a^2 z}{(z-a)^3} = \dfrac{az^{-1} + a^2 z^{-2}}{(1 - az^{-1})^3}$

NOTE: $a = e^{-2}$

3β

L.P. BUTTERWORTH
$N=2$
$\omega_c=\sqrt{2}$
[IMPULSE INVARIANCE]
$\Rightarrow H(z)=?$ D.F. II

- $$H_c(s)=\frac{\Omega_c^2}{(s-s_0)(s-s_1)}=\frac{\Omega_c^2}{\left(s-\Omega_c e^{j\frac{3\pi}{4}}\right)\left(s-\Omega_c e^{j\frac{5\pi}{4}}\right)}$$

$$=\frac{\Omega_c^2}{s^2-s\Omega_c\left(e^{j\frac{3\pi}{4}}+e^{-j\frac{3\pi}{4}}\right)+\Omega_c^2 e^{j\frac{8\pi}{4}}}=\frac{\Omega_c^2}{s^2+\sqrt{2}\,\Omega_c s+\Omega_c^2} \quad (1)$$

- IMPULSE INVARIANCE $\Rightarrow$ $\Omega_c=\omega_c=\sqrt{2}$ (2) (με $T=1$)

- Από (1),(2) $\Rightarrow$ $$H_c(s)=\frac{2}{s^2+2s+2}=\frac{2\cdot 1}{(s+1)^2+1}\Rightarrow$$

$$\xrightarrow[\text{ΕΥΣΤΑΘΕΙΑ}]{\mathcal{L}^{-1}(\cdot)} h_c(t)=2e^{-t}\sin(t)u(t) \xrightarrow[\text{IMPULSE INVARIANCE}]{(T=1)} h[n]=2e^{-n}\sin(n)u(n)$$

$h[n]=h_c(T)$

$$\xrightarrow[\substack{a=e^{-1}\\ \omega_0=1}]{\mathcal{Z}(\cdot)} H(z)=\frac{2e^{-1}z\sin(1)}{z^2-2e^{-1}z\cos(1)+e^{-2}}\Rightarrow$$

$$\Rightarrow \boxed{H(z)=\frac{[2e^{-1}\sin(1)]\cdot z^{-1}}{1-[2e^{-1}\cos(1)]z^{-1}+e^{-2}z^{-2}}}$$

- ΔΙΑΓΡΑΜΜΑ ΥΛΟΠΟΙΗΣΗΣ