Optimum Modulation and Multicode Formats in CDMA Systems with Multiuser Receivers

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Abstract—**We study the problem of maximizing the total system throughput under a bit error rate constraint for all users in the uplink of a single-cell synchronous CDMA system. Users realize variable bit rates by using a combination of multicode transmission and adaptive QAM modulation. We assume random signature sequences for all users, and perform an asymptotic analysis. We parametrize each user's resource allocation scheme by two parameters, viz., the number of signatures the user transmits with and the number of signal points in the user's QAM constellation, and optimize the total throughput over this parameter space. We examine four different settings: single-user matched filter (SUMF) and minimum-mean-square error (MMSE) receiver at the base station, with and without maximum-power constraints. For a single user system, we describe the jointly optimum number of multicodes and constellation sizes for these four different system models. When multiple users are present, we show that the total throughput is maximized when only one user transmits: with no maximum power constraints this user can be chosen arbitrarily, otherwise it should be the one with the largest SNR. This solution, although optimal in the sense of maximizing the total throughput, is unfair to all but one user: thus, we examine a scheduling mechanism that assigns equal time frames to all users, thus yielding maximum fairness, and discuss the resulting total throughput loss.**

Keywords—**CDMA, adaptive modulation, multicode, throughput maximization, power control, MMSE.**

I. Introduction

One of the key objectives of future wireless systems is to deliver flexible, variable-data-rate services with high spectral efficiency. Code Division Multiple Access (CDMA) is a promising candidate to realize this objective [1], [2]. Current CDMA systems, such as those based on the IS-95 standard, are designed primarily for voice communications, which is a real-time, constant-bit-rate service in which each user needs to maintain a constant quality of service (QoS) when communicating with the base station. By choosing the signal-to-interference ratio (SIR) as the QoS measure, the resource allocation problem in the context of voice communications becomes that of assigning transmit powers so that each user achieves its target SIR at the base station. No use is made of an excess SIR in this context. In data communications, on the other hand, users may require variable bit rates, and a higher SIR allows transmission at a higher rate. Thus, optimal distribution of the resources among the users in the context of data communications calls for maximization of the total system throughput at any given instant. This is precisely the focus of this paper.

There are several approaches for supporting variable bit rates in a CDMA system. Among those are:
(a) Fixed chip rate, variable processing gain [3], [4],
(b) Fixed processing gain, variable chip rate [3], [5],
(c) Fixed chip rate and processing gain, multiple signatures [6].
In (a), the symbol duration is varied over a fixed-rate chip sequence. As the symbol duration is shortened the bit rate is increased, but at the same time the processing gain is decreased. Consequently, the immunity to multiaccess interference is decreased as the bit rate is increased. In (b), the chip rate is varied along with the symbol duration to keep the processing gain of the user fixed. In this method, as the duration of a symbol is decreased, the bit rate of the user is increased while its immunity to multiaccess interference remains unchanged. However, the transmission bandwidth of the signal is increased and therefore bandwidth requirement is increased. Reference [3] comments that (b) is not very attractive, since this unequal bandwidth expansion among the users complicates the frequency planning task. A comparative study of schemes (a) and (b) can be found in [5]. In (c), the symbol duration and the processing gain are kept fixed, and the user's bit rate is increased by assigning it multiple signatures (commonly called "multicode"). This method entails self-interference, that is, simultaneous transmission of several nonorthogonal signatures from the same user creates multiaccess interference among themselves. It is also possible to use multilevel modulation schemes, as in the case of TDMA-based systems [7], [8], [9], in combination with any one of the spreading methods mentioned above to deliver variable bit rates.

In this work we investigate the combined use of multiple signatures and adaptive multilevel modulation to enable users to transmit at variable bit rates. We consider the cases of single-user matched filter (SUMF) and minimum-mean-square error (MMSE) receivers (see for example [10], [11]) at the base station. We restrict the structure of the multilevel modulation to QAM. We assume that all of the signatures are created randomly and that each of the K_i signatures of user i are modulated by the same number of constellation points M_i. Therefore the parameter pairs $\{K_i, M_i\}_{i=1}^{J}$, where J denotes the number of users, completely characterize the combined multicode/multilevel-modulation scheme.

We maximize the achievable system throughput, defined as the total number of bits that can be transmitted per second per hertz, under a bit error rate (BER) constraint for all users. Specifically, we study the joint assignment to all users of the number of signatures and the constellation sizes that maximizes the total system throughput, subject to the constraint that the BERs of all users are below a given level. Clearly, the throughput of a user i is an increasing function of K_i and M_i: however, one cannot increase K_i and/or M_i indefinitely, as the total interference in the system would increase and eventually make the transmit power assignment problem infeasible. Therefore, the feasibility of the transmit power control problem dictates constraints that $\{K_i, M_i\}_{i=1}^{J}$ must jointly satisfy. To describe these constraints, we use recent results on the user capacity of very large systems [12].

II. BACKGROUND

Let K, N, and β denote the number of users, processing gain, and the common SIR target for all users, respectively. Let P denote the common maximum received power and $\eta = P/\sigma^2$ be the common maximum signal-to-noise ratio (SNR), where σ^2 is the variance of the AWGN. The user capacity of a single-cell synchronous CDMA system was studied in [12], for two linear receiver structures, viz., SUMF and MMSE receivers. The results are asymptotic, in the sense that both $K \to \infty$ and $N \to \infty$, while the number of users per dimension $\alpha = K/N$ is fixed and finite. The results show that, for a fixed common SIR target of β, the system capacity must satisfy the following inequalities when the base station is equipped with SUMF receivers, with and without maximum received power constraints (MRPC):

$$\alpha \le \frac{1}{\beta} \qquad \text{Without MRPC} \qquad (1)$$

$$\alpha \le \frac{1}{\beta} - \frac{1}{\eta} \qquad \text{With MRPC} \qquad (2)$$

When the base station is equipped with MMSE receivers the system capacity is given as

$$\alpha \le \frac{1+\beta}{\beta} \qquad \text{Without MRPC} \qquad (3)$$

$$\alpha \le (1+\beta)\left(\frac{1}{\beta} - \frac{1}{\eta}\right) \qquad \text{With MRPC} \qquad (4)$$

Note that when there is no MRPC, $P \to \infty$, $\eta \to \infty$, and (2) and (4) reduce to (1) and (3), respectively.

These results can be generalized to multiple classes of users [12]. Classes differ only in terms of their required SIR targets, numbers of users per dimension, and their MRPCs. Let J denote the number of different classes, and let K_j, α_j, β_j, P_j and η_j denote the number of users, number of users per dimension ($\alpha_j = K_j/N$), required SIR target, maximum received power, and the SNR corresponding to the maximum received power ($\eta_j = P_j/\sigma^2$), respectively, in class j. For this case as well, [12] requires that $K_j \to \infty$ and $N \to \infty$ and $\alpha_j = K_j/N$ to be fixed and finite. The system capacity with SUMF receivers is given as

$$\sum_{j=1}^{J} \alpha_j \beta_j \le 1 \qquad \text{Without MRPC} \qquad (5)$$

$$\sum_{j=1}^{J} \alpha_j \beta_j \le \min_{1\le i\le J} \left[1 - \frac{\beta_i}{\eta_i}\right] \qquad \text{With MRPC} \qquad (6)$$

And the system capacity for the case of MMSE receivers is given as

$$\sum_{j=1}^{J} \alpha_j \frac{\beta_j}{1+\beta_j} \le 1 \qquad \text{Without MRPC} \qquad (7)$$

$$\sum_{j=1}^{J} \alpha_j \frac{\beta_j}{1+\beta_j} \le \min_{1\le i\le J} \left[1 - \frac{\beta_i}{\eta_i}\right] \qquad \text{With MRPC} \qquad (8)$$

As in the case of a single class of users, when $P_j \to \infty$ we have $\eta_j \to \infty$ for all j, and (6) and (8) reduce to (5) and (7). Also, when $J = 1$, (5)–(8) reduce to (1)–(4).

The capacity results summarized above can be interpreted in two ways. The results for a single class of users given in (1)–(4) can be seen as capacity results for a large system where the number of users and the processing gain go to infinity while their ratio is fixed and finite, or they can be seen as capacity results for a *single user* system with multiple (simultaneous) signatures, where the number of signatures and the processing gain grow to infinity while their ratio is fixed and finite. Similarly, the results for multiple classes of users given in (5)–(8) can be seen as capacity results for a finite number of classes with infinitely many users in each class, or they can be seen as capacity results for a system with multiple (but finite number of) users with infinitely many CDMA signatures per user. In this work, we will adopt the latter interpretation of a finite number of users with several (simultaneous) signatures.

The BER of an ith user using a QAM constellation with M_i signal points can be approximated as [7], [8, Appendix A]

$$\text{BER}_i \approx 0.2 e^{-1.5 \frac{\text{SIR}_i}{M_i - 1}} \qquad (9)$$

In particular, for $M_i = 2$, (9) is a lower bound to the exact BER, and for $M_i \ge 4$, (9) is an upper bound. We can express the QoS of the user in terms of its BER, and require the user to maintain a BER below a target value ϵ_i^*. Some simple algebra reveals that $\text{BER}_i \le \epsilon_i^*$ is equivalent to

$$\text{SIR}_i \ge \frac{-\ln(5\epsilon_i^*)}{1.5}(M_i - 1) \qquad (10)$$

The right-hand-side of (10) denotes the SIR target of user i. Note that this is a (linear) increasing function of constellation size for a fixed BER target ϵ_i^*, and a decreasing function of the BER target for a fixed constellation size. That is, the user needs a higher SIR if it requires a lower BER for a fixed constellation size, or if it requires a larger constellation size for a fixed desired BER. In this work, we will assume fixed BER targets for all users, and treat the number of signal points in the constellation of each user as a variable we optimize. Without loss of generality, we also assume that all users have the same BER target, $\epsilon_i^* = \epsilon^*$ for all i, and define

$$k \triangleq \frac{-\ln(5\epsilon^*)}{1.5} \qquad (11)$$

For instance, $\epsilon^* = 10^{-3}$ yields $k \cong 3.53$. With this definition of k, we can rewrite (10) as

$$\text{SIR}_i \ge k(M_i - 1) \qquad (12)$$

Thus, the SIR target of the ith user, β_i, has the following linear relationship to the modulation constellation size of user i, M_i,

$$\beta_i = k(M_i - 1) \qquad (13)$$

In this paper, we will allow M_i to take any real value larger than 2. In reality, however, M_i is an integer number and cannot take arbitrarily high values due to practical constraints. Since $\log_2(M_i)$ bits per symbol can be transmitted using a constellation size of M_i, we can conclude from (13) that user i can transmit

$$\log_2(1 + \frac{1}{k}\beta_i) \quad \text{bits/two-dimensional symbol} \qquad (14)$$

with an SIR of β_i. Note the similarity of this expression to the information-theoretic capacity as a function of the SIR: $\log_2(1+\beta_i)$. Later on we shall elaborate upon this similarity.

III. A Single User with Multiple Signature Sequences

Assume that we have a single user transmitting with K simultaneous signature sequences, each of which is randomly generated and modulated with the same QAM scheme with M signal points. The user can transmit $\log_2 M$ bits per symbol on each one of the signature sequences, and the SIR necessary for each parallel transmission to achieve the required BER is $\beta = k(M-1)$. We can express the bits/symbol that each signature sequence can carry in terms of β using (14) as $\log_2(1+\beta/k)$. The throughput T is defined as the total number of bits per second per hertz:

$$\begin{aligned} T &= K \times \log_2\left(1+\frac{1}{k}\beta\right) \times \frac{1}{T_s} \times \frac{1}{W} \\ &= \alpha \log_2\left(1+\frac{1}{k}\beta\right) \end{aligned} \tag{15}$$

where T_s is the symbol duration, W is the bandwidth of the system, and $\alpha = K/N$.

For this single-user system, we will maximize the throughput in terms of α and β. The optimum value of α will denote the optimum number of simultaneous signatures as a fraction of the processing gain, and the optimum β will determine the optimum constellation size through (13). We solve this problem for the following four cases:

(A) SUMF receivers without MRPC,
(B) SUMF receivers with MRPC,
(C) MMSE receivers without MRPC, and
(D) MMSE receivers with MRPC.

In all cases, the cost function will be the throughput (15), while our constraint set will be given by one of inequalities (1)–(4).

A. SUMF receivers without MRPC

The optimization problem in this case is

$$\begin{aligned} \max_{\alpha,\beta} \quad & \alpha \log_2\left(1+\frac{1}{k}\beta\right) \\ \text{s. t.} \quad & \alpha \leq \frac{1}{\beta} \end{aligned} \tag{16}$$

where the "subject to" part is exactly (1). Note that the objective function of the optimization problem is an increasing function of α. Therefore, the constraint must be satisfied with equality. Inserting this into the objective function yields the following unconstrained optimization problem:

$$\max_{\beta} \frac{1}{\beta} \log_2\left(1+\frac{1}{k}\beta\right) \tag{17}$$

This objective function is monotonically decreasing in β, as can be shown by taking its derivative. A quicker method of verifying that the function in (17) is decreasing in β consists of recalling that the information-theoretic capacity of an AWGN channel with noise power spectral density N_0 and bandwidth B is $C = B\log_2(1+P/N_0 B)$, and that C increases monotonically with B and converges to the so-called C_∞ as $B \to \infty$ [13]. By substituting $1/\beta$ for B we see that (17) is a decreasing function of β. Therefore, the maximum of the objective function is achieved when β takes its least possible value. Since $\beta = k(M-1)$, the maximum throughput T_{opt} is

$$T_{\text{opt}} = \frac{1}{k} \tag{18}$$

and is achieved when M takes the smallest possible value, i.e., $M = 2$, which implies that the optimum number of simultaneous signature sequences as a function of the processing gain is $\alpha = 1/k$. As a specific example, consider a BER target of $\epsilon^* = 10^{-3}$. In this case we have $k \cong 3.53$, and the throughput is maximized using BPSK modulation ($M = 2$) and choosing the number of signature sequences as approximately 28% of the processing gain ($\alpha \cong 0.28$).

B. SUMF receivers with MRPC

The optimization problem in this case is

$$\begin{aligned} \max_{\alpha,\beta} \quad & \alpha \log_2\left(1+\frac{1}{k}\beta\right) \\ \text{s. t.} \quad & \alpha \leq \frac{1}{\beta} - \frac{1}{\eta} \end{aligned} \tag{19}$$

where the "subject to" part this time is exactly (2). Similar to (16), the constraint must be satisfied with equality. Inserting this into the objective function yields the following unconstrained optimization problem:

$$\max_{\beta} \left(\frac{1}{\beta} - \frac{1}{\eta}\right) \log_2\left(1+\frac{1}{k}\beta\right) \tag{20}$$

Note that this objective function is also monotonically decreasing in β. This fact can be easily verified by noting that the derivative of the objective function in (20) is equal to the derivative of that in (17) plus the derivative of the term $-1/\eta \log_2(1+\beta/k)$ which is also negative for all β. Therefore, the maximum is achieved when β takes its least possible value. Since $\beta = k(M-1)$, the maximum throughput T_{opt} is

$$T_{\text{opt}} = \frac{1}{k} - \frac{1}{\eta} \tag{21}$$

and the optimum is achieved when M takes the smallest possible value, i.e., $M = 2$, which implies that the optimum number of simultaneous signature sequences as a function of the processing gain is $\alpha = 1/k - 1/\eta$. If we again consider the example case of $\epsilon^* = 10^{-3}$, we see that the throughput is maximized using BPSK modulation ($M = 2$) and choosing the number of signature sequences as $1/\eta$ "backed off" from 28% of the processing gain ($\alpha \cong 0.28 - 1/\eta$).

Note that the optimum throughput given in (21) is an increasing function of η, as we would expect. This result will be important in Section IV.

C. MMSE receivers without MRPC

The optimization problem in this case is

$$\begin{aligned} \max_{\alpha,\beta} \quad & \alpha \log_2\left(1+\frac{1}{k}\beta\right) \\ \text{s. t.} \quad & \alpha \le \frac{1+\beta}{\beta} \end{aligned} \tag{22}$$

where the "subject to" part is exactly (3). Again, the constraint must be satisfied with equality. Inserting this into the objective function yields the following unconstrained optimization problem:

$$\max_{\beta} \ \frac{1+\beta}{\beta}\log_2\left(1+\frac{1}{k}\beta\right) \tag{23}$$

It can be proved that this objective function is monotonically increasing in β, and hence its maximum is achieved as $\beta \to \infty$. Since $\beta = k(M-1)$, the maximum throughput T_{opt} is

$$T_{\text{opt}} \to \infty \tag{24}$$

and the optimum is achieved as $M \to \infty$. For this case, $\alpha = 1$; that is, the throughput is maximized by using the largest possible constellation and choosing the number of signature sequences equal to the processing gain ($\alpha = 1$).

D. MMSE receivers with MRPC

The optimization problem in this case is

$$\begin{aligned} \max_{\alpha,\beta} \quad & \alpha \log_2\left(1+\frac{1}{k}\beta\right) \\ \text{s. t.} \quad & \alpha \le (1+\beta)\left(\frac{1}{\beta}-\frac{1}{\eta}\right) \end{aligned} \tag{25}$$

where the "subject to" part is exactly (4). Again, the constraint must be satisfied with equality. Inserting this into the optimization problem yields the following unconstrained optimization problem:

$$\max_{\beta} \ (1+\beta)\left(\frac{1}{\beta}-\frac{1}{\eta}\right)\log_2\left(1+\frac{1}{k}\beta\right) \tag{26}$$

In this case the objective function is not a monotone function of β. Let us denote by β^* the value of β where it reaches its maximum. Corresponding to this β^*, the optimum M, say M^*, can be found by using $\beta^* = k(M^*-1)$, and the optimum value of the number of users per dimension, α^*, can be found by using $\alpha^* = (1+\beta^*)(1/\beta^* - 1/\eta)$. The optimum value of β, β^*, which in turn determines the optimum constellation size and the optimum number of users per dimension through M^* and α^*, depends only on the maximum SNR, η. Figure 1 shows the objective function in (26) for $\eta = 100, 500, 1000$ when $\epsilon^* = 10^{-3}$. Figures 2, 3, 4, 5 show the optimum β^*, M^*, α^* and T_{opt} as a function of η, respectively, for several different ϵ^* values.

We now derive a simple result which will be important in the next section, where we will investigate the throughput maximization problem for the multiple-user case. Let us denote the objective function of (26) by $f(\eta, \beta)$

$$f(\eta,\beta) = (1+\beta)\left(\frac{1}{\beta}-\frac{1}{\eta}\right)\log_2\left(1+\frac{1}{k}\beta\right), \tag{27}$$

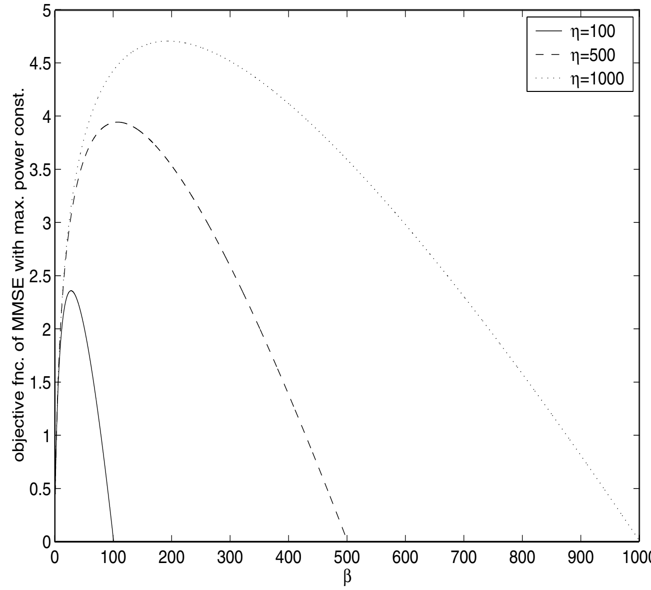

Fig. 1. Objective function in (26) as a function of β for several values of η.

and the objective function after optimization with respect to β by $T_{\text{opt}}(\eta)$.

$$T_{\text{opt}}(\eta) = \max_{\beta} f(\eta,\beta) \tag{28}$$

That is, if we denote the optimum value of β in the optimization problem of (26) by β^*, then $T_{\text{opt}}(\eta) = f(\eta, \beta^*)$ denotes the maximum achievable throughput for a given η, or equivalently, the optimized objective function. In the following we will show that $T_{\text{opt}}(\eta)$ is an increasing function of η. That is, if the user's MRPC η is increased, then the throughput achieved by using optimal number of signature sequences and modulation constellation size (α^* and β^*), is increased too (see Figure 5 as an example).

Lemma 1: Let $f(x,y)$ be a nondecreasing function of x. Then $\max_y f(x,y)$ is a nondecreasing function of x.

Proof: Since $f(x,y)$ is a nondecreasing function of x, when $x_2 \ge x_1$ we have

$$f(x_2,y) \ge f(x_1,y) \tag{29}$$

for all y. Let us denote by y_2 and y_1 the values of y that maximize $f(x_2,y)$ and $f(x_1,y)$, respectively. Since y_2 maximizes $f(x_2,y)$, by definition we have

$$f(x_2,y_2) \ge f(x_2,y) \tag{30}$$

for all y: in particular, $f(x_2,y_2) \ge f(x_2,y_1)$. But (29) implies that $f(x_2,y_1) \ge f(x_1,y_1)$, and therefore

$$f(x_2,y_2) \ge f(x_1,y_1) \tag{31}$$

giving the desired result. ■

IV. Multiple Users with Multiple Signature Sequences

In this section we assume that there exist J users in the system. The jth user transmits with K_j randomly generated simultaneous signature sequences where $K_j \to \infty$ and the common

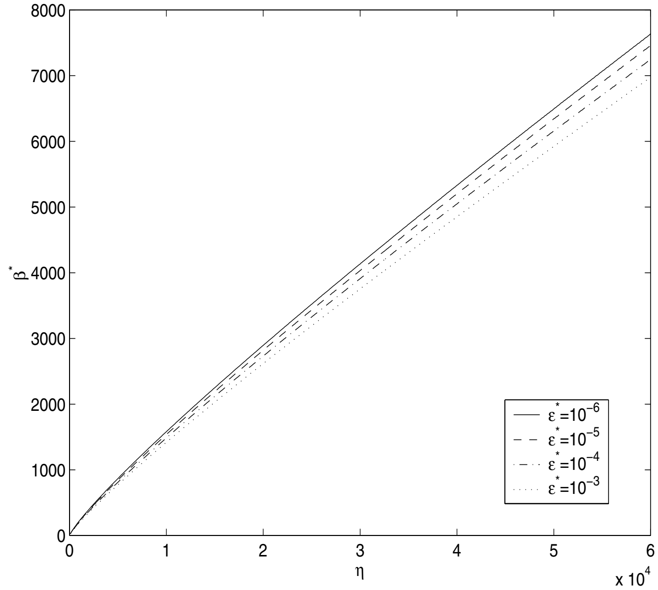

Fig. 2. Optimum SIR, β^*, versus maximum SNR η.

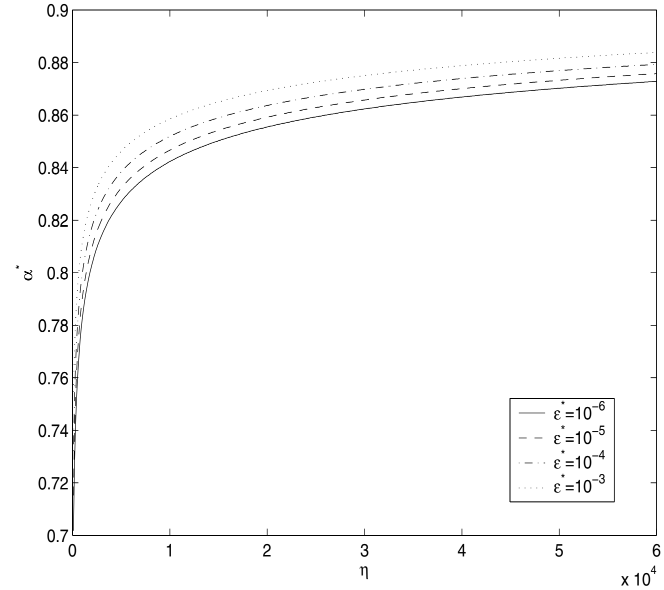

Fig. 4. Optimum number of users per dimension, α^*, versus maximum SNR η.

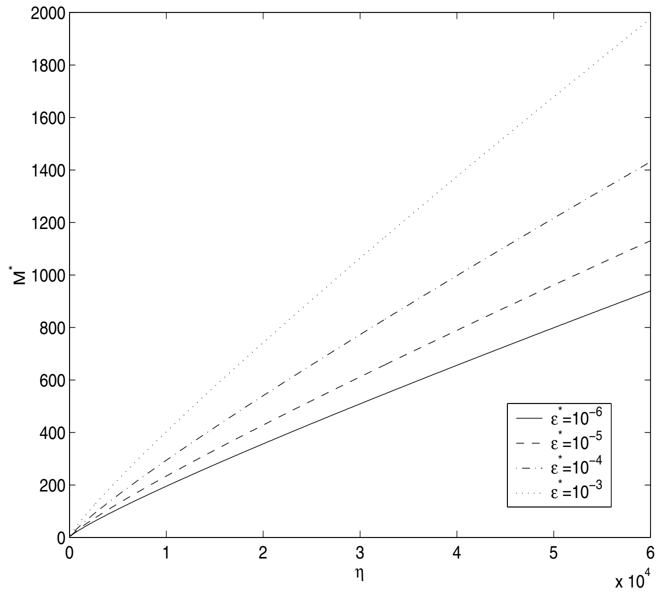

Fig. 3. Optimum constellation size, M^*, versus maximum SNR η.

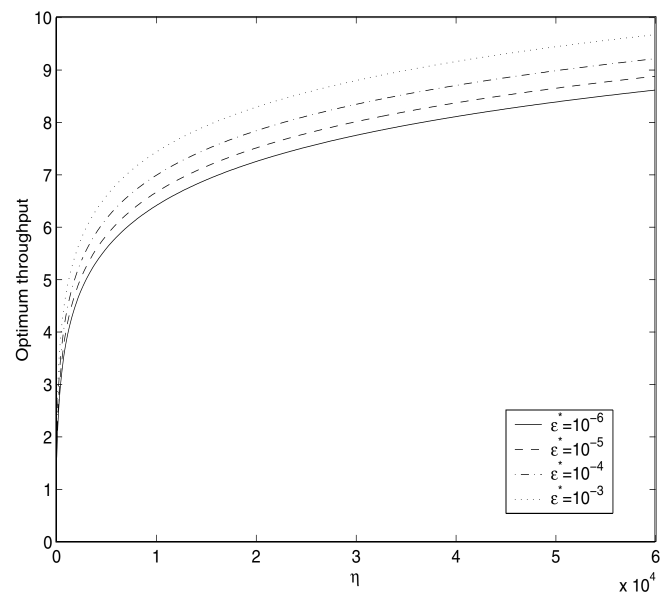

Fig. 5. Optimum throughput versus maximum SNR η.

processing gain $N \to \infty$, but $\alpha_j = K_j/N$ is fixed and finite, for $j = 1, \cdots, J$. The users are assumed to use QAM constellations which are parameterized with M_j, $j = 1, \cdots, J$. For this multiuser case, we define the total throughput as

$$T \triangleq \sum_{j=1}^{J} \alpha_j \log_2 \left(1 + \frac{1}{k}\beta_j\right) \tag{32}$$

Our aim is to maximize this objective function with respect to all of the free variables: $\{\alpha_j, \beta_j\}_{j=1}^{J}$, conditioned only on one of the constraints given in (5), (6), (7) or (8) depending on which one of the system configurations we assume: SUMF or MMSE receivers at the base station, and with MRPC or without MRPC.

We will follow a two-step method to solve this problem. First we will fix $\{\beta_j\}_{j=1}^{J}$ to some arbitrary values and find the optimum $\{\alpha_j\}_{j=1}^{J}$ in terms of the fixed β_j's. The second-stage optimization problem will be a function of only the set $\{\beta_j\}_{j=1}^{J}$, and we shall solve this second problem to find the optimum β_j's.

For fixed $\{\beta_j\}_{j=1}^{J}$, the optimization problem becomes:

$$\begin{aligned} \max_{\{\alpha_j\}_{j=1}^{J}} \quad & \sum_{j=1}^{J} d_j \alpha_j \\ \text{s. t.} \quad & \sum_{j=1}^{J} c_j \alpha_j \leq b \\ & \alpha_j \geq 0 \qquad\qquad j = 1, \cdots, J \end{aligned} \tag{33}$$

where the coefficients in the objective function (d_j's) are fixed and defined by

$$d_j \triangleq \log_2\left(1+\frac{1}{k}\beta_j\right) \tag{34}$$

Similarly, the coefficients (c_j's and b) in the constraint set are also fixed and given by one of the following equalities corresponding to the four different system models we consider:

$$\begin{array}{lll}
c_j = \beta_j & b = 1 & \text{SUMF, no MRPC} \\
c_j = \beta_j & b = \min\limits_{1\le i\le J}\left[1-\dfrac{\beta_i}{\eta_i}\right] & \text{SUMF, MRPC} \\
c_j = \dfrac{\beta_j}{1+\beta_j} & b = 1 & \text{MMSE, no MRPC} \\
c_j = \dfrac{\beta_j}{1+\beta_j} & b = \min\limits_{1\le i\le J}\left[1-\dfrac{\beta_i}{\eta_i}\right] & \text{MMSE, MRPC}
\end{array} \tag{35}$$

Therefore, in the first stage we have a linear programming problem. The feasible set in J dimensional space is a convex body delimited by the J coordinate axes and by the hyperplane described by the first constraint. With the exception of the origin, at all of the other J extreme (corner) points of the feasible set, all but one α_j is equal to zero: $(b/c_1, 0, \cdots, 0), (0, b/c_2, \cdots, 0), \cdots, (0, 0, \cdots, b/c_J)$. It is well-known that the optimum value of a linear program is achieved at an extreme point of the feasible set [14, pp. 81–82]. Note that the origin, although an extreme point of the feasible set, is not a candidate for the solution of the optimization problem, since the objective function is minimized at the origin. Therefore, the objective function must be maximized at one of the remaining J corner points of the feasible set. That is, the solution of (33) is achieved at one of the J extreme points of the feasible set where $J-1$ of the α_j's are zero and one of them, say α_i, is equal to b/c_i. Although this is a known result in linear programming literature, for the purposes of completeness we will outline its proof here.

First note that in order to maximize the objective function, the first inequality in the constraint set must be satisfied with equality. If it is not satisfied with equality, then one can increase one of the α_j's to increase the objective function since $d_j \ge 0$ for all j. Define new variables $\lambda_j \triangleq \alpha_j c_j / b$, for $j = 1, \cdots, J$. Therefore, we can express the optimization problem in (33) in terms of λ_j's as

$$\begin{array}{lll}
\max\limits_{\{\lambda_j\}_{j=1}^J} & \displaystyle\sum_{j=1}^{J} \lambda_j \frac{d_j}{c_j} & \\
\text{s. t.} & \displaystyle\sum_{j=1}^{J} \lambda_j = 1 & \\
 & \lambda_j \ge 0 & j = 1, \cdots, J
\end{array} \tag{36}$$

The objective function of (36) is a convex combination (weighted average) of the positive numbers d_j/c_j, $j = 1, \cdots, J$. In order to maximize this objective function one should choose $\lambda_i = 1$ and $\lambda_j = 0$ for $j \ne i$, if $i = \arg\max_j d_j/c_j$. Therefore, the optimum α_j's are found as $\alpha_i = b/c_i$, and $\alpha_j = 0$ for $j \ne i$, which confirms the observation above that the optimum is achieved at one of the extreme points of the feasible set.

In summary, we have found that for any given set of $\{\beta_j\}_{j=1}^J$, the optimum set $\{\alpha_j\}_{j=1}^J$ has only one nonzero component. Since this is true for any given set $\{\beta_j\}_{j=1}^J$, it is also true for the optimum set. Since the optimum α_j's depend only on β_j's, one can now express the second-stage optimization problem solely in terms of the β_j's, and solve it. We will take a slightly different approach here. Since we know that at the global optimum point we will have all α_j's equal to zero except for one of them, we will hypothetically consider solving each one of the J possible subproblems, defined as those in which only one user is active. We will index the subproblems as problems 1 through J, and denote the throughput achievable in each problem as T_j for $j = 1, \cdots, J$. In subproblem j, we will assume $\alpha_j > 0$ and $\beta_j > 0$ and all other $\alpha_l = 0$ and $\beta_l = 0$ for $l \ne j$. The solutions for all of these subproblems can be deduced from the solutions in Sections III-A through III-D.

When there is no MRPC, the throughputs achievable, T_j's in all the J, scenarios will be the same. The reason for this is that, when there is no MRPC, the achievable throughput is independent of any parameter a user might have; the achievable throughput depends either only on the common k value (see Section III-A), or it increases without bound (see Section III-C). Therefore, when there is no MRPC, we can choose any user to be the one who transmits and turn all remaining users' transmissions off (by assigning no signatures to them). The optimum number of signatures and constellation sizes for the only user who transmits can be derived as in Sections III-A and III-C.

When there is MRPC, the optimized (achievable) throughput in the jth subproblem depends only on the maximum SNR, η_j, of the jth user. We showed in III-B and III-D that the achievable throughput increases monotonically with η_j. Therefore, if the ith user's maximum SNR is larger than the jth user's maximum SNR, i.e., $\eta_i \ge \eta_j$, then we have $T_i \ge T_j$. Therefore, the user which has the highest η_j must be chosen to be the user which transmits, and all other users must turn their transmissions off.

In conclusion, although we allowed arbitrary α_j and β_j values for each user and optimized the throughput over all possible values of $\{\alpha_j, \beta_j\}_{j=1}^J$, we have found that, in order to maximize the total system throughput, all but one user must turn their transmissions off, or equivalently choose their number of signature sequences equal to zero.

V. Further comments

A. Capacity results

As mentioned before, if $k = 1$, (14) expresses the information-theoretic capacity of the channel: this implies the presence of coding, with no constraints on the signal constellation used (this becomes Gaussian). Thus, our results with $k = 1$ can be interpreted in terms of the best tradeoff between coding and spreading if a fixed amount of bandwidth expansion is allowed (see [15]). We see that with a SUMF receiver the throughput is maximum for $\alpha \to \infty$, which can be interpreted by saying that the whole bandwidth expansion should be allocated to low-rate coding. With MMSE, the optimum choice is $\alpha \le 1$, which implies that part of the bandwidth expansion should be allocated to coding and part to spreading, the balance between the two being determined by the power constraint.

B. Resource allocation for maximum fairness

Although the scheme described above maximizes the total number of bits the users can collectively transmit to the base station, it has an obvious shortcoming in terms of the fairness among the users. With this scheme a user never gets to transmit any of its data unless it has the highest channel gain. In order to alleviate this problem, we examine a scheduling mechanism whereby each one of J users is allowed to transmit in a preassigned time frame (we may assume equal-duration time frames). In each time frame, the throughput is maximized in the sense that only the user with the highest channel gain is allowed to transmit its data: however, a user who has already transmitted in the previous $J-1$ frames cannot contend for the resources again. This scheduling mechanism gives users with lower channel gains a chance to access the system.

VI. Simulation Results

For sake of illustration, in this section we present some simple simulation results. We consider a single circular cell with a radius of $R_0 = 1000$m. The positions of the users are independently and uniformly distributed in the cell. The distance of a user from the base station, d, has the probability density function of $f(d) = 2d/R_0^2$. The channel gain of user i has two components: a distance-based component and a shadow fading component:

$$h_i = \kappa(d_i/100)^{-\alpha} s_i \tag{37}$$

where $\kappa = 10^{-8}$, $\alpha = 2$ if $d_i \leq 100$ meters and $\alpha = 4$ otherwise; and s_i denotes the log-normal (shadow) fading factor. The distance of the user to the base station, d_i, is calculated by taking the height of the base station antenna into account (base station antenna height is assumed to be 40 meters). The standard deviation of $10\log s_i$ is 8 dB. In the simulations, we choose ratio of the maximum *transmit* power to the variance of the AWGN so that the median received SNR of a user located at the edge of the cell is 0 dB. We consider the case where the base station is equipped with MMSE receivers and where we have MRPCs defined by η_j's.

First, we consider a single user system. We generate the channel gain of the user for 100,000 different times, solve the throughput maximization problem for each case and plot the cumulative distribution function (CDF) of the throughput in Figures 6 and 7 for four different values of ϵ^*. The difference between Figures 6 and 7 is that the channel gains in Figure 6 are calculated using the distance-based component only, and the channel gains in Figure 7 are calculated using both the distance-based and shadow fading components.

Figure 8 shows a comparison of the CDFs of the throughputs that can be achieved by the optimum and the fair schemes. There are $J = 3$ users in the system. The optimum scheme allows only the highest-channel-gain user to transmit in all 3 time slots. The fair scheme on the other hand lets the highest channel gain user to transmit in the first slot, the second highest channel gain user to transmit in the second slot, and the lowest channel gain user to transmit in the third slot. Again, we generated 100,000 different scenarios to obtain the CDFs in Figure 8. This figure shows the total-throughput loss caused by the introduction of a fairness constraint.

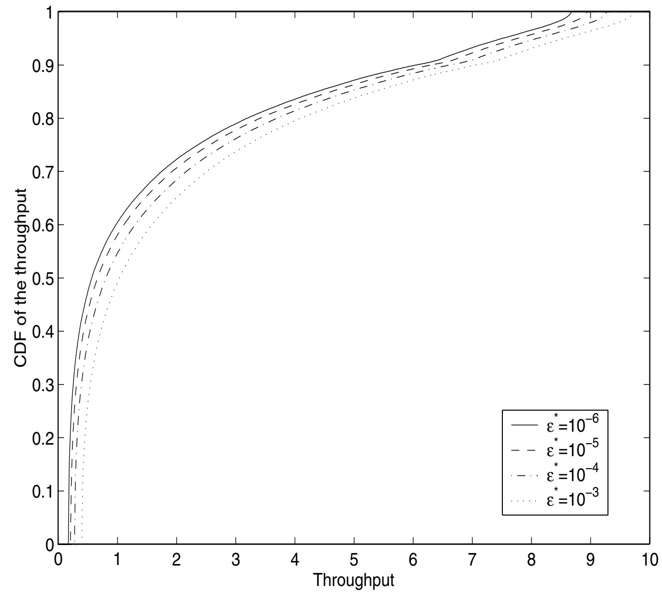

Fig. 6. CDF of the throughput for a single user for four different BER targets. Distance-based channel gains.

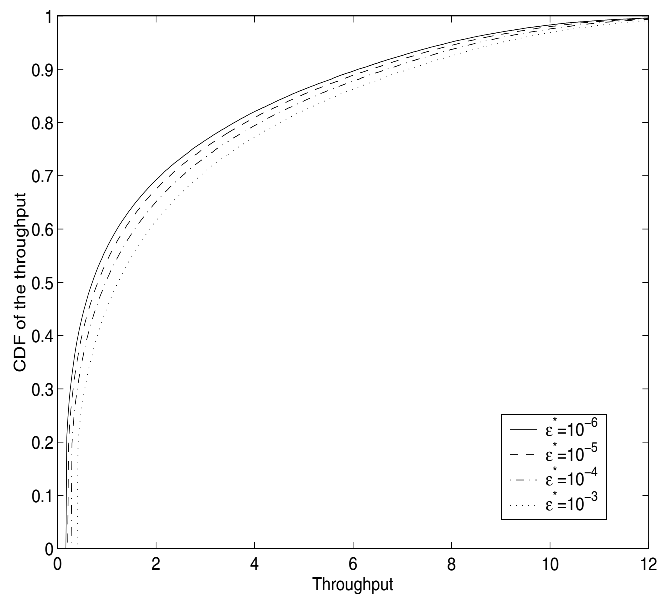

Fig. 7. CDF of the throughput for a single user for four different BER targets. Distance and shadow fading based channel gains.

VII. Conclusions

We have studied the problem of maximizing the total system throughput under a bit error rate constraint for all users in the uplink of a single-cell synchronous CDMA system. The following settings were examined: SUMF and MMSE receivers, with and without maximum-power constraints. With a single user and SUMF receiver, the throughput is maximized when the user chooses the smallest QAM constellation. With an MMSE receiver and no maximum received power constraint, the user can increase its throughput indefinitely by increasing the constellation size. If there is a maximum received power constraint, there exist an optimum constellation size and a corresponding opti-

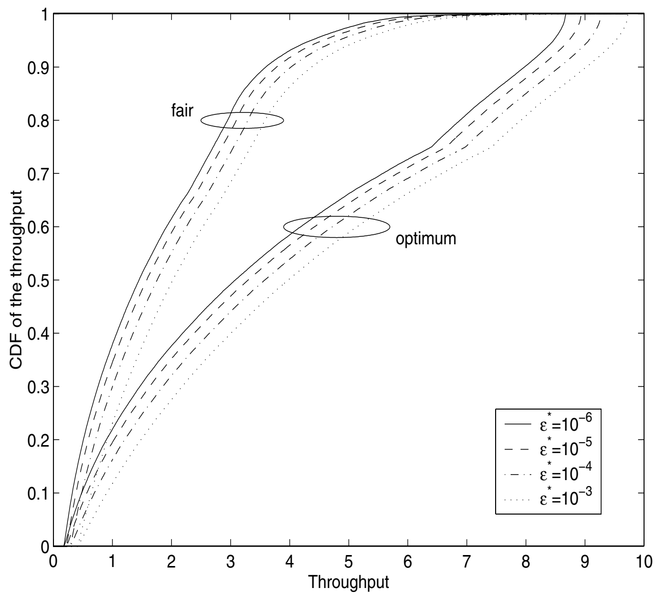

Fig. 8. CDF of the throughput of the system. $N = 3$. Optimum and Fair schemes. Distance based channel gains.

mum number of simultaneous signatures. With multiple users, the total throughput is maximized when only one user transmits: with no maximum power constraints this can be chosen arbitrarily, otherwise it should be the one with the largest SNR. Since this solution is unfair to all but one user, we have examined a scheduling mechanism that assigns equal-duration time frames to all users, thus enhancing fairness, and we have discussed the resulting total throughput loss.

In this work we considered a single-cell, synchronous CDMA system and a non-dispersive wireless channel similar to [12] to simplify our analysis. Selection of optimum modulation and multicode formats in multi-cell, asynchronous CDMA systems in dispersive channels needs to be investigated.

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