# Balancing Transport and Physical Layers in Wireless Multihop Networks: Jointly Optimal Congestion Control and Power Control

Mung Chiang

*Abstract*—**In a wireless network with multihop transmissions and interference-limited link rates, can we balance power control in the physical layer and congestion control in the transport layer to enhance the overall network performance while maintaining the architectural modularity between the layers? We answer this question by presenting a distributed power control algorithm that couples with existing transmission control protocols (TCPs) to increase end-to-end throughput and energy efficiency of the network. Under the rigorous framework of nonlinearly constrained utility maximization, we prove the convergence of this coupled algorithm to the global optimum of joint power control and congestion control, for both synchronized and asynchronous implementations. The rate of convergence is geometric and a desirable modularity between the transport and physical layers is maintained. In particular, when congestion control uses TCP Vegas, a simple utilization in the physical layer of the queueing delay information suffices to achieve the joint optimum. Analytic results and simulations illustrate other desirable properties of the proposed algorithm, including robustness to channel outage and to path loss estimation errors, and flexibility in trading off performance optimality for implementation simplicity.**

**This paper presents a step toward a systematic understanding of "layering" as "optimization decomposition," where the overall communication network is modeled by a generalized network utility maximization problem, each layer corresponds to a decomposed subproblem, and the interfaces among layers are quantified as the optimization variables coordinating the subproblems. In the case of the transport and physical layers, link congestion prices turn out to be the optimal "layering prices."**

*Index Terms*—**Congestion control, convex optimization, cross-layer design, energy-aware protocols, Lagrange duality, power control, transmission control protocol, utility maximization, wireless ad hoc networks.**

## I. Introduction

WE CONSIDER wireless networks with multihop transmissions and interference-limited link rates. In order to achieve high end-to-end throughput in an energy efficient manner, congestion control and power control need to be jointly designed and distributively implemented. Congestion control mechanisms, such as those in transmission control protocol (TCP), regulate the allowed source rates so that the total traffic load on any link does not exceed the available capacity. At the same time, the attainable data rates on wireless links depend on the interference levels, which in turn depend on the power control policy. This paper proposes and analyzes a distributed algorithm for *jointly* optimal end-to-end congestion control and per-link power control. The algorithm utilizes the coupling between the transport and physical layers to increase end-to-end throughput and energy efficiency in a wireless multihop network.

Congestion avoidance mechanisms in TCP variants have recently been shown to approximate distributed algorithms that implicitly solve network utility maximization problems. Traditionally, this class of optimization problems are linearly constrained by link capacities that are assumed to be fixed quantities. However, network resources can sometimes be allocated to change link capacities, therefore change TCP dynamics, and the optimal solution to network utility maximization. For example, in code-division multiple-access (CDMA) wireless networks, transmit powers can be controlled to induce different signal-to-interference ratios (SIRs) on the links, changing the attainable throughput on each link.

This formulation of network utility maximization with "elastic" link capacities leads to a new approach of congestion avoidance in wireless multihop networks. The current approach of congestion control in the Internet is to avoid the development of a bottleneck link by reducing the allowed transmission rates from all the sources using this link. Intuitively, an alternative approach is to build, in real-time, a larger transmission "pipe" and "drain" the queued packets faster on a bottleneck link (i.e., a link where traffic load is larger than capacity). Indeed, a smart power control algorithm would allocate just the "right" amount of power to the "right" nodes to alleviate the bottlenecks, which may then induce an increase in end-to-end TCP throughput. However, there are two major difficulties in making this idea work: defining which link constitutes a bottleneck *a priori* is infeasible, and changing the transmit power on one link also affects the data rates available on other links. Due to interference in wireless CDMA networks, increasing the capacity on one link reduces those on other links. We need to find an algorithm that distributively and adaptively detects the bottleneck links and optimally "shuffles" them around in the network.

This intuitive approach is made precise and rigorous in this paper. After reviewing the background materials in Section II and specifying the problem formulation in Section III, we propose in Section IV a distributed power control algorithm that couples with existing TCP algorithms to solve the joint problem of congestion control and power control. The joint algorithm can be distributively implemented on a multihop network, despite the fact that the data rate on a wireless link is a global

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The author is with the Electrical Engineering Department, Princeton University, Princeton, NJ 08544 USA (e-mail: chiangm@princeton.edu).

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function of all the interfering powers. Interpretations in terms of data rate demand-supply coordination through shadow prices are presented, as well as numerical examples illustrating that end-to-end throughput and energy efficiency of the network can indeed be significantly increased.

It is certainly not a surprise that performance can be enhanced through a cross-layer design. The more challenging task is to analyze the algorithm rigorously and to make it attractive according to other important design criteria. In Section VI, we prove that, under very mild conditions, the proposed algorithm converges to the joint and global optimum of the nonlinear congestion-power control problem. In Section VII-A, we provide the sufficient conditions under which convergence to the global optimum is maintained despite errors in path loss estimation or packet losses due to channel outage. Cross-layer designs usually improve performance at the expense of higher complexity in communication and computation. In Section VII-B, we propose a suite of simplified versions of the optimal algorithm to flexibly tradeoff performance with complexity. In Section VII-C, we prove that the algorithm converges under any finite asynchronism in practical implementation, and characterize a condition under which asynchronous implementation does not induce a reduction in convergence speed. In Section VII-D, we show that the rate of convergence of the algorithm is geometric, and provide a simple bound on convergence speed. Further suggestions on choosing algorithm parameters and achieving convergence speedup are made in Section VII-E. Even after crossing the layers, architectural modularity is desirable for practical implementation and future network evolution. In this paper, the desirable convergence is achieved as power control uses the same link prices that are already generated by TCP for regulating distributed users. Performance enhancement from the jointly optimal design is achieved without modifying the existing TCP protocol stack.

Assumptions behind the models and limitations on the results are stated throughout the paper, while extensions are outlined in Section V. This paper presents a step towards understanding "layering" as "optimization decomposition," where the overall communication network is modeled by a generalized utility maximization problem, each layer corresponds to a decomposed subproblem, and the interfaces among layers are quantified as the optimization variables coordinating the subproblems. In the case of the transport and physical layers, link congestion prices turn out to be the optimal "layering prices." Future research directions are discussed in Section VIII.

## II. Background and Related Work

Both power control in CDMA wireless networks and congestion control in the Internet are extensively researched topics. Many power control algorithms have been proposed in the literature, but the effects of power control on source rate regulation through end-to-end congestion control have not been characterized.

TCP is one of the two widely used transport layer protocols on the Internet. A main function performed by TCP is network congestion control and end-to-end rate allocation. Roughly speaking, there are two phases of TCP congestion control: slow start and congestion avoidance. Long-lived flows spend most of the time in congestion avoidance. Similar to recent work on utility maximization models of TCP, we assume a deterministic fluid model for the average equilibrium behavior of the congestion avoidance phase. TCP uses sliding windows to adjust the allowed transmission rate in each source based on implicit or explicit feedback of the congestion signals generated by Active Queue Management (AQM). Among the variants of TCP, such as Tahoe, Reno, Vegas, and FAST, some use loss as congestion signal and others use delay. Most of this paper focuses on delay-based congestion signal because of the nice properties on convergence, stability, and fairness [22], and the simulation examples use TCP Vegas [5] at the sources.

The basic rate allocation mechanism of TCP Vegas is as follows. Let $d_s$ be the propagation delay along the path originating from source $s$, and $D_s$ be the propagation plus congestion-induced queueing delay. Obviously, $d_s = D_s$ when there is no congestion on all the links used by source $s$. The window size $w_s$ is updated at each source $s$ according to whether the difference between the expected rate $(w_s)/(d_s)$ and the actual rate $(w_s)/(D_s)$, where $D_s$ is estimated by the timing of acknowledgment (ACK) packets, is smaller than a parameter $\alpha_s$

$$w_s(t+1) = \begin{cases} w_s(t) + \frac{1}{D_s(t)}, & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} < \alpha_s \\ w_s(t) - \frac{1}{D_s(t)}, & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} > \alpha_s \\ w_s(t), & \text{else} \end{cases}.$$

The end-to-end throughput for each path is the allowed source rate $x_s$, which is proportional to the window size $x_s(t) = (w_s(t))/(D_s(t))$.

Following the seminal work by Kelly *et al.* [16], [17] that analyze network rate allocation as a distributed solution of utility maximization, TCP congestion control mechanisms have recently been analyzed as approximated distributed algorithms solving appropriately formulated network utility maximization problems (e.g., [18], [21]–[23], and [25]). The key innovation in this series of work is to interpret source rates as *primal variables*, link congestion measures as *dual variables*, and a TCP-AQM protocol as a distributed algorithm over the Internet to implicitly solve the following network utility maximization problem. Consider a wired communication network with $L$ links, each with a *fixed* capacity of $c_l$ b/s, and $S$ sources, each transmitting at a source rate of $x_s$ b/s. Each source emits one flow, using a fixed set $L(s)$ of links in its path, and has an increasing, strictly concave, and twice differentiable utility function $U_s(x_s)$. Network utility maximization is the problem of maximizing the total utility $\sum_s U_s(x_s)$ over the source rates $\mathbf{x}$, subject to linear flow constraints $\sum_{s:l\in L(s)} x_s \le c_l$ for all links $l$

$$\begin{aligned} \text{maximize} \quad & \sum_s U_s(x_s) \\ \text{subject to} \quad & \sum_{s:l\in L(s)} x_s \le c_l, \quad \forall l \\ & \mathbf{x} \succeq 0. \end{aligned} \tag{1}$$

Different TCP-AQM protocols solve for different utility functions using different types of congestion signals. For example, TCP Vegas is shown [23] to be implicitly solving (1) for logarithmic utility functions $U_s(x_s) = \alpha_s d_s \log x_s$, using queueing delays as the dual variables. Although TCP and AQM protocols were designed and implemented without regard to utility maximization, now they can be *reverse-engineered* to determine the underlying utility functions and to rigorously characterize many important properties.

An underlying assumption in the utility maximization models of TCP is that each communication link is a fixed-size transmission "pipe" provided by the physical layer. This assumption is invalid when the sizes of the "pipes" depend on time-varying channel conditions and adaptive physical layer resource allocation, such as transmit power control in interference-limited wireless networks. Different cases of utility maximization jointly over rates and powers have been studied for wireless cellular networks, e.g., in [9] and [24], and, in general, optimization-theoretic or game-theoretic studies of wireless network resource allocation using the utility framework have been reported, e.g., in [20], [26]–[28], [32], and [33]. This paper focuses on jointly optimal congestion control and power control in wireless *multihop* networks.

Augmenting the utility maximization framework to include layers other than the transport layer may lead to a general methodology for cross-layer design. Cross-layer issues in communication networks have attracted the attention of many researchers, forming a literature that is too large to be exhaustively reviewed here. Complementing these cross-layer investigations, we examine the balance between the transport and physical layers and provide a quantitative framework of joint design across layers 1 and 4, under which theorems of global convergence can be proved for nonlinearly coupled dynamics. This cross-layer issue is particularly interesting because congestion control is conducted end-to-end while power control is link-based. The resulting jointly optimal congestion control and power control algorithm increases end-to-end throughput and energy efficiency in wireless multihop networks. Echoing some of the cautionary notes on cross-layer designs, we also put special emphasis on the practical implementation issues of robustness, asynchronism, complexity, and the rate of convergence.

We note that there are at least two possible interpretations of the phrase "balancing transport and physical layers in wireless networks."

- Characterize the impacts of physical layer resource allocation on TCP throughput, which is the focus of this paper.
- Characterize the impacts of wireless channel variations on TCP throughput and try to distinguish between packet losses due to congestion and those due to fading. This problem, which has been actively researched in both academia and industry, is not the subject of this paper. However, we will investigate the robustness of our algorithm to fading. The nonlinear convex optimization methods used here, as well as in [14], can also be used for power control to guarantee certain levels of packet loss necessary to sustain a desired TCP throughput.

It should be noted that we do not consider joint optimization over routing or medium-access control in this paper. However, a generalized utility maximization problem is proposed at the end of this paper as a possible vehicle to rigorously and systematically study "layering" as "optimization decomposition."

## III. Problem Formulation

Consider a wireless multihop network with $N$ nodes and an established logical topology, where some nodes are sources of transmission and some nodes act as "voluntary" relay nodes. A sequence of connected links $l \in L(s)$ forms a route originating from source $s$. Let $x_s$ be the transmission rate of source $s$, and $c_l$ be the "capacity," in terms of the attainable data rate rather than the information-theoretic multiterminal channel capacity, on logical link $l$. Note that each physical link may be regarded as multiple logical links. Source nodes are indexed by $s$ and logical links by $l$.

Revisiting the network utility maximization formulation (1), for which TCP congestion control solves, we observe that in an interference-limited wireless network, data rates attainable on wireless links are not fixed numbers $\mathbf{c}$ as in (1), and instead can be written, for a large family of modulations, as a global and nonlinear function of the transmit power vector $\mathbf{P}$ and channel conditions

$$c_l(\mathbf{P}) = \frac{1}{T}\log(1 + K\mathrm{SIR}_l(\mathbf{P})).$$

Here, constant $T$ is the symbol period, which will be assumed to be one unit without loss of generality, and constant $K = (-\phi_1)/(\log(\phi_2 \mathrm{BER}))$, where $\phi_1, \phi_2$ are constants depending on the modulation and BER is the required bit-error rate [13]. The signal-to-interference ratio for link $l$ defined as $\mathrm{SIR}_l = (P_l G_{ll})/(\sum_{k\neq l} P_k G_{lk} + n_l)$ for a given set of path losses $G_{lk}$ (from the transmitter on logical link $k$ to the receiver on logical link $l$) and a given set of noises $n_l$ (for the receiver on logical link $l$). The $G_{lk}$ factors incorporate propagation loss, spreading gain, and other normalization constants. Notice that $G_{ll}$ is the path gain on link $l$ (from the transmitter on logical link $l$ to the intended receiver on the same logical link). With reasonable spreading gain, $G_{ll}$ is much larger than $G_{lk}, k \neq l$, and assuming that not too many close-by nodes transmit at the same time, $K\mathrm{SIR}$ is much larger than 1. In this case, $c_l$ can be approximated as $\log(\mathrm{SIR}_l)$, where $K$ is absorbed into $G_{ll}$ in $\log(\mathrm{SIR}_l)$.

This wireless channel model has several limitations. First, it assumes fixed target decoding error probabilities and coding modulation schemes. Transmit power is the only resource that is being adapted. Second, the assumption that $K\mathrm{SIR}$ is much larger than 1 is not always true. With this assumption, it will be shown that while $\log(K\mathrm{SIR}_l(\mathbf{P}))$ is a nonlinear *nonconcave* function of $\mathbf{P}$, it can be converted into a nonlinear *concave* function through a log transformation, leading to a critical convexity property that establishes the global optimality of the proposed algorithm. The important role played by convexity in utility maximization will be further discussed in Section VIII. Last but not least, simple decoding is not the only option for a wireless channel. Either multiuser decoding that does not treat all interferences as noise or simple "amplify-and-forward" signaling strategies will lead to different physical layer models.

The network model is also limited by the assumptions on fixed nodes, fixed single-path routing, and perfect CDMA-based medium access. In addition to rate and power controls, two other mechanisms to reduce bottleneck congestion are scheduling over different time slots and routing through alternate paths. Indeed, adaptive routing for mobile networks, and scheduling or contention-based medium access for broadcast wireless transmissions are important research topics in their own rights. While neither will be optimized jointly with the algorithm in this paper, a preliminary framework to incorporate these networking aspects will be presented in Section VIII.

With the above assumptions, we have specified the following network utility maximization with "elastic" link capacities:

$$\begin{aligned} \text{maximize} \quad & \sum_s U_s(x_s) \\ \text{subject to} \quad & \sum_{s:l\in L(s)} x_s \le c_l(\mathbf{P}), \quad \forall l \\ & \mathbf{x}, \mathbf{P} \succeq 0 \end{aligned} \tag{2}$$

where the optimization variables are both source rates $\mathbf{x}$ and transmit powers $\mathbf{P}$. The key difference from the standard utility maximization (1) is that each link capacity $c_l$ is now a function of the new optimization variables: the transmit powers $\mathbf{P}$. The design space is enlarged from $\mathbf{x}$ to both $\mathbf{x}$ and $\mathbf{P}$, which are clearly coupled in (2). Linear flow constraints on $\mathbf{x}$ become nonlinear constraints on $(\mathbf{x}, \mathbf{P})$. In practice, problem (2) is also constrained by the maximum and minimum transmit powers allowed at each transmitter on link $l$: $P_{l,\min} \le P_l \le P_{l,\max}, \forall l$.

The nonlinearly constrained optimization problem (2) may be solved by centralized computation using the interior-point method for convex optimization [4], after the log transformation that converts it into a convex optimization problem as will be shown in Section VI. However, in the context of wireless ad hoc networks, new *distributive* algorithms are needed to solve (2). Thus, the major challenges are the two global dependencies in (2).

- Source rates $\mathbf{x}$ and link capacities $\mathbf{c}$ are globally coupled across the network, as reflected in the range of summation $\{s : l \in L(S)\}$ in the constraints in (2).
- Each link capacity $c_l(\mathbf{P})$, in terms of the attainable data rate under a given power vector, is a global function of all the interfering powers.

Our primary goal in this paper is to distributively find the joint and globally optimal solution $(\mathbf{x}^*, \mathbf{P}^*)$ to problem (2) by breaking down these two global dependencies.

## IV. Optimal Algorithm, Pricing Interpretation, and Numerical Example

We propose the following distributive algorithm and later prove that it converges to the joint and global optimum of (2) and possesses several other desirable properties of a cross-layer design. We first present the ideal form of the algorithm, assuming synchronized discrete time slots, no propagation delay, and full-scale message passing. Practical issues on asynchronism, propagation delay, complexity, robustness, and the rate of convergence will be investigated in Section VII. To make the algorithm and its analysis concrete, we will focus on delay-based price and TCP Vegas window update (as reflected in items 1 and 2 in the algorithm, respectively) and the corresponding logarithmic utility maximization over $(\mathbf{x}, \mathbf{P})$

$$\begin{aligned} \text{maximize} \quad & \sum_s \alpha_s d_s \log x_s \\ \text{subject to} \quad & \sum_{s:l\in L(s)} x_s \le c_l(\mathbf{P}), \quad \forall l \\ & \mathbf{x}, \mathbf{P} \succeq 0. \end{aligned} \tag{3}$$

Similar to the general problem (2), in practice problem (3) is also constrained by the maximum and minimum transmit powers allowed at each transmitter on link $l$. Extensions to other TCP variants and congestion prices will be discussed in Section V.

*Jointly Optimal Congestion-Control and Power-Control (JOCP) Algorithm:* During each time slot $t$, the following four updates are carried out simultaneously until convergence.

1) At each intermediate node, a weighted queueing delay $\lambda_l$ is implicitly updated,[1] where $\gamma > 0$ is a constant

$$\lambda_l(t+1) = \left[\lambda_l(t) + \frac{\gamma}{c_l(t)}\left(\sum_{s:l\in L(s)} x_s(t) - c_l(t)\right)\right]^+. \tag{4}$$

2) At each source, total delay $D_s$ is measured and used to update the TCP window size $w_s$. Consequently, the source rate $x_s$ is updated

$$w_s(t+1) = \begin{cases} w_s(t) + \frac{1}{D_s(t)}, & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} < \alpha_s \\ w_s(t) - \frac{1}{D_s(t)}, & \text{if } \frac{w_s(t)}{d_s} - \frac{w_s(t)}{D_s(t)} > \alpha_s \\ w_s(t) & \text{else} \end{cases}$$

$$x_s(t+1) = \frac{w_s(t+1)}{D_s(t)}. \tag{5}$$

3) Each transmitter $j$ calculates a message $m_j(t) \in \mathbf{R}_+$ based on locally measurable quantities, and passes the message to all other transmitters by a flooding protocol

$$m_j(t) = \frac{\lambda_j(t)\mathrm{SIR}_j(t)}{P_j(t)G_{jj}}.$$

4) Each transmitter updates its power based on locally measurable quantities and the received messages, where $\kappa > 0$ is a constant

$$P_l(t+1) = P_l(t) + \frac{\kappa\lambda_l(t)}{P_l(t)} - \kappa\sum_{j\ne l} G_{lj} m_j(t). \tag{6}$$

With the maximum and minimum transmit power constraint $(P_{l,\min}, P_{l,\max})$ on each transmitter, the updated power is projected onto the interval $[P_{l,\min}, P_{l,\max}]$.

We present some intuitive arguments on this algorithm before proving the convergence theorem and discussing the practical implementation issues. Item 2 is simply the TCP Vegas window update [5]. Item 1 is a modified version of queueing

[1] This is using an average model for deterministic fluids. The difference between the total ingress flow intensity and the egress link capacity, divided by the egress link capacity, gives the average time that a packet needs to wait before being sent out on the egress link.

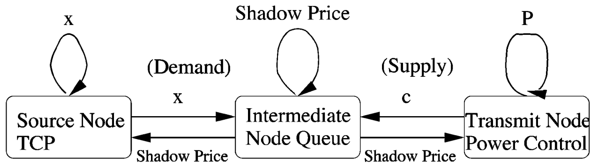


Fig. 1. Nonlinearly coupled dynamics of joint congestion and power control.

delay price update [23] (and the original update [5] is an approximation of item 1). Items 3 and 4 describe a new power control using message passing [7]. Taking in the current values of $(\lambda_j(t)\mathrm{SIR}_j(t))/(P_j(t)G_{jj})$ as the messages from other transmitters indexed by $j$, the transmitter on link $l$ adjusts its power level in the next time slot in two ways: first increase power directly proportional to the current price (e.g., queueing delay in TCP Vegas) and inversely proportional to the current power level, then decreases power by a weighted sum of the messages from all other transmitters, where the weights are the path losses $G_{lj}$. Intuitively, if the local queueing delay is high, transmit power should increase, with more moderate increase when the current power level is already high. If queueing delays on other links are high, transmit power should decrease in order to reduce interference on those links.

Note that to compute $m_j$, the values of queueing delay $\lambda_j$, signal-interference-ratio $\mathrm{SIR}_j$, and received power level $P_jG_{jj}$ can be directly measured by node $j$ locally. This algorithm only uses the resulting message $m_j$ but *not* the individual values of $\lambda_j, \mathrm{SIR}_j, P_j$, and $G_{jj}$. Each message is simply a real number. To conduct the power update, $G_{lj}$ factors are assumed to be estimated through training sequences. In practical wireless ad hoc networks, $G_{lj}$ are stochastic rather than deterministic and path loss estimations can be inaccurate. The effects of the fluctuations of $G_{lj}$ will be discussed in Section VII-A.

We also observe that the power control part of the joint algorithm can be interpreted as the selfish maximization of a local utility function of power by the transmitter of each link $l$

$$\text{maximize}_{P_l} \quad U_l(P_l)$$

where $U_l(P_l) = \lambda_l c_l - \beta_l P_l$ and $\beta_l = \sum_{j \neq l} G_{lj} m_j$. This complements the standard interpretation of congestion control as the selfish maximization of a local utility function $U_s(x_s)$ by each source $s$.

The known source algorithm (5) and queue algorithm (4) of TCP-AQM, together with the new power control algorithm (6), form a set of distributed, joint congestion control and resource allocation in wireless multihop networks. As the transmit powers change, SIR and, thus, data rate also change on each link, which in turn change the congestion control dynamics. At the same time, congestion control dynamics change the dual variables $\boldsymbol{\lambda}(t)$, which in turn change the transmit powers. Fig. 1 shows this nonlinear coupling of "supply" (regulated by power control) and "demand" (regulated by congestion control), through the shadow prices $\boldsymbol{\lambda}$ that are currently used by TCP to regulate distributed demand. Now $\boldsymbol{\lambda}$ serves the second function of cross-layer coordination in the JOCP algorithm. Theorem 1 in Section VI proves that this globally coupled, nonlinear dynamic converges to the jointly optimal $(\mathbf{x}^*, \mathbf{P}^*)$.

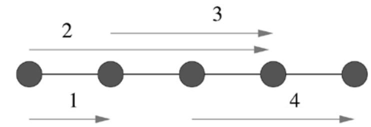


Fig. 2. Logical topology and connections for an illustrative example.

It is important to note that there is no need to change the existing TCP congestion control and queue management algorithms. All that is needed to achieve the joint and global optimum of (3) is to utilize the values of weighted queueing delay in designing power control algorithm in the physical layer. This approach is complementary to some recent suggestions in the Internet community to pass physical layer information for a better control of routing and congestion in upper layers. Notice that the problem we seek to solve is *jointly* optimal transport and physical layer design. The conclusion that physical layer algorithm needs to adapt according to transport layer prices is reached after the derivation, rather than presumed as a restrictive assumption before the derivation.

Much recent work has been done on opportunistic scheduling at the MAC layer based on the physical layer channel conditions. The JOCP algorithm complements such work by considering how can physical layer resource allocation be adapted to enhance the end-to-end utilities. Transport layer utilities guide how power control should be conducted, using very little information exchange across the layers and requiring no change within the transport layer.

Using the JOCP algorithm (4)–(6), we simulated the above joint power and congestion control for various wireless networks with different topologies and fading environments. The advantage of such a joint control can be captured even in a small illustrative example, where the logical topology and routes for four multihop connections are shown in Fig. 2. Sources at each of the four flows use TCP Vegas window updates with $\alpha_s$ ranging from 3 to 5. The path loss $G_{ij} = G_{ji}$ is determined by the relative physical distance $d_{ij}$, which we vary in different experiments, by $G_{ij} = d_{ij}^{-4}$. The target BER is $10^{-3}$ on each logical link.

Transmit powers, as regulated by the proposed distributed power control, and source rates, as regulated through TCP Vegas window update are shown in Fig. 3. The initial conditions of the graphs are based on the equilibrium states of TCP Vegas with fixed power levels of 2.5 mW. With power control, the transmit powers $\mathbf{P}$ distributively adapt to induce a "smart" capacity $\mathbf{c}$ and queueing delay $\boldsymbol{\lambda}$ configuration in the network, which in turn increases end-to-end throughput as indicated by the rise in all the allowed source rates. Notice that some link capacities actually decrease, while the capacities on the bottleneck links rise to maximize the total network utility. This is achieved through a distributive adaptation of power, which lowers the power levels that cause most interference on the links that are becoming a bottleneck in the dynamic demand-supply matching process. Confirming our intuition, such a "smart" allocation of power tends to reduce the spread of queueing delays, thus preventing any link from becoming a bottleneck. Queueing delays on the four links do not become the same though, due to the asymmetry in traffic load on the links and different weights $\alpha_s d_s$ in the logarithmic utility objective functions.

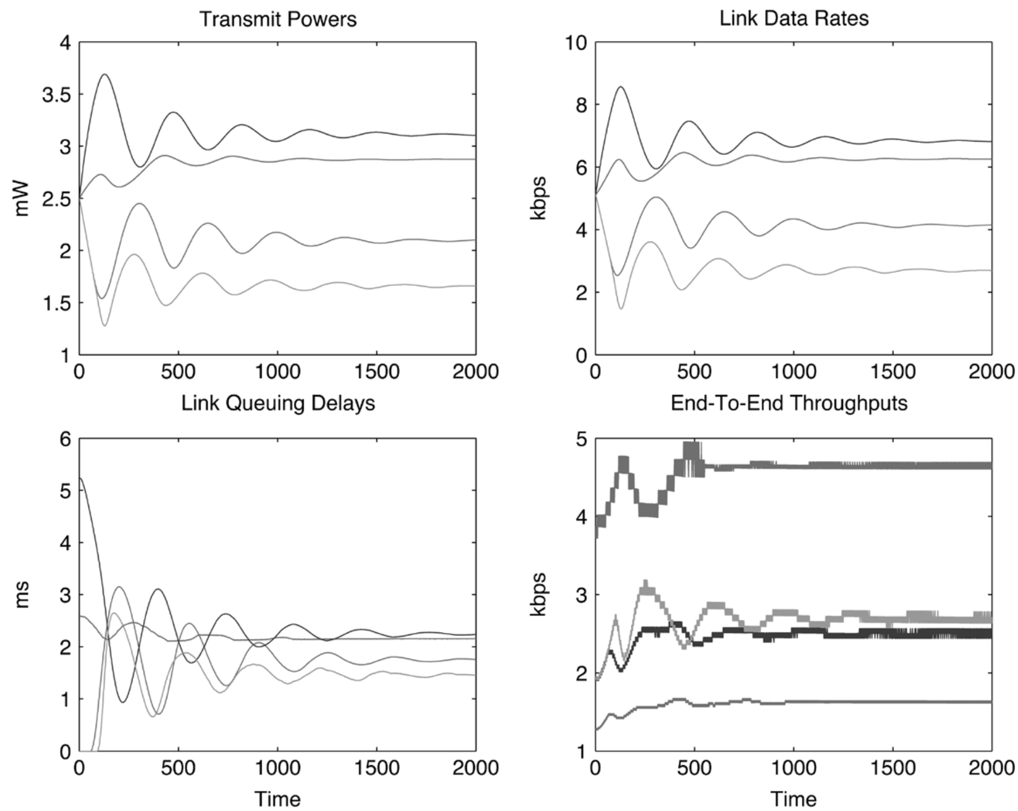


Fig. 3. A typical numerical example of joint TCP Vegas congestion control and power control. The top left graph shows the primal variables $\mathbf{P}$. The lower left graph shows the dual variables $\boldsymbol{\lambda}$. The lower right graph shows the primal variables $\mathbf{x}$, i.e., the end-to-end throughput. In order of their $y$ axis values after convergence, the curves in the top left, top right, and bottom left graphs are indexed by the third, first, second, and fourth links in Fig. 2. The curves in the bottom right graph are indexed by flows 1, 4, 3, and 2.

We indeed achieve the primary goal of this joint design across the transport and physical layers. The end-to-end throughput per watt of power transmitted, i.e., the throughput power ratio (TPR), is 82% higher with power control. A series of simulations are conducted based on different fading environments and TCP Vegas parameter settings. Based on the resulting statistics of TPR, we see that power control (6) increases TCP throughput and TPR in all experiments, and in 78% of the instances, energy efficiency rises by 75%–115%, compared with TCP without power control. Power control and congestion control, each running distributively and coordinated through the dual variables, work together to enhance the energy efficiency of multihop transmissions across a wireless multihop networks.

## V. Extensions

The last section only describes the basic version of the JOCP algorithm. Many variations can be readily accommodated without substantial changes in the algorithm and its analysis.

For example, the source utilities can be any increasing, strictly concave functions $U_s$ other than the logarithm function. Different utilities represent different types of TCP variants. As will be shown in the proof in Section VI, (5) and (6) are solving two decomposed subproblems that are coordinated by the congestion prices $\boldsymbol{\lambda}$. Instead of updating $\boldsymbol{\lambda}$ after moving only one step along the solution path in these two subproblems, we could have waited for the convergence of the subproblems for a given $\boldsymbol{\lambda}$. In that case, each source would be solving the following problem: $x_s^*(\boldsymbol{\lambda}) = U_s'^{-1}(\sum_{l \in L(s)} \lambda_l)$, and the power update in (6) would be allowed to converge before $\boldsymbol{\lambda}$ are updated by (4) (which is more practical if the time scale of power update is much smaller than that of link price update). The convergence theorem in Section VI remains valid with the above generalizations.

If metrics other than queueing delay are used as congestion price, e.g., packet loss in TCP Reno, then the price update (4) will look different. Any link prices with the following equilibrium property can act as the dual variables coordinating congestion control and power control: $\lambda_l^*(\sum_{s:l \in L(s)} x_s^* - c(\mathbf{P}^*)) = 0, \forall l$. However, depending on the specific price update equation, convergence may not be guaranteed.

If energy efficiency is desired to be modeled explicitly in the objective function, we can subtract a sum of increasing, convex power cost functions $\sum_l V_l(P_l)$ from the network utility $\sum_s U_s(x_s)$, and accordingly modify the power update equation.

In addition to end-to-end rate allocation over a fixed single-path route, multicommodity flow type of routing can easily be jointly optimized with power control (6) [33]. If the relay nodes require incentives to help relay traffic originating from the source nodes, joint optimization over source rate, total relay rate, incentive pricing, and transmit power can be conducted using a similar message passing approach.

The balance between the transport and physical layers is important not only for wireless ad hoc or cellular networks, but also to the wired Internet. For example, physical layer resource allocation, in terms of adaptation of coding, modulation, and interleaving parameters, in digital subscriber lines (DSLs) at the access part of the network can also be optimized based on TCP parameters and variables, in order to enhance the end-to-end performance [19].

## VI. Performance Evaluation: Convergence Theorem and Equilibrium State

It is not too surprising that allowing cross-layer interactions improves the performance of wireless multihop networks. The rest of this paper is devoted to the more interesting and challenging task of proving that the JOCP algorithm also has the following desirable properties: global convergence to the jointly optimal $(\mathbf{x}^*, \mathbf{P}^*)$, robustness to parameter perturbation and asynchronism, graceful tradeoff between complexity and performance, and geometric rate of convergence.

We first show that convergence of the nonlinearly coupled system, formed by the JOCP algorithm and shown in Fig. 1, is guaranteed under two mild assumptions. First, $P_l$ are within a range between $P_{l,\min} > 0$ and $P_{l,\max} < \infty$ for each link $l$. Second, when link prices are high enough, source rates can be made very small: for any $\epsilon > 0$, there exists a $\lambda_{\max}$ such that if $\lambda_l > \lambda_{\max}$, then $x_s(\boldsymbol{\lambda}) < \epsilon$ for all sources $s$ that use link $l$. To make the analysis concrete, we again focus on the case of TCP Vegas with logarithmic source utilities. But the proof technique is applicable to the interaction between other TCP sources with different utilities and the power control algorithm (6), as long as the congestion price update converges.

It is also interesting to note that the two decomposed problems in the proof are both geometric programming problems, a class of nonlinear optimization that was invented in the 1960s [12] and recently found many applications in communication systems, e.g., in [8], [10], [14], and [15]. The JOCP algorithm can be viewed as a distributed solution to a class of geometric programs.

*Theorem 1:* For small enough positive constants $\gamma$ and $\kappa$, the distributed JOCP algorithm (4)–(6) converges to the global optimum of the joint congestion control and power control problem (3).

*Proof:* We first associate a Lagrange multiplier $\lambda_l$ for each of the constraints $\sum_{s:l\in L(s)} x_s \leq c_l(\mathbf{P})$. Using the KKT optimality conditions for convex optimization [2], [4], solving problem (3) [or (2)] is equivalent to satisfying the complementary slackness condition and finding the stationary points of the Lagrangian.

Complementary slackness condition states that at optimality, the product of each dual variable and the associated primal constraint must be zero. This condition is satisfied since the equilibrium queueing delay must be zero if the total equilibrium ingress rate at a router is strictly smaller than the egress link capacity.

We now find the stationary points of the Lagrangian $I_{\text{system}}(\mathbf{x}, \mathbf{P}, \boldsymbol{\lambda}) = (\sum_s U_s(x_s) - \sum_l \lambda_l \sum_{s:l\in L(s)} x_s) + (\sum_l \lambda_l c_l(\mathbf{P}))$. By linearity of the differentiation operator, this can be decomposed into two separate maximization problems

$$\text{maximize}_{\mathbf{x}\succeq 0} \quad \sum_s U_s(x_s) - \sum_s \sum_{l\in L(s)} \lambda_l x_s$$
$$\text{maximize}_{\mathbf{P}\succeq 0} \quad I_{\text{power}}(\mathbf{P}, \boldsymbol{\lambda}) = \sum_l \lambda_l c_l(\mathbf{P}).$$

The first maximization is already implicitly solved by the congestion control mechanism for different $U_s$ (such as TCP Vegas for $U_s(x_s) = \alpha_s d_s \log x_s$), but we still need to solve the second maximization, using the Lagrange multipliers $\boldsymbol{\lambda}$ as the shadow prices to allocate exactly the right power to each transmitter, thus increasing the link data rates and reducing congestion at the network bottlenecks. For scalability in ad hoc networks, this power control must also be implemented distributively, just like the congestion control part. Since the data rate on each wireless link is a global function of all the transmit powers, the power control problem cannot be nicely decoupled into local problems for each link as in [32]. However, we show that distributed solution is still feasible, as long as an appropriate set of limited information is passed among the nodes.

We first establish that, if the algorithm converges, the convergence is indeed toward the global optimum. We will show that the partial Lagrangian to be maximized $I_{\text{power}}(\mathbf{P}) = \sum_l \lambda_l \log(\text{SIR}_l(\mathbf{P}))$ is a strictly concave function of a logarithmically transformed power vector. Let $\tilde{P}_l = \log P_l, \forall l$, we have

$$\begin{aligned} I_{\text{power}}(\tilde{\mathbf{P}}) &= \sum_l \lambda_l \log \frac{G_{ll} e^{\tilde{P}_l}}{\sum_k G_{lk} e^{\tilde{P}_k} + n_l} \\ &= \sum_l \lambda_l \left[ \log\left(G_{ll} e^{\tilde{P}_l}\right) - \log\left(\sum_k G_{lk} e^{\tilde{P}_k} + n_l\right)\right] \\ &= \sum_l \lambda_l \left[ \log\left(G_{ll} e^{\tilde{P}_l}\right) \right. \\ &\qquad \left. - \log\left(\sum_k \exp(\tilde{P}_k + \log G_{lk}) + n_l\right)\right]. \end{aligned}$$

The first term in the square bracket is linear in $\tilde{\mathbf{P}}$, and the second term is concave in $\tilde{\mathbf{P}}$ because the log of a sum of exponentials of linear functions of $\tilde{\mathbf{P}}$ is convex, as verified below.

Taking the derivative of $I_{\text{power}}(\tilde{\mathbf{P}})$ with respect to $\tilde{P}_l$, we have

$$\begin{aligned} \nabla_l I_{\text{power}}(\tilde{\mathbf{P}}) &= \lambda_l - \sum_{j\neq l} \frac{\lambda_j G_{jl} e^{\tilde{P}_l}}{\sum_{k\neq j} G_{jk} e^{\tilde{P}_k} + n_j} \\ &= \lambda_l - P_l \sum_{j\neq l} \frac{\lambda_j G_{jl}}{\sum_{k\neq j} G_{jk} P_k + n_j}. \end{aligned}$$

Taking derivatives again, for each of the nonlinear $-\lambda_l \log(\sum_k \exp(\tilde{P}_k + \log G_{lk}) + n_l)$ terms in $I_{\text{power}}(\tilde{\mathbf{P}})$, we obtain the Hessian

$$\mathbf{H}^l = \frac{-\lambda_l}{(\sum_k z_{lk} + n_l)^2}\left(\left(\sum_k z_{lk} + n_l\right)\mathbf{diag}(\mathbf{z}_l) - \mathbf{z}_l\mathbf{z}_l^T\right)$$

where $z_{lk} = \exp(\tilde{P}_k + \log G_{lk})$ and $\mathbf{z}_l$ is a column vector $[z_{l1}, z_{l2}, \ldots, z_{lN}]^T$.

Matrix $\mathbf{H}^l$ is indeed negative definite: for all vectors $\mathbf{v}$

$$\mathbf{v}^T\mathbf{H}^l\mathbf{v} = \frac{-\lambda_l\left(\left(\sum_k z_{lk} + n_l\right)\left(\sum_k v_k^2 z_{lk}\right) - \left(\sum_k v_k z_{lk}\right)^2\right)}{\left(\sum_k z_{lk} + n_l\right)^2} < 0. \quad (7)$$

This is because of the Cauchy Schwarz inequality $(\mathbf{a}^T\mathbf{a})(\mathbf{b}^T\mathbf{b}) \geq (\mathbf{a}^T\mathbf{b})^2$, where $a_k = v_k\sqrt{z_{lk}}$ and $b_k = \sqrt{z_{lk}}$ and the fact that $n_l > 0$. Therefore, $I_{\text{power}}(\tilde{\mathbf{P}})$ is a strictly concave function of $\tilde{\mathbf{P}}$, and its Hessian is a negative definite block diagonal matrix $\mathbf{diag}(\mathbf{H}^1, \mathbf{H}^2, \ldots, \mathbf{H}^L)$. Interestingly, some of the statements in later propositions depend on the invertibility of $\mathbf{H}$, which are provided for by the nonzero noise terms.

Coming back to the $\mathbf{P}$ solution space instead of $\tilde{\mathbf{P}}$, it is easy to verify that the derivative of $I_{\text{power}}(\mathbf{P})$ with respect to $P_l$ is

$$\nabla_l I_{\text{power}}(\mathbf{P}) = \frac{\lambda_l}{P_l} - \sum_{j\neq l} \frac{\lambda_j G_{jl}}{\sum_{k\neq j} G_{jk}P_k + n_j}.$$

Therefore, the logarithmic change of variables simply scales each entry of the gradient by $P_l$ : $\nabla_l I_{\text{power}}(\mathbf{P}) = (1)/(P_l)\nabla_l I_{\text{power}}(\tilde{\mathbf{P}})$. Power update can be conducted in either $\mathbf{P}$ or $\tilde{\mathbf{P}}$ domain.

We now use the gradient method [4], with a constant step size $\kappa$, to maximize $I_{\text{power}}(\mathbf{P})$

$$\begin{aligned} P_l(t+1) &= P_l(t) + \kappa \nabla_l I_{\text{power}}(\mathbf{P}) \\ &= P_l(t) + \kappa\left(\frac{\lambda_l(t)}{P_l(t)} - \sum_{j\neq l}\frac{\lambda_j(t) G_{jl}}{\sum_{k\neq j} G_{jk}P_k(t) + n_j}\right). \end{aligned}$$

Simplifying the equation and using the definition of SIR, we can write the gradient steps as the following distributed power control algorithm with message passing:

$$P_l(t+1) = P_l(t) + \frac{\kappa\lambda_l(t)}{P_l(t)} - \kappa\sum_{j\neq l} G_{lj} m_j(t)$$

where $m_j(t)$ are messages passed from node $j$

$$m_j(t) = \frac{\lambda_j(t)\text{SIR}_j(t)}{P_j(t)G_{jj}}.$$

These are exactly items 3 and 4 in the JOCP algorithm.

It is known [2] that when the step size along the gradient direction is optimized, the gradient-based iterations converge. Such an optimization of step size $\kappa$ in (6) would require global coordination in a wireless ad hoc network, and is undesirable or infeasible. However, in general, gradient-based iterations with a constant step size may not converge.

By the descent lemma [2], convergence of the gradient-based optimization of a function $f(\mathbf{x})$, with a constant step size $\kappa$, is guaranteed if $f(\mathbf{x})$ has the Lipschitz continuity property: $\|\nabla f(\mathbf{x}_1) - \nabla f(\mathbf{x}_2)\| \leq L\|\mathbf{x}_1 - \mathbf{x}_2\|$ for some $L > 0$, and the step size is small enough: $\epsilon \leq \kappa \leq (2-\epsilon)/(L)$ for some $\epsilon > 0$. It is known that $f(\mathbf{x})$ has the Lipschitz continuity property if it has a Hessian bounded in $l_2$ norm.

The Hessian $\mathbf{H}$ of $\sum_l \lambda_l c_l(\mathbf{P})$ can be verified to be

$$H_{ll} = \sum_{j\neq l}\lambda_j\left(\frac{G_{jl}}{\sum_{k\neq j} G_{jk}P_k + n_j}\right)^2 - \frac{\lambda_l}{P_l^2} \tag{8}$$

$$H_{li} = \sum_{j\neq l,i}\frac{\lambda_j G_{jl}G_{ji}}{\left(\sum_{k\neq j} G_{jk}P_k + n_j\right)^2}, \quad i\neq l. \tag{9}$$

The second assumption for Theorem 1 leads to the conclusion that $\boldsymbol{\lambda}$ are upper bounded [30], which, together with the first assumption for Theorem 1, shows that $\|\mathbf{H}\|_2$ is upper bounded. The upper bound can be estimated by the following inequality:

$$\|\mathbf{H}\|_2 \leq \sqrt{\|\mathbf{H}\|_1\|\mathbf{H}\|_\infty}$$

where $\|\mathbf{H}\|_1$ is the maximum column-sum matrix norm of $\mathbf{H}$, and $\|\mathbf{H}\|_\infty$ is the maximum row-sum matrix norm.

Therefore, the power control part (6) converges for a small enough step size $\kappa$

$$\epsilon \leq \kappa \leq \frac{2-\epsilon}{L'}$$

where

$$\begin{aligned} (L')^2 = \max_i &\left(\sum_l\sum_{j\neq l,i}\frac{\lambda_j G_{jl}G_{ji}}{\left(\sum_{k\neq j} G_{jk}P_k + n_j\right)^2} + \left|\sum_{j\neq l}\lambda_j\left(\frac{G_{jl}}{\sum_{k\neq j} G_{jk}P_k + n_j}\right)^2 - \frac{\lambda_l}{P_l^2}\right|\right) \\ \times \max_l &\left(\sum_i\sum_{j\neq l,i}\frac{\lambda_j G_{jl}G_{ji}}{\left(\sum_{k\neq j} G_{jk}P_k + n_j\right)^2} + \left|\sum_{j\neq l}\lambda_j\left(\frac{G_{jl}}{\sum_{k\neq j} G_{jk}P_k + n_j}\right)^2 - \frac{\lambda_l}{P_l^2}\right|\right) \end{aligned}$$

and $\epsilon$ can be any small positive number $\leq (2)/(1+L')$.

It is known [23] that TCP Vegas converges for a small enough step size $0 < \gamma \leq (2\alpha_{\min}d_{\min}c_{\min})/(L_{\max}S_{\max}x_{\max}^2)$, where $\alpha_{\min}$ and $d_{\min}$ are the smallest TCP source parameters $\alpha_s$ and $d_s$ among the sources, respectively, $x_{\max}$ is the largest possible number of source rates, $c_{\min}$ is the smallest link data rate, $L_{\max}$ is the largest number of links any path has, and $S_{\max}$ is the largest number of sources sharing a link.

Convergence of TCP Vegas assumes that $c_{\min} \neq 0$. Since $\text{SIR}_l$ is lower bounded by $(P_{l,\min}G_{ll})/(\sum_{j\neq l} P_{j,\max}G_{lj} + n_l)$, each $c_l$ is lower bounded by a strictly positive number. (In fact, the formulation in (2) assumes high SIR in the first place.) Consequently, TCP Vegas (4) and (5) also converge. By the convergence result of simultaneous gradient-method to the saddle point of minmax problems [3], [29] (in this case, minimizing the Lagrangian over dual variables and the maximizing it over the primal variables to the saddle point of the Lagrangian, which is the optimal $(\mathbf{x}^*, \mathbf{P}^*)$), the JOCP algorithm converges.

Since $c_l$ can be turned into a concave function in $\tilde{\mathbf{P}}$, each constraint $\sum_{s:l\in L(s)} x_s - c_l(\mathbf{P}) \leq 0$ in (2) is an upper bound constraint on a convex function in $(\mathbf{x}, \tilde{\mathbf{P}})$. So problem (2) can be turned into maximizing a strictly concave objective function over a convex constraint set. The established convergence is towards the global optimum. ■

In addition to convergence guarantee, total network utility $\sum_s U_s(x_s)$ with power control can never be smaller than that without power control, because by allowing power adaptation, we are optimizing over a larger constraint set. Note that an increase in network utility $\sum_s U_s(x_s)$ is not equivalent to a higher total throughput $\sum_s x_s$, since the utility functions are not iden-

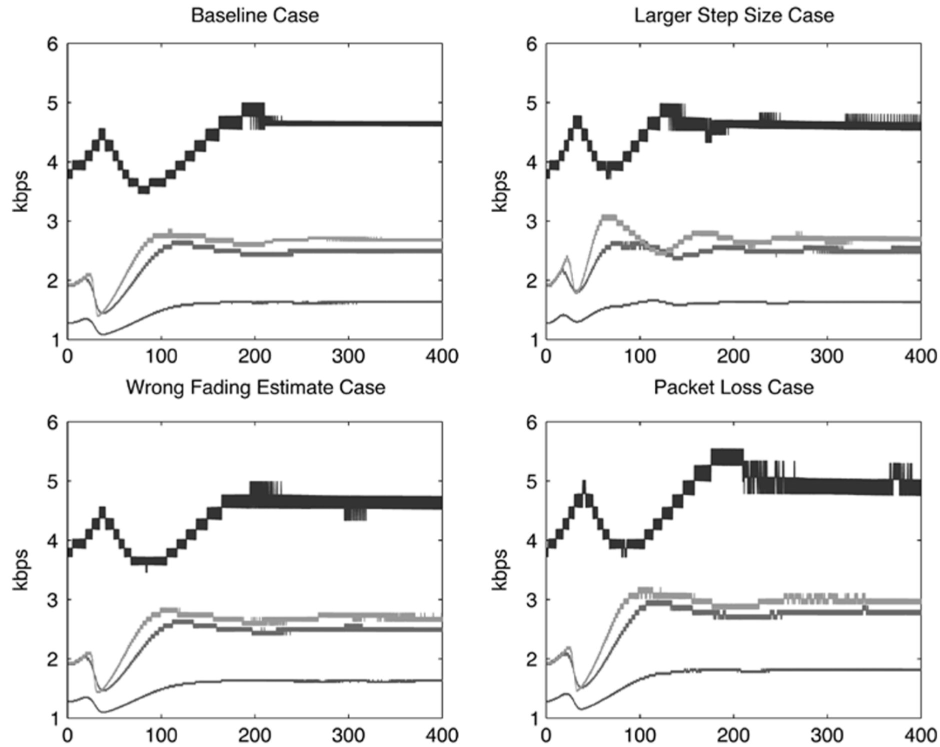


Fig. 4. Robustness of joint power control and TCP Vegas. Top left graph is the baseline performance of the four end-to-end throughput (the same as in Fig. 3). Top right graph shows that a larger step size in the algorithm accelerates convergence but also leads to larger variances. Bottom left graph shows that the algorithm is robust to wrong estimates of path losses. Bottom right graph shows robustness to packet losses on links with wireless channel outage.

tity functions, but strictly concave functions. However, empirical evidence from simulation suggests that, at least in the logarithmic utility case of TCP Vegas, both throughput and energy efficiency will indeed rise significantly after power control (6) regulates data rate supply, and dual variables $\boldsymbol{\lambda}$ coordinate data rate demand with supply.

## VII. Some Practical Issues: Robustness, Complexity Reduction, Asynchronous Implementation, and Convergence Speed

We use various tools from nonlinear optimization, distributed algorithm, and linear algebra to rigorously study other important properties of the JOCP algorithm. Some proofs of the propositions can be found in [6].

### A. Robustness

Robustness is often as important as optimality of an algorithm. We focus on the following robustness properties of the JOCP algorithm.

1) The effects of inaccurate estimation of the path losses at various nodes. Even with an accurate estimation, mobility of the nodes and fast variation of the fading process may lead to a mismatch between the $G_{ij}$ used in the power update algorithm and the $G_{ij}$ that actually appear in the link data rate formula.
2) The effects of packet loss due to wireless channel outage during deep fading.

First, it is assumed in the power control algorithm (6) that the pass loss factors $G_{ij}$ are perfectly estimated by the receivers. It is useful to know how much error in the estimation of $G_{ij}$ can be tolerated without losing the convergence of joint power control and TCP congestion control.

Denoting the error in the estimation of $G_{ij}$ at time $t$ as $\Delta G_{ij}(t)$, and suppressing the time index on $\boldsymbol{\lambda}(t), \mathbf{P}(t), \mathrm{SIR}(t), \Delta G_{ij}(t)$, we provide a sufficient condition using results about gradient algorithm with errors in gradient circulation [2].

*Proposition 1:* Convergence to the global optimum of (3) is achieved through the JOCP algorithm (4)–(6) with $G_{ij}$ estimation errors, if there exists a $T$ such that for all times $t \geq T$, the following inequality holds:

$$\sum_l \sum_{j \neq l} \sum_{k \neq l} (G_{jl}G_{kl} - \Delta G_{jl} \Delta G_{kl}) \frac{\lambda_j \lambda_k \mathrm{SIR}_j \mathrm{SIR}_k}{P_j P_k G_{jj} G_{kk}} > 2 \sum_l \sum_{j \neq l} \frac{\lambda_l \lambda_j G_{jl}}{P_l P_j G_{jj}} \mathrm{SIR}_j - \frac{\lambda_l^2}{P_l^2}.$$

While Proposition 1 gives an analytic condition of convergence with inaccurate estimations of $G_{ij}$ for any network, numerical experiments can also be carried out in simulations, where the $G_{ij}$ factors in (6) are perturbed randomly within a range. Results of one typical experiment are shown in the lower left graph in Fig. 4, for the same network topology and connections as in Fig. 2. In this simulation, the $G_{ij}$ factors are generated at random between $+25\%$ and $-25\%$ of their true values. The algorithm converge to the same global optimum after a much longer and wider transient period.

Another peculiar feature of wireless transmissions is that during deep fading, SIR on a link may become too small for correct decoding at the receiver. This channel outage induces packet losses on the link. Consequently, the queue buffer sizes become smaller than they should have been. Analysis of TCP in such a lossy environment has been carried out, for example, in [1]. In our framework of nonlinear optimization, since queueing delays are implicitly used as the dual variables $\boldsymbol{\lambda}$ in TCP Vegas, such channel variations lead to incorrect values of the dual variables. Sources will mistake the decreases in total queueing delay as indications of reduced congestion levels and boost their source rates through TCP update accordingly. Having incorrect pricing on the wireless links may, thus, prevent the joint system from converging to the global optimum. We have the following sufficient condition for convergence, where outage-induced packet loss on link $l$ is denoted as $\Delta y_l$.

*Proposition 2:* Convergence to the global optimum of (3) is achieved through the JOCP algorithm (4)–(6) with packet losses, if there exists a $T$ such that for all times $t \geq T$, the following inequality holds:

$$\sum_l \left[ \frac{1}{P_l^2} \left( \lambda_l^2 - \left( \frac{\Delta y_l}{c_l} \right)^2 \right) + \sum_{j \neq l} \left( \frac{G_{jl} \mathrm{SIR}_j}{G_{jj} P_j} \right)^2 \left( \lambda_j - \left( \frac{\Delta y_j}{c_j} \right)^2 \right) \right] > 2 \sum_l \sum_{j \neq l} \left( \lambda_j \lambda_l - \frac{\Delta y_l \Delta y_j}{c_l c_j} \right) \frac{G_{jl} \mathrm{SIR}_j}{G_{jj} P_l P_j}.$$

Because the chance of having simultaneous channel outages at all links is small, it is reasonable to expect that only a few $\Delta y_l$ are nonzero at any time. We again numerically experiment with channel outage induced packet loss on various links and a typical result is shown in the lower right graph in Fig. 4, where the underlying outage probability is 20%. The convergence is much slower but still maintained toward the optimal solution.

## B. Complexity Reduction

Another practical issue concerning the JOCP algorithm is the tradeoff between performance optimality and implementation simplicity. The increases in TCP throughput and energy efficiency have been achieved with a rise in the communication complexity of message passing. There can be many terms in the $\sum_{j \neq l} G_{lj} m_j(t)$ sum in (6) as the number of transmitters increases. Fortunately, those transmitters farther away from transmitter $l$ will have their messages be correspondingly multiplied by a much smaller $G_{lj} \propto d_{lj}^{-\alpha}$, where $\alpha$ ranges between 2 and 6. Their messages $m_j$ will, therefore, be given much smaller weights in the power update.

This leads to a simplified power control algorithm, where each transmitter $l$ uses the path loss estimations to form a small set $J_l$ of neighbors whose messages will be needed and used in the power update. Naturally, if there are $V$ elements in set $J_l$, they should correspond to the nodes with the $V$ largest $G_{lj}$ toward node $l$. The power update equation becomes

$$P_l(t+1) = P_l(t) + \frac{\kappa \lambda_l(t)}{P_l(t)} - \kappa \sum_{j \in J_l} G_{lj} m_j. \tag{10}$$

The following sufficient condition of convergence with the simplified algorithm can be shown.

*Proposition 3:* Convergence to the global optimum of (3) is achieved through the simplified version of the JOCP algorithm (4), (5), and (10), if there exists a $T$ such that for all time $t \geq T$, the following inequality holds:

$$\sum_l \sum_{j \in J_l} \left( \frac{G_{jl} \lambda_j \mathrm{SIR}_j}{G_{jj} P_j} \right)^2 > 2 \sum_l \sum_{j \neq l} \frac{\lambda_l \lambda_j G_{jl}}{P_l P_j G_{jj}} \mathrm{SIR}_j - \frac{\lambda_l^2}{P_l^2}.$$

The reduction in complexity can be measured by the ratio

$$\Delta \mathrm{COM} = \frac{\sum_l |J_l|}{M(M-1)}$$

where $M$ is the total number of transmitters in the network. Obviously, $0 \leq \Delta\mathrm{COM} \leq 1$, and a smaller $\Delta\mathrm{COM}$ represents a simpler and less optimal message passing and power update. The effectiveness of complexity reduction through partial message passing depends on the path loss matrix $\mathbf{G}$. While the intuition is clear: the reduced-complexity versions do not work well for network topologies where nodes are evenly spread out, we do not yet have an analytic characterization on the tradeoff between $\Delta\mathrm{COM}$ and energy efficiency enhancement or the maximized network utility.

## C. Asynchronous Implementation

The algorithmic analysis thus far has been limited to the case where propagation delay is insignificant and all the local clocks are synchronized, which is not practical in large wireless ad hoc networks. In this subsection, we investigate the convergence of the algorithm under asynchronous implementation, with variable propagation delays and clock asynchronism.

Suppose each source updates $x_s$ and each transmitter updates $P_l$ at asynchronous time slots, using possibly outdated variables, such as $\lambda_l$ and $m_j$, in their update. At least one local update is carried out sometime within a window of $D$ time slots, and the variables used in the update can be outdated by up to $D$ time slots. We have the following.

*Proposition 4:* The asynchronous JOCP algorithm converges if and only if $D$ is finite.

This result shows that the proposed algorithm is able to support asynchronous implementation as long as the constants $\kappa, \gamma$ are small enough. An adverse effect of asynchronism is the reduction of the maximum step sizes allowed for convergence to be maintained, which reduces the convergence speed. However, in the case of sufficiently small asynchronism

$$D \leq \min \left\{ \frac{L'}{2-\epsilon}, \frac{L_{\max} S_{\max} x_{\max}^2}{2\alpha_{\min} d_{\min} c_{\min}} \right\}$$

we can show that propagation delay, delay in message passing, and clock asynchronism in rate-power updates become the loose constraints on the maximum step sizes, and do not cause a reduction in the rate of convergence.

## D. Rate of Convergence

So far, we have focused on the equilibrium behaviors of the JOCP algorithm. In general, very little is understood on the transient behaviors of the dynamics of JOCP algorithm, or of just

TCP congestion control alone. This section provides a preliminary analysis on the rate of convergence of the power control algorithm. The rate of convergence for any distributive algorithm on wireless ad hoc networks is particularly important because the network topology and source traffic are dynamic and source traffic may exhibit a low degree of stability. A key question for practical implementation of the proposed cross-layer design is whether the coupled nonlinear dynamics between TCP and power control can proceed close to the equilibrium before the network topology, routing, and source characteristics change dramatically.

Convergence analysis for distributive nonlinear optimization can take several different approaches. We focus on the more practical local analysis approach, which investigates the rate of convergence after the algorithm reaches a point reasonably close to the optimum. Because our algorithm nonlinearly depends on the path loss matrix $\mathbf{G}$, exact and closed-form results on the rate of convergence are very difficult to obtain. Nonetheless, the following result on the geometric convergence property and a loose bound on the convergence speed can be proved.

Let $U^{(k)}$ be the network utility at the $k$th iteration of the JOCP algorithm, and $U^*$ be the maximized network utility. Let $e^{(k)} = |U^{(k)} - U^*|$ be the error. Let $\mathbf{P}^{(k)}$ be the power vector at the $k$th iteration, and $\mathbf{P}^*$ be the optimizer. Assume that the limit of the Hessian $I_{\text{power}}(\mathbf{P}^{(k)})$ as $k \to \infty$ exist and is denoted by $\mathbf{H} = \{H_{ij}\}$.

Using local analysis that characterizes the rate of convergence in terms of Hessian matrix eigenvalues [2], and the Gersgorin Theorem on eigenvalue location [34], we can show the following result.

*Proposition 5:* The JOCP algorithm converges geometrically, i.e., there exist $q > 0$ and $\beta \in (0,1)$ such that for all $k, e^{(k)} \leq q\beta^k$. With an appropriate constant parameter $\kappa$, the rate of convergence (of the power control part) is at least $(M' - m')/(M' + m')$, where

$$M' = \max_i \left( H_{ii} + \sum_{j \neq i} |H_{ij}| \right)$$
$$m' = \min_i \left( H_{ii} - \sum_{j \neq i} |H_{ij}| \right).$$

A similar result holds for the rate of convergence of the congestion control part. However, we add the cautionary note that the above lower bound on the rate of convergence is based on the worst case scenario and can be orders of magnitude loose, depending on the path loss environment in the network. Numerical simulations show that the actual convergence speed is much faster than the bound in Proposition 5.

### E. Further Algorithmic Enhancements

In concluding our performance analysis of the JOCP algorithm, we briefly outline a couple of algorithmic enhancements that can be readily accomplished.

It is desirable to choose a constant step size that is neither so large that the algorithm diverges (e.g., violating the conditions in Section IV), nor so small that the convergence is too slow. One way to accomplish this is to let each source and each transmitter autonomously decrease the step sizes at each time slot $t$ according to the following rule:

$$\gamma(t) = \kappa(t) = \frac{\omega}{t}, \quad \omega > 0.$$

Such a diminishing sequence of step sizes also makes the algorithm even more robust: errors in queueing delays $\boldsymbol{\lambda}$ and path losses $\mathbf{G}$ that are proportional to the magnitudes of $\boldsymbol{\lambda}$ and $\mathbf{G}$ can be tolerated.

It is also possible to speed up the convergence of the algorithm by diagonally scaling the distributed gradient method

$$P_l(t+1) = P_l(t) + \kappa \mathbf{W} \nabla_l I_{\text{power}}(\mathbf{P})$$

where $\mathbf{W}$ ideally should be the inverse of the Hessian $\mathbf{H}$ of $I_{\text{power}}(\mathbf{P})$. Since forming this inverse will require extensive global coordination and centralized computation, we approximate the inverse by letting

$$\mathbf{W} = \mathbf{diag}\left(H_{ii}^{-1}\right).$$

Substituting the expression for $H_{ii}$ in (8) and simplifying the expressions, we arrive at the following accelerated algorithm:

$$P_l(t+1) = P_l(t) + \kappa \frac{\frac{\lambda_l(t)}{P_l(t)} - \sum_{j \neq l} G_{lj} m_j(t)}{\frac{\lambda_l(t)}{P_l^2(t)} - \sum_{j \neq l} \frac{(G_{lj} m_j(t))^2}{\lambda_j(t)}}.$$

Therefore, by passing an additional message: the explicit value of price $\lambda_j(t)$ from node $j$, the jointly optimal congestion control and power control algorithm can converge faster.

## VIII. To Layer or Not to Layer: Layering as Optimization Decomposition

Like [9], [19], [27], [31], and [32], this paper can be viewed as a case study of the "layering as optimization decomposition" approach, which may allow us to integrate many layers in wired and wireless networks, and to rigorously quantify the general *architectural principles* and inherent tradeoffs of layering. If a mapping can be found from different *decompositions* of a generalized utility maximization problem to different *layering* schemes, and from primal or Lagrange dual *variables* coordinating the subproblems to the *interfaces* among the layers, then we can tackle the question "how to and how not to layer" by investigating the pros and cons of decomposition techniques. By comparing the objective values under optimal decompositions, suboptimal decompositions, and decompositions with some layering variables fixed, we can seek "separation theorems" among layers: conditions under which strict layering incurs no loss of optimality. Robustness of these separation theorems can be further characterized by sensitivity analysis in optimization theory: how much will the differences in the objective value (between different layering schemes) fluctuate as constant parameters in the utility maximization problem are perturbed. In addition to "vertical decomposition" across layers of functional modules, "horizontal decomposition" across geographically diverse nodes may also be conducted via functions of the layering variables.

If layering schemes are viewed as decompositions of some global optimization problems, the price of layering or relayering, the price of no layering, and the price of cross-layering may be quantified in both reverse and forward engineering directions.

- Reverse engineering: Given a layered protocol stack, what is the optimization problem that it implicitly solves?
- Forward engineering: Given a utility maximization formulation, how to decompose it into subproblems, solve each subproblem individually, then solve the overall problem?

In the case of wireless networks, the transmission medium is an untethered, unshielded, broadcast one, with time-varying channels that have attenuation, shadowing, and fading. A wireless network is essentially a space with electromagnetic energy propagating in it. There is no *a priori* definition of "link capacity" or even of "link." Therefore, all of the following issues complicate the utility maximization model substantially: signal interference and power control, packet collision and medium access control (MAC), rate-reliability tradeoff and coding, and spatial diversity and multiple-antenna transmissions. For example, the following utility maximization problem and its decomposition and distributed algorithms needs to be studied:

$$\begin{aligned} \text{maximize} \quad & \sum_s U_s(x_s, P_{e,s}) + \sum_j V_j(w_j) \\ \text{subject to} \quad & \mathbf{R}\mathbf{x} \preceq \mathbf{c}(\mathbf{w}, \mathbf{P}_e), \\ & \mathbf{x} \in \mathcal{C}_1(\mathbf{P}_e) \bigcap \mathcal{C}_2(\mathbf{F}), \\ & \mathbf{R} \in \mathcal{R}, \quad \mathbf{F} \in \mathcal{F}, \quad \mathbf{w} \in \mathcal{W}. \end{aligned} \tag{11}$$

Here, $x_s$ denotes the rate for source $s$ and $w_j$ denotes the physical layer resource at network element $j$. The utility functions $U_s$ and $V_j$ may be any nonlinear, monotonic functions. $\mathbf{R}$ is the routing matrix and $\mathbf{c}$ are the logical link capacities as functions of both physical layer resources $\mathbf{w}$ and the desired decoding error probabilities $\mathbf{P}_e$. The issue of signal interference and power control can be captured in this functional dependency. The rates must also be constrained by the interplay between physical layer decoding reliability and upper layer error control mechanisms like ARQ in the link layer. This constraint set is denoted as $\mathcal{C}_1(\mathbf{P}_e)$, and captures the issue of rate-reliability tradeoff and coding. Constraint on the rates by the medium access success probabilities is represented by the constraint set $\mathcal{C}_2(\mathbf{F})$, where $\mathbf{F}$ is the contention matrix [26]. The issue of packet collision and MAC is captured in this constraint. The set of possible physical layer resource allocation schemes is represented by $\mathcal{W}$, that of possible scheduling or contention based medium access schemes by $\mathcal{F}$, and that of single-path or multipath routing schemes by $\mathcal{R}$. The optimization variables are $\mathbf{x}, \mathbf{w}, \mathbf{P}_e, \mathbf{R}, \mathbf{F}$.

Five layers in the current standard protocol stack are modeled in (11), although the decompositions of (11) do *not* have to be along the lines dictated by the current layering structure.

- *Application Layer*: Utility functions $U_i$ and $V_j$ model the application needs.
- *Transport Layer*: The end-to-end throughput is represented as the source rate $x_s$ for each end user $s$.
- *Network Layer*: The routing matrix can be designed by varying $\mathbf{R}$ within the constraint set $\mathcal{R}$.
- *Link Layer*: Through scheduling, antenna beamforming, and spreading code assignment, the contention matrix $\mathbf{F}$ can be designed within the constraint set $\mathcal{F}$. The rates are then constrained by contention-free or contention-based access schemes as described by the constraint set $\mathcal{C}_2$.
- *Physical Layer*: Adaptive resource allocations, e.g., power control, adaptive modulation, coding with embedded diversity, will lead to different logical link capacities $\mathbf{c}$ as functions of decoding error probabilities $\mathbf{P}_e$ and the physical layer resources $\mathbf{w}$.

The generic formulation (11) can be specialized in different cases. The two most difficult issues are time-scale and nonzero duality gap. In this paper, we have assumed that the time scale of power control and congestion control is longer than the time scale needed for channel coding to achieve $c_l$, and shorter than the time scale of dynamic changes in network topology and routing. Using the approximation that $K\mathrm{SIR}$ is much larger than 1 and a log transformation, we have turned problem (2) into a convex optimization with strictly feasible solutions, thus having zero duality gap.

Finally, this paper balances the transport and physical layers only from a network performance viewpoint. Other than the discussion on how the JOCP algorithm maintains the modularity between the two layers, this paper is not examining the most important reason for layering. Layering, like many other networking principles, is *not* established only for *efficiency* of performance metrics in terms of throughput, latency, distortion, or energy efficiency, but also for *robustness* in terms of important *X-ities*: evolvability, scalability, verifiability, manageability, deployability, adaptability ... Compared with standard performance metrics, these X-ities are much less well-understood, often without any theoretical foundations, quantitative frameworks, or even units of measurement. Yet, X-ities are crucial if we are to analyze current layering and design future ones properly. Initial results on quantifying some basic aspects of evolvability have recently been obtained [11]. It will be most interesting and challenging to investigate how the X-ities aspects may be understood through the general framework of "layering as optimization decomposition."

## IX. Conclusion

We present a distributed power control algorithm that couples with the existing TCP congestion control algorithms to increase end-to-end throughput and energy efficiency of multihop transmissions in wireless multihop networks. No modification to TCP is needed to achieve the optimal balancing between data rate demand (regulated through TCP) and supply (regulated through power control). We prove that the nonlinearly coupled system converges to the global optimum of the joint congestion control and power control problem. The convergence is geometric and can be maintained under finite asynchronism. The proposed algorithm is robust to wireless channel variations and path loss estimation errors. Suboptimal but much simplified versions of the algorithm are presented for scalable architectures.

As a step toward "layering as optimization decomposition," this paper expands the scope of the network utility maximization methodology to handle nonlinear, elastic link capacities. This extension enables us to rigorously prove that the proposed JOCP algorithm has the above desirable properties in achieving the optimal balance between the transport and physical layers in wireless multihop networks.

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**Mung Chiang** received the B.S. degree (honors) in electrical engineering and mathematics, and the M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, in 1999, 2000, and 2003, respectively.

He is an Assistant Professor of Electrical Engineering at Princeton University, Princeton, NJ. He conducts research in the areas of nonlinear optimization of communication systems, network resource allocation and congestion control algorithms, broadband access networks, and information theory and stochastic analysis of communication systems.

Dr. Chiang has been awarded a Hertz Foundation Fellow, a Stanford Graduate Fellow, and a National Science Foundation (NSF) Graduate Fellow. He received the Stanford University School of Engineering Terman Award and the SBC Communications New Technology Introduction Contribution Award. He is the Lead Guest Editor of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS Special Issue on Nonlinear Optimization of Communication Systems, a Guest Editor of the IEEE TRANSACTIONS ON INFORMATION THEORY Special Issue on Information Theory and Networking, and the Program Co-Chair of the 38th Conference on Information Sciences and Systems.