# Jointly Optimal Congestion Control and Power Control in Wireless Multihop Networks

Mung Chiang Electrical Engineering, Princeton University Rosanna Man Computer Science, Stanford University

Abstract—Power control can significantly enhance the performance of congestion control mechanisms, such as TCP, in wireless networks. We present a distributed power control algorithm that works together with the original TCP protocol to increase endto-end throughput and energy efficiency of multihop data transmissions in CDMA wireless ad hoc networks. We prove that the resulted nonlinear coupled system converges to the global optimality of network utility maximization with elastic link capacities. This cross-layer algorithm can be interpreted as using link queuing delays as shadow prices to coordinate bandwidth demand and supply. Various simulations show desirable properties of the algorithms, including robustness to channel variations and fading estimation errors, and flexibility in the tradeoff between performance optimality and algorithmic simplicity.

## I. INTRODUCTION

In order to achieve in a power efficient manner high endto-end throughput in wireless ad hoc networks with multihop transmissions, both congestion control and power control need to be optimally designed and distributively implemented. Congestion control mechanisms, such as those in TCP, regulate allowed source rates so that total traffic load on any link does not exceed the available capacity. At the same time, link capacities depend on the signal to interference ratios, which in turn depend on the power control policy. This paper derives, analyzes, and simulates a distributed algorithm for joint congestion control and power control that can significantly increases end-toend throughput and energy efficiency in a wireless multihop network. This performance enhancement is achieved without having to modify the existing TCP protocol stack.

It has been shown in Kelly et. al. [4] that congestion control mechanisms can be viewed as distributed algorithms solving the following network utility maximization problem:

maximize 
$$\sum_{s} U_s(x_s)$$
  
subject to  $\sum_{s \in L(s)} x_s \le c_l, \quad \forall l,$  (1)  
 $\mathbf{x} \succeq 0$ 

where source rates  $\mathbf{x} = \{x_s\}$  are the optimization variables, link capacities  $\{c_l\}$  are the constant parameters, L(s) denotes the set of links l traversed by the connection originating from source s, and the utility  $U_s$  for each source can be any increasing, concave function.

In particular, versions of TCP have recently been modelled and analyzed (*e.g.*, [5], [6], [7], [8]) as primal-dual distributed algorithms implicitly solving the above utility maximization for different utility functions. As each source updates its allowed rate (the primal variable) through a TCP algorithm, each link updates a congestion indicator (the dual variable, which can be interpreted as the 'shadow price' of using the link) through a queue management algorithm, and implicitly feeds it back to all the sources using this link.

However, in this standard formulation of network utility maximization, link capacities  $\{c_l\}$  are assumed to be fixed constants. This is not true in wireless networks where power control changes attainable data rates on the links. Intuitively, a proper power control algorithm would allocate the right amount of power at the right nodes to alleviate the bandwidth bottlenecks by increasing capacity on the appropriate links, which will then induce an increase in end-to-end TCP throughput. What complicates this approach is that changing the transmit power on one link also affects the data rates available on other links, due to the interference in wireless networks.

In this paper, we make precise the above intuition in the framework of the following utility maximization with elastic link capacities:

$$\begin{array}{ll} \text{maximize} & \sum_{s} U_{s}(x_{s}) \\ \text{subject to} & \sum_{s \in L(s)} x_{s} \leq c_{l}(\mathbf{P}), \quad \forall l, \\ & P_{l} \leq P_{l,max}, \quad \forall l, \\ & \mathbf{P}, \mathbf{x} \succeq 0 \end{array}$$

$$(2)$$

where the optimization variables are both source rates  $\mathbf{x} = \{x_s\}$  and transmit powers  $\mathbf{P} = \{P_l\}$ . The key difference from the standard utility maximization (1) is that link capacities  $\{c_l\}$  are now nonlinear and global functions of the transmit powers  $\mathbf{P}$ . We will show that in a wireless CDMA multihop network, as power control changes bandwidth supply and congestion control regulates bandwidth demand, they can be distributively coordinated through 'shadow prices' to globally solve (2). The advantage is a significant increase in TCP throughput and energy efficiency of multihop data transmissions.

There has been an extensive research literature on wireless network power control for different objectives, such as minimizing total power. This paper shows that if the objective is to maximize network utility for traffic running on TCP, then power control can be carried out simultaneously with congestion control to increase end-to-end throughput using less power. This can be viewed as an example of co-design across physical layer and transport layer. The power control algorithm proposed here can be used together with any congestion control algorithm, and we focus on TCP Vegas in this paper.

In section II, we briefly review the source algorithm and queue algorithm that solve the utility maximization (1) for TCP Vegas. In section III, utility maximization with elastic link capacities (2) is solved through a combination of a new power control algorithm and the original TCP Vegas algorithms. The algorithms can be distributively implemented on a multihop network, despite the fact that link capacity on a wireless link is a global function of all the interfering powers. Section III also offers interpretations in terms of demand-supply coordination through shadow prices. In section IV, we prove that the nonlinear coupled system formed by the proposed algorithms will converge to the global optimality of utility maximization with elastic link capacities, and illustrate through simulations that end-to-end throughput and energy efficiency can be significantly increased. Results on robustness and simplified versions of the algorithms are presented in section V.

### II. BACKGROUND: TCP VEGAS ALGORITHMS

TCP Vegas [2] is a sliding window based transport protocol that regulates the allowed source rates in a mesh network. Let  $d_s$  be the propagation delay for the path originating from source s, and  $D_s$  be the propagation plus queuing delay. When there is no congestion along all the links used by source s, we have  $d_s = D_s$ . The window size  $w_s$  is updated depending on whether the difference between the expected rate  $\frac{w_s}{d_s}$  and the actual rate  $\frac{w_s}{D_s}$  is smaller than a parameter  $\alpha_s$ :

$$w_{s}(t+1) = \begin{cases} w_{s}(t) + \frac{1}{D_{s}(t)} & \text{if } \frac{w_{s}(t)}{d_{s}} - \frac{w_{s}(t)}{D_{s}(t)} < \alpha_{s} \\ w_{s}(t) - \frac{1}{D_{s}(t)} & \text{if } \frac{w_{s}(t)}{d_{s}} - \frac{w_{s}(t)}{D_{s}(t)} > \alpha_{s} \\ w_{s}(t) & \text{else.} \end{cases}$$
(3)

The end-to-end throughputs are the allowed source rates  $x_s(t) = \frac{w_s(t)}{D_s(t)}$ , which are the primal variables of the utility maximization problem (1) where  $U_s(x_s) = \alpha_s d_s \log x_s$  [7].

The dual variables (or shadow prices)  $\lambda_l$  for TCP Vegas are shown [7] to be the queuing delays along each link l, updated as follows:

$$\lambda_l(t+1) = \left[\lambda_l(t) + \frac{\gamma}{c_l} \left(\sum_{s:l \in L(s)} x_s(t) - c_l\right)\right]^+ \quad (4)$$

where  $\gamma$  is a constant step size, and the term  $\frac{1}{c_l}(\sum_{s \in L(s)} x_s(t) - c_l)$  represents the queuing delay as the ratio between packet backlog and link capacity.

# III. DISTRIBUTED POWER CONTROL ALGORITHM: DERIVATION AND INTERPRETATION

Since throughput performance of TCP Vegas is determined by the underlying utility maximization (1), we pose the following question: can end-to-end throughput over a wireless ad hoc network be increased by solving the utility maximization with elastic link capacities (2)? In the next two sections, we show that the answer is positive, and that throughput, as well as throughput per watt of power transmitted, can be increased by augmenting congestion control with a distributed power control, without modifying the existing TCP algorithms (3,4).

We first specify the form of network utility problem with elastic link capacities (2), using the attainable data rate of a link as its practical 'capacity'. For a wireless CDMA multihop network with logical links indexed by l, the data rates attainable can be written for a large set of modulation techniques as

$$c_l = \frac{1}{T}\log(1 + K \mathrm{SIR}_l)$$

where T is the symbol period, K is a constant depending on the modulation and required bit error rate, and SIR<sub>l</sub> is the signal to interference ratio for link l defined as SIR<sub>l</sub> =  $\frac{P_l G_{ll}}{\sum_{k \neq l} P_k G_{lk} + n_l}$ for a given set of path losses  $G_{lk}$  (from the transmitter on link k to the receiver on link l) and a given set of noises  $n_l$  (for the receiver on link l). For high SIR,  $c_l$  can be approximated as  $\frac{1}{T} \log(K \text{SIR}_l)$ .

Now associate a Lagrange multiplier  $\lambda_l$  for each of the constraints  $\sum_{s \in L(s)} x_s \leq c_l(\mathbf{P})$  in (2). Using the KKT optimality conditions from optimization theory [1], solving problem (2) is equivalent to satisfying the complementary slackness condition (which is satisfied here since the equilibrium queuing delays must be zero if the total equilibrium ingress rates at a router is strictly smaller than the egress link capacity) and finding the stationary points of the Lagrangian  $I_{system}(\mathbf{x}, \mathbf{P}, \boldsymbol{\lambda}) =$  $(\sum_s U_s(x_s) - \sum_l \lambda_l \sum_{s \in L(s)} x_s) + (\sum_l \lambda_l c_l(\mathbf{P}))$ . By the linearity of the differentiation operator, this can be decomposed into two separate maximizations:

$$\begin{aligned} & \text{maximize}_{\mathbf{x} \succeq 0} \qquad \sum_{s} U_{s}(x_{s}) - \sum_{s} \sum_{l \in L(s)} \lambda_{l} x_{s}, \\ & \text{aximize}_{\mathbf{P}_{max} \succeq \mathbf{P} \succeq 0} \qquad I_{power}(\mathbf{P}, \boldsymbol{\lambda}) = \sum_{l} \lambda_{l} c_{l}(\mathbf{P}). \end{aligned}$$

The first maximization is already implicitly solved by the congestion control mechanism [7]. But we still need to solve the second maximization, and use the Lagrange multipliers  $\lambda$  as the shadow prices to allocate exactly the right power to each transmitter, thus increasing the link data rates and reducing congestion at the bottlenecks of the overall network. For scalability, this power control must also be implemented distributively, just like the congestion control part. Since the data rate on each wireless link is a global function of all transmit powers that may cause interference, the power control problem cannot be nicely decoupled into local problems for each link as in the case treated in [9]. However, we show that distributed solution is still feasible, as long as message passing [3] is allowed where an appropriate set of limited information is passed among the nodes.

By taking the derivative of  $I_{power}(\mathbf{P}, \boldsymbol{\lambda})$  with respect to  $P_l$ , we evaluate the *l*th component of the gradient  $\nabla I_{power}$  to be

$$\frac{\lambda_l(t)}{P_l} - \sum_{j \neq l} \frac{\lambda_j(t)G_{jl}}{\sum_{k \neq j} G_{jk} P_k + n_j}.$$

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Consequently, the gradient descent method [1] with a constant step size  $\kappa$  to maximize  $I_{power}(P, \lambda)$ :

$$P_{l}(t+1) = P_{l}(t) + \kappa \nabla I_{power}$$
$$= P_{l}(t) + \kappa \left( \frac{\lambda_{l}(t)}{P_{l}} - \sum_{j \neq l} \frac{\lambda_{j}(t)G_{jl}}{\sum_{k \neq j} G_{jk}P_{k} + n_{j}} \right)$$

can be written (after some simplifications) as the following distributed power control algorithm:

$$P_l(t+1) = P_l(t) + \frac{\kappa \lambda_l(t)}{P_l(t)} - \kappa \sum_{j \neq l} G_{lj} m_j(t)$$
 (5)

where  $m_j$  are messages passed from node j to the transmitter on link l:

$$m_j(t) = \frac{\lambda_j(t) \operatorname{SIR}_j(t)}{P_j(t) G_{jj}}.$$

Therefore, taking in the current values of  $\frac{\lambda_i(t) \text{SIR}_j(t)}{P_j(t)G_{jj}}$  as the messages, the transmitter on link *l* adjusts its power level in the next time slot in two ways: first increases it directly proportional to the current shadow price (*i.e.*, queuing delay in TCP Vegas) and inversely proportional to the current power level, then decreases it by a weighted sum of the messages from all neighbors, where the weights are the path losses. All powers are capped by  $P_{l,max}$  during the iterations. Note that the values of  $\lambda_j$ , SIR<sub>j</sub> and  $P_j$  can be directly measured by node j locally, and the path loss  $G_{jj}$  usually estimated through periodic training sequences.

Intuitively, if the local queuing delay is high, transmit power should increase, with more moderate increase when the current power level is already high. If queuing delays on other links are high, transmit power should decrease in order to reduce interference on those links. The unmodified source algorithm (3) and queue algorithm (4) of TCP Vegas, together with the new power control algorithm (5), form a set of distributed, joint congestion control and resource allocation for wireless ad hoc networks.



Fig. 1. Coupled dynamics of joint congestion and power control.

We conclude this section by emphasizing that as transmit powers change, SIR and thus data rate also change on each link, which in turn change the congestion control dynamics. At the same time, congestion control dynamics change the queuing delays  $\lambda$ , which in turn change transmit powers. Figure 1 shows this nonlinear coupling of supply (regulated by power control) and demand (regulated by congestion control) through the shadow prices  $\lambda$ . As shown in (3) and (5) respectively, a higher price  $\lambda_l$  induces an increase in supply (higher  $P_l$ ) and a decrease in demand (lower  $x_s, \forall s : l \in L(s)$ ).

#### IV. PERFORMANCE AND SIMULATION

In general, distributed gradient descent with a constant step size may not converge [1]. However, we prove that convergence of the coupled nonlinear system as shown in Figure 1 is guaranteed, as long as link data rates do not become arbitrarily small and link queuing delays do not become arbitrarily large. These are reasonable engineering assumptions under any normal network operations, since a link with arbitrarily small data rate is essentially disconnected, and a queue with finite buffer cannot support arbitrarily large queuing delay. We have

Theorem 1: Assuming that transmit powers  $P_l$  are within a range between  $P_{l,min}$  and  $P_{l,max}$  for all links l, and link queuing delays  $\lambda_l$  are upper bounded. For small enough step sizes  $\gamma$  and  $\kappa$ , the distributed power control (5) coupled with TCP Vegas algorithm (3,4) converge to the global optimality of joint congestion control and power control (2).

In addition to convergence guarantee, total utility  $\sum_{s} U_s(x_s)$  for the network with power control can never be smaller than that without power control, simply because by allowing power adaptation, we are optimizing over a larger constraint set. Note that an increase in total utility is not equivalent to a higher total throughput, since  $U_s$  can be any increasing, concave functions of  $x_s$ . However, empirical evidence from simulation shows that in many cases, both throughput and energy efficiency (measured by the total Source rate to total Power Ratio (SPR)) will indeed rise significantly after power control (5) regulates bandwidth supply and the dual variables  $\lambda$  balance supply with demand.

Using TCP Vegas (3,4) and the new power control (5), we simulated the above joint power and congestion control for various wireless multihop networks with different topologies and fading environments. The advantage of such a joint control can be captured even in a small illustrative example, where the network topology and routes for four multi-hop connections are shown in Figure 2. The fading coefficients are determined by the relative distances.



Fig. 2. Network topology and routes for an illustrative simulation example.

Transmit powers, as regulated by the proposed distributed power control, and source rates, as regulated through TCP Vegas window udpate, are shown in Figure 3. The initial conditions of the graphs are based on the equilibrium states of TCP Vegas with fixed power levels. With power control, it can be seen that transmit powers  $\mathbf{P}$  distributively adapt so as to induce a favorable capacity and queuing delay configurations on the overall network, which in turn lead to increases in end-toend throughput as indicated by the rise in all the allowed source rates. Notice that some link capacities actually decrease while the capacities on the bottleneck links rise to maximize the total utility. This is achieved through a distributive adaptation of powers, which lowers the power levels that cause most interference on the bottleneck links. Confirming our intuition, a 'smart' allocation of power tends to reduce the spread of queuing delays, thus preventing any link from becoming the bottleneck. As expected from distributed gradient methods, convergence to equilibrium can be accelerated by using a larger constant step size  $\kappa$ , at the expense of wider variances around equilibrium. This is shown in the top right graph in Figure 4.

We indeed achieve what this co-design across physical and transport layers is aiming at. The end-to-end throughput per watt of power transmitted (*i.e.*, SPR) is 82% higher with power control. A series of simulations are conducted based on different fading environments and TCP Vegas parameter settings, where we see that power control (5) increases TCP throughput and SPR in all experiments, and in 78% of the instances, energy efficiency rises by 75% to 105%, compared to TCP without power control. Power control and congestion control, each running distributively and coordinated through the dual variables of queuing delay, work together to increase the energy efficiency of multi-hop transmission across the entire network.



Fig. 3. Performance of joint congestion control and power control.

#### V. ROBUSTNESS AND COMPLEXITY

In this section, we present results on three issues related to the practical implementation of the joint congestion and power control algorithms in a wireless multihop network:

- 1) Effect of wrong estimates of path losses at various nodes.
- Effect of packet loss due to wireless channel outage during deep fading.
- Reduction of implementation complexity through partial message passing.

First, it is assumed in our power control algorithm (5) that the path losses  $G_{ij}$  are perfectly estimated by the receiver, and  $G_{ii}$  are perfectly estimated by the sender and receiver. While training sequence can help accurately estimate the  $G_{ii}$  factors, the assumption of perfect tracking of the  $G_{ij}$  factors is stronger than most practical systems can support. It is interesting to know how much error in the estimation of  $G_{ij}$ , or fluctuations in  $G_{ij}$  themselves, can be tolerated while maintaining the convergence of joint power control and TCP congestion control.

Denoting the error in the estimation of  $G_{ij}$  at time t as  $\Delta G_{ij}(t)$ , and suppressing the time index on  $\lambda(t)$ ,  $\mathbf{P}(t)$ , SIR(t),  $\Delta G_{ij}(t)$ , we provide a sufficient condition in the following

Corollary 1: Convergence to global optimality of (2) through (3,4,5) is maintained if there exists a T such that for all times  $t \ge T$ , the following inequality holds:

$$\sum_{l} \sum_{j \neq l} \sum_{k \neq l} (G_{jl}G_{kl} - \Delta G_{jl}\Delta G_{kl}) \frac{\lambda_j \lambda_k \operatorname{SIR}_j \operatorname{SIR}_k}{G_{jj}G_{kk}P_jP_k}$$
  
>  $2\sum_{l} \sum_{j \neq l} \frac{\lambda_l \lambda_j G_{jl}}{P_l P_j G_{jj}} \operatorname{SIR}_j - \frac{\lambda_l^2}{P_l^2}.$ 

While Corollary 1 gives a test of convergence under wrong estimates of  $G_{ij}$  for any network, empirical experiments can be carried out in simulations where the  $G_{ij}$  factors in (5) are perturbed randomly within a range. The result of one such experiment is shown in the bottom left graph in Figure 4, where the  $G_{ij}$  factors are generated at random between +25% and -25% of their true values. Compared to the baseline throughput results shown in the top left graph, we see that the algorithms converge to the same global optimality after a longer and wider transient period. A series of such experiments show that convergence is maintained when the estimated  $G_{ij}$  are within 30% of their true values, which suggests that the algorithms are robust within reasonable bounds.

Another peculiar feature of wireless transmissions is that during deep fading, SIR on a link may become too small for correct decoding at the receiver. This channel outage induces packet loss on the link. Consequently the queue buffer size becomes smaller than it should have been. Since queuing delay is implicitly used as the dual variable in TCP Vegas, such channel variations lead to incorrect values of the dual variables. Sources will mistake the decrease in a total queuing delay as an indication of reduced congestion level, and increase their source rates through TCP update accordingly. Having incorrect pricing on the wireless links may thus prevent the joint system from converging to optimality.

Similar to Corollary 1, we have the following sufficient condition for convergence, with outage induced packet loss on link l denoted as  $\Delta y_l$ :

Corollary 2: Convergence to global optimality of (2) through (3,4,5) is maintained if there exists a T such that for all times  $t \ge T$ , the following inequality holds:

$$\sum_{l} \left[ \frac{1}{P_{l}^{2}} \left( \lambda_{l}^{2} - \left( \frac{\Delta y_{l}}{c_{l}} \right)^{2} \right) + \sum_{j} \left( \frac{G_{jl} \mathbf{SIR}_{j}}{G_{jj} P_{j}} \right)^{2} \left( \lambda_{j} - \left( \frac{\Delta y_{j}}{c_{j}} \right)^{2} \right) \right]$$
$$> 2 \sum_{l} \sum_{j \neq l} \left( \lambda_{j} \lambda_{l} - \frac{\Delta y_{l} \Delta y_{j}}{c_{l} c_{j}} \right) \frac{G_{jl} \mathbf{SIR}_{j}}{G_{jj} P_{l} P_{j}}.$$



Fig. 4. Robustness of joint power control and TCP Vegas. Top left case is the baseline performance of the four end-to-end throughput. Top right case shows that a larger step size in the algorithm accelerates convergence but also leads to larger variance. Bottom left case shows that the algorithm is robust to wrong estimates of fading coefficients. Bottom right case shows robustness against packet losses on links with wireless channel outage.

Because the chance of having channel outage simultaneously at all links is small, it is reasonable to expect that only few of  $\Delta y_l$  are nonzero at any time. We again empirically experiment with channel outage induced packet loss at various links, and a typical result is shown in the bottom right graph in Figure 4, where the underlying outage probability is 20%. For the topology in Figure 2, we find that the algorithms are robust up to 25% of outage probability, a level that most systems rarely exceed.

Another issue concerning the practical implementation of the joint power control and TCP congestion control is on trading-off performance optimality with implementation simplicity. The increases in TCP throughput and energy efficiency has been achieved with a rise in the communication complexity of message passing and the computational complexity of power update. Although the shadow prices  $\lambda$  efficiently coordinate bandwidth demand and supply, there can still be many terms in the  $\sum_{j \neq l} G_{lj} m_j$  sum in (5) as the number of nodes increases. Fortunately, those transmitters far away from transmitter j will be correspondingly multiplied by a smaller  $G_{lj}$ . Their messages  $m_j$  will therefore be given smaller weights in the power update.

This leads to a simplified power control algorithm, where only messages from a set  $J_l$  of other transmitters are passed to the transmitter on link l. Naturally, if there are M elements in set  $J_l$ , they should correspond to nodes with the M largest  $G_{lj}$ toward node l. Power update equation now becomes:

$$P_{l}(t+1) = P_{l}(t) + \frac{\kappa \lambda_{l}(t)}{P_{l}(t)} - \kappa \sum_{j \in J_{l}} G_{lj} m_{j}.$$
 (6)

The reduction in complexity is measured by the ratio  $\Delta \text{COM} = \frac{\sum_{l} |J_{l}|}{N(N-1)}$  where N is the total number of transmitters in the network. Obviously,  $0 \le \Delta \text{COM} \le 1$ , and a smaller  $\Delta \text{COM}$  represents a simpler and less optimal message passing and power update. Similar to Corollary 1, the following sufficient condition of convergence with the simplified algorithm (6) can be shown:

Corollary 3: Convergence to global optimality of (2) through (3,4,6) is maintained if there exists a T such that for all times  $t \ge T$ , the following inequality holds:

$$\sum_{l} \sum_{j \in J_l} \left( \frac{G_{jl} \lambda_j \operatorname{SIR}_j}{G_{jj} P_j} \right)^2 > 2 \sum_{l} \sum_{j \neq l} \frac{\lambda_l \lambda_j G_{jl}}{P_l P_j G_{jj}} \operatorname{SIR}_j - \frac{\lambda_l^2}{P_l^2}.$$

## VI. CONCLUSIONS

We present a distributed power control algorithm that works together with the original TCP algorithms to significantly increase end-to-end throughput and energy efficiency of multihop transmission in wireless ad hoc networks. No modification to current TCP protocols is needed to achieve the optimal balancing between bandwidth demand (regulated through congestion control) and supply (regulated through power control). We prove that the coupled nonlinear system converges to global optimality of the joint congestion control and power control problem. The proposed algorithms are robust to wireless channel variations and errors in fading coefficient estimations. Slightly suboptimal but much simplified versions of the algorithms are also presented for scalable architectures. Further study is being carried out to investigate the transient properties of the new algorithms, such as the rate of convergence.

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