Opportunistic Power Control for Throughput Maximization in Mobile Cellular Systems

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***Abstract*— When considering mobile communication systems we see that for services which can tolerate delay do not need to maintain a minimum signal-to-interference ratio (SIR). In fact, to achieve maximum throughput transmission power should be increased when the interference level is low, and information transmission rate adjusted accordingly through adaptive modulation and coding. This approach is called opportunistic communications. In this paper, we introduce an opportunistic power control algorithm, which exploits the channel fluctuation of wireless channels. The algorithm is distributed and is proven to converge to a unique fixed point. Simulation results show that a significant increase in system capacity can be achieved, when compared with the traditional target tracking approach.**

I. Introduction

In the literature, power control is mainly used to maintain a pre-define signal-to-interference ratio (SIR) (e.g. [2], [5], [8], [11]. This approach is called *target tracking* and is suitable for real-time applications like mobile phone services. In view of the proliferation of wireless data, it is necessary to re-examine the power control problem. In [3], the problem is formulated as a non-cooperative game with data and voice having different utility functions. Other attempts are made in [9], [10], in which different utility functions are proposed. These works relax the requirement of maintaining a target SIR. However, it remains unclear how system capacity can be increased, especially for multi-cell systems.

The objective of this paper is to increase system throughput by means of power control. As data can tolerate a much larger delay than voice, the fluctuation of wireless channels can be exploited to maximize overall system throughput. In this regard, power control becomes a valuable tool. It can be used to support *opportunistic communications*, in the sense that a user increases his transmit power whenever the gain of his channel is large or the interference at his receiver is low. This idea is related to the work by Knopp and Humblet [4], which shows that the system throughput of a single-cell CDMA system is maximized if only one user transmits at a time; the one with largest instantaneous channel gain should transmit.

In this paper, we propose a new paradigm called *opportunistic power control*. Instead of compensating for fading, we do just the opposite. The capacity gain is huge; our simulation results show that the throughput can be increased by twenty times when compared with target tracking. This is made possible by the novel design of a new distributed power control algorithm, which is proven to converge for any given link gain matrix.

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II. Opportunistic Power Control

Consider a wireless system with N mobile users. Let $\boldsymbol{p} = (p_1, p_2, \ldots, p_N)$, where p_i is the transmit power of user i. Each user determines his own power in order to maximize his utility function, $u_i(\boldsymbol{p})$. In general, the utility of a user depends on the signal-to-interference ratio (SIR). Denote the link gain between transmitter j and receiver i by G_{ij} and the noise power at receiver i by η_i; then the SIR of user i, Γ_i is defined as

$$\Gamma_i = \frac{G_{ii} p_i}{\sum_{j \neq i} G_{ij} p_j + \eta_i}. \tag{1}$$

To maximize his throughput, a user should transmit as large his power as possible, since Γ_i is an increasing function of p_i. On the other hand, energy is a valuable resource for mobile devices. To save energy, one would like to minimize one's power consumption. To model this conflicting objective, we propose the following utility function:

$$u_i = \Gamma_i^{\beta_i/2} - \lambda_i p_i, \tag{2}$$

where β_i and λ_i are tunable parameters satisfying $0 < \beta_i \leq 1$ and $\lambda_i > 0$. These two terms represent respectively the user satisfaction and the cost of user i.

Assuming the power of other users are fixed, the optimal power of user i, p_i^*, can be obtained by differentiating u_i with respect to p_i. Thus, we have

$$p_i^* = \left(\frac{\beta_i}{2\lambda_i}\right)^{\frac{2}{2-\beta_i}} R_i^{-\frac{\beta_i}{2-\beta_i}}, \tag{3}$$

where

$$R_i = (\sum_{j \neq i} G_{ij} p_j + \eta_i)/G_{ii} \tag{4}$$

is called the *effective interference* of user i.

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Consider the following iterative power control algorithm. At each iteration, each user sets his power to the optimal value p_i^*, assuming that the power of other users are fixed, that is:

$$\boldsymbol{p}^{(n+1)} = \boldsymbol{I}(\boldsymbol{p}^{(n)}), \tag{5}$$

where $\boldsymbol{I}(\boldsymbol{p}) = (I_1(\boldsymbol{p}), I_2(\boldsymbol{p}), \ldots, I_N(\boldsymbol{p}))$ and

$$I_i(\boldsymbol{p}) = \frac{c_i}{[R_i(\boldsymbol{p})]^{\phi_i}}. \tag{6}$$

Here we have simplified the notation by introducing two variables:

$$c_i = \left(\frac{\beta_i}{2\lambda_i}\right)^{\frac{2}{2-\beta_i}} \tag{7}$$

and

$$\phi_i = \frac{\beta_i}{2-\beta_i}. \tag{8}$$

Since $0 < \beta_i \leq 1$, we have $0 < \phi_i \leq 1$.

Note that the power update of user i depends only on the effective interference of user i, which is locally available at the base station serving user i. Hence it can be implemented in a distributed way. Alternatively, (6) can be expressed as

$$I_i(\boldsymbol{p}) = c_i\left(\frac{\Gamma_i(\boldsymbol{p})}{p_i}\right)^{\phi_i}. \tag{9}$$

Therefore, it belongs to the class of SIR-based power control. Furthermore, we classify it as *opportunistic* according to the following:

Definition 1. *A power control algorithm for user i is said to be opportunistic if the iterative function, $I_i(\boldsymbol{p})$, is a decreasing function of the effective interference, $R_i(\boldsymbol{p})$.*

III. A New Framework

Our proposed algorithm does not fall into Yates' framework [11], since the iterative function is not *standard*. In [7], a key condition for convergence of an iterative function is identified:

- *Two-sided Scalability:* For all $\alpha > 1$, $\frac{1}{\alpha}\boldsymbol{p} \leq \boldsymbol{p}' \leq \alpha\boldsymbol{p}$ implies $\frac{1}{\alpha}\boldsymbol{I}(\boldsymbol{p}) < \boldsymbol{I}(\boldsymbol{p}') < \alpha\boldsymbol{I}(\boldsymbol{p})$.

A point $\boldsymbol{p}$ is said to be a fixed point of $\boldsymbol{I}(\boldsymbol{p})$ if $\boldsymbol{p} = \boldsymbol{I}(\boldsymbol{p})$. A two-sided scalable iterative function has the following property:

Theorem 1. *Given a two-sided scalable iterative function, if a fixed point $\boldsymbol{p}^*$ exists, then that fixed point is unique and the power vector $\boldsymbol{p}^{(n)}$ converges to $\boldsymbol{p}^*$ under both synchronous and totally asynchronous update model.*

The proof of this theorem is lengthy and is omitted here (see [7]). Note that this result is more general than Yates's, since a standard iterative function is two-sided scalable, but the converse is not true.

Theorem 1 readily applies to our case, since it is straightforward to show that the iterative function defined by (9) is two-sided scalable. Furthermore, the iterative function can be proven to have a fixed point by means of Brouwer's fixed-point theorem:

Theorem 2 (Brouwer's fixed-point theorem [1]). *Let $Z \subseteq \mathbb{R}^m$ be compact and convex and $F : Z \to Z$ a continuous function. Then there exists $z \in Z$ such that $z = F(z)$.*

Theorem 3. *A fixed point of $\boldsymbol{I}(\boldsymbol{p})$ always exists.*

Proof: For each user i, denote the closed interval $[0, c_i(G_{ii}/\eta_i)^{\phi_i}]$ by $\mathcal{R}_i$. Let $\mathcal{R}$ be the Cartesian product $\mathcal{R}_1 \times \mathcal{R}_2 \cdots \times \mathcal{R}_N$. Note that $\boldsymbol{I}(\boldsymbol{p}) : \mathcal{R} \to \mathcal{R}$ is a continuous function and $\mathcal{R}$ is compact. By Browser's fixed-point theorem, a fixed point exists. □

As a consequence, the proposed algorithm has the following property:

Theorem 4. *$\boldsymbol{I}(\boldsymbol{p})$ defined in (6) has a unique fixed point, $\boldsymbol{p}^*$. Furthermore, given any initial power vector, $\boldsymbol{p}^{(n)}$ converges to $\boldsymbol{p}^*$.*

The following result presents a property of the fixed point:

Theorem 5. *At the fixed point $\boldsymbol{p}^*$, we have*

$$\Gamma_i(\boldsymbol{p}^*) = \frac{c_i}{[R_i(\boldsymbol{p}^*)]^{1+\phi_i}}$$

for all i.

Proof: If $\boldsymbol{p}^*$ is a fixed point, (6) implies that

$$p_i^* = \frac{c_i}{R_i(\boldsymbol{p}^*)^{\phi_i}}. \tag{10}$$

The statement follows by dividing both sides by R_i. □

$\Gamma_i(\boldsymbol{p}^*)$ may be interpreted as the target SIR of user i. Theorem 5 then implies that the target adjusts automatically in response to changes in effective interference. This is in contrast to target tracking, which assumes a fixed target SIR.

Besides, the target SIR is inversely proportional to $R_i^{1+\phi_i}$, where ϕ_i increases when β_i increases according to (8). Hence, for large values of β_i, the target SIR is more sensitive to changes in interference. In other words, the communication is more "opportunistic", as will be shown by simulation.

IV. Simulation Results

In this section, we compare the performance of opportunistic power control and target tracking For target tracking, we employ the algorithm proposed by Foschini and Miljanic [2]:

$$p_i^{(n+1)} = \frac{\gamma_i}{\Gamma_i(\boldsymbol{p}^{(n)})} p_i^{(n)}. \tag{11}$$

First of all, we use a simple two-user single-cell system to illustrate the idea of opportunistic communications. The two users are located at a distance of 0.55 and 0.45 from the base station. Path loss exponent is assumed to be 4 while shadow fading is ignored. The large-scale link gain is given by C/d^4, where C is a constant equal to 0.001, and d is the distance of the mobile from the serving base station. Both users experience Rayleigh fading. Flat Doppler spectrum with maximum Doppler frequency of 20 Hz is assumed, which models a typical indoor wireless environment Noise power is assumed to be 10^{-4}. We simulate the system for 1,000 power control iterations. Each iteration corresponds to 1 msec.

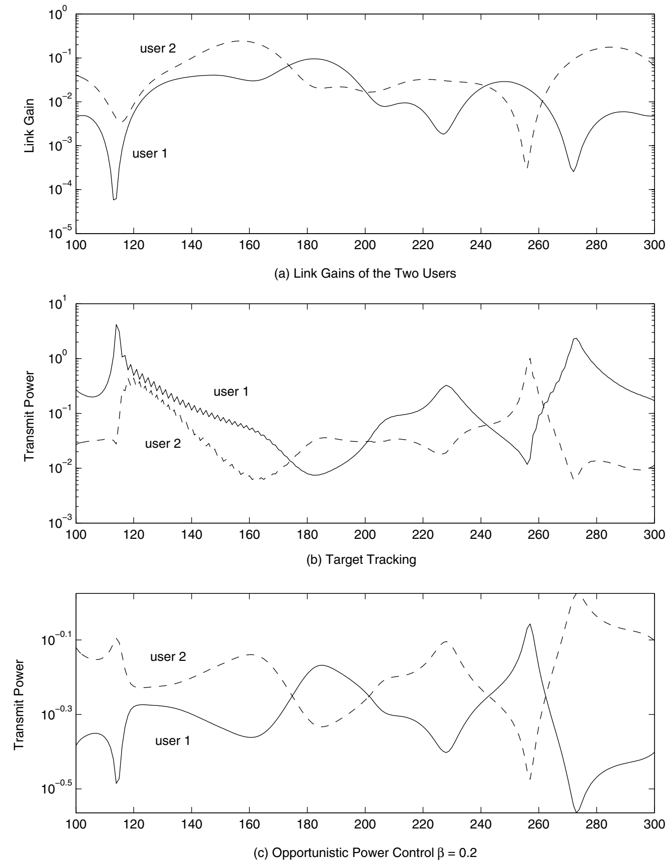

Fig. 1. Evolution of power levels in a two-user system – a comparison between target tracking and opportunistic power control ($\beta = 0.2$).

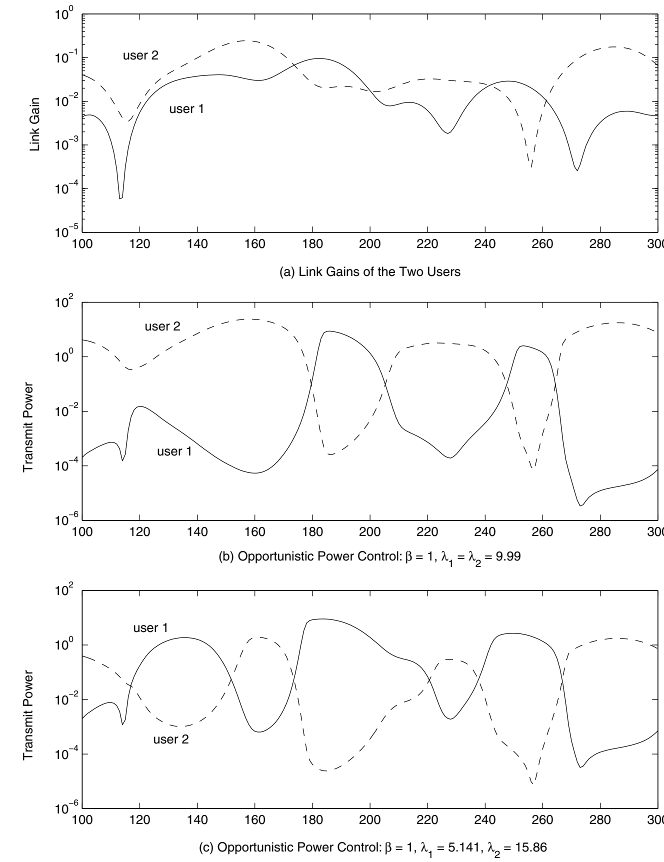

Fig. 2. Evolution of power levels in a two-user system – opportunistic power control with $\beta = 1$.

Figure 1 shows the change of link gains and the power evolution of the two users from iteration 100 to 300. A comparison between target tracking and opportunistic power control ($\beta = 0.2$) is made. For target tracking, we set $\gamma_t = 0.9$ for both users. Since the aim of target tracking is to maintain a constant SIR, the power levels of the users are somehow reciprocal of their corresponding link gains. In contrary, opportunistic power control do just the opposite. Roughly speaking, a user transmits less power when he experiences a deep fade. He transmits more when his link gain is relatively large. This is the essence of opportunistic communications.

Table I summarizes the average performances of target tracking and opportunistic power control. C_1 and C_2 are the throughputs of user 1 and 2 respectively. The total throughput, C_T, is equal to $C_1 + C_2$. Similarly, p_1 and p_2 are the average transmit powers of user 1 and user 2, whereas $p_T = p_1 + p_2$. For opportunistic power control, four different sets of parameters are considered. We first focus on case (a), where $\beta = 0.2$. We set $\lambda_1 = \lambda_2 = 0.178$ so that it consumes the same amount of power as target tracking. With this setting, the total throughput obtained by opportunistic power control is 78.3% greater than that obtained by target tracking – a very significant gain.

Next we set $\beta = 1$. The evolution of power levels is shown in Figure 2. We first consider Figure 2(b), in which both users have the same pricing factor. This corresponds to case (b) in Table I. As in Figure 1(c), the transmission is opportunistic. However, the major difference between these two cases is that with $\beta = 1$, the power level difference between the two users is generally much larger. In other words, at any time, one user transmits at very high power while the other one transmits at very low power. This behavior approximates closely the theoretically optimal result in [4]. From Table I, the total throughput is increased by 119%, when compared with case (a). It implies that using a larger value of β is more effective in maximizing system throughput.

In this example, the throughput of the two users are 1.140 and 6.572 respectively. Since the first user is farther away from the base station, his throughput is much lower. This phenomenon is also identified in [10] and is termed *near-fair unfairness*.

The opportunistic power control algorithm provides flexibility for a system to achieve fairness. User i can increase his throughput by decreasing his pricing factor, λ_i. Figure 2(c) shows the outcome when λ_1 and λ_2 are changed to 5.141 and 15.86 respectively. This corresponds to case (c). The reason of choosing these parameters is to keep the total power consumption the same as in case (a) and (b) and to equalize

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TABLE I

PERFORMANCE RESULTS IN THE TWO-USER EXAMPLE.

Algorithms and their Parameters	C_1	C_2	C_T	p_1	p_2	p_T
Target Tracking ($\gamma_t = 0.9$)	0.987	0.987	1.974	0.947	0.210	1.157
(a) Opportunistic ($\beta = 0.2, \lambda_1 = \lambda_2 = 0.178$)	0.650	2.870	3.520	0.489	0.668	1.157
(b) Opportunistic ($\beta = 1, \lambda_1 = \lambda_2 = 9.99$)	1.140	6.572	7.712	0.115	1.042	1.157
(c) Opportunistic ($\beta = 1, \lambda_1 = 5.141, \lambda_2 = 15.86$)	3.350	3.350	6.700	0.900	0.257	1.157
(d) Opportunistic ($\beta = 1, \lambda_1 = \lambda_2 = 5$)	1.410	8.068	9.478	0.423	4.105	4.528

the throughput of user 1 and 2. It can be seen that user 1 is now more aggressive in transmitting his signal. About half of the time user 1 transmits at a larger power than user 2. The throughput of the two users now become the same, both of them equal to 3.35. As a result, the total throughput drops by 13.1%. Since the power consumption is kept constant, we can see that fairness is achieved at the expense of system throughput in this example. Compared with target tracking, a much larger throughput is obtained.

From the example, we can see that the pricing factor governs the desire of a user to transmit at a high power. Therefore, it may be used to provide a tradeoff between throughput and power consumption. Suppose in the example, both λ_1 and λ_2 are changed from 9.99 to 5. This corresponds to case (d). The power evolution is similar to Figure 2(b) and is ignored. From Table I, the total throughput increases by 22.9% at the expense of an increase of 291% in power consumption.

Finally we evaluate the capacity improvement by opportunistic power control. We assume that seven users are stationary and randomly located in a 2×2 square cellular system. Ten different scenarios are generated. In each scenario, the large-scale component of each link gain between a mobile and a base station is subject to independent path loss and shadow fading. Path loss exponent is assumed to be 4 and shadow fading component is log-normally distributed with standard deviation equal to 6 dB. A user is assumed to connect to the base station that have greatest large-scale link gain. On top of the large-scale effect, we assume that each link gain has a small-scale component due to Rayleigh fading. As before, Flat Doppler spectrum with maximum Doppler frequency of 20 Hz is assumed. For each scenario, we simulate the system for 10,000 power control iterations. The performance result is averaged over the ten scenarios.

Figure 3 shows the power consumption against total throughput for target tracking and opportunistic power control. For target tracking, the curve is obtained by adjusting the target SIR, γ_t. The larger the value of γ_t, the larger the throughput but the more the power is consumed. For opportunistic power control, we consider two cases: $\beta = 0.2$ and $\beta = 1$. In each case, we adjust the pricing factor, λ. The larger the value of λ, the less the power is consumed and the smaller the throughput.

From the graph, we see that opportunistic power control dramatically increases the system capacity. For the same power consumption, the throughput is increased by more than

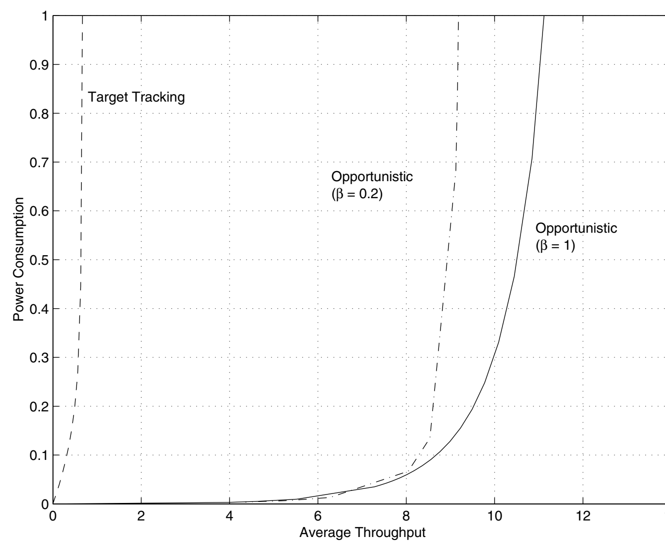

Fig. 3. Power consumption against average throughput

20 times. Furthermore, a larger value of β yields a higher throughput, which agrees with our previous result.

V. DISCUSSION AND CONCLUSION

We propose an opportunistic power control algorithm for mobile cellular systems. It is flexible in the sense that it provides tunable parameters to have tradeoff between throughput, power consumption and fairness. The algorithm is proven to converge to a unique fixed point when the link gains are fixed. We show by simulation that it is able to increase system capacity by an order of magnitude when compared with target tracking. We conclude that a tremendous gain in capacity can be obtained by exploiting the fluctuation of wireless channels.

The power control algorithm does not depend on the stochastic model of the link gains. Hence it can be applied to the case where mobility is included. Furthermore, it can be extended to include soft handoff and maximum power constraints by modifying the iterative function. It can be shown that with these modifications, the algorithm still converges and possesses the opportunistic property. Details can be found in [6].

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