Demo

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Abstract

Xcas + TeXmacs are presented to demonstrate their interaction. Watch out how images are created by Xcas and inserted into TeXmacs.

Exercise: For the function $e^x \ln(1 + e^{-x})$ do the following:
(a) Compute its limit as $x \to +\infty$;
(b) Plot the function and explain why the plot becomes 0 at $x \approx 33$;
(c) Compute the machine-epsilon of your computer.

Solution: We start a session with Giac from Texmacs and we have:

a) The limit is 1 as can be seen from the result below:

```
> limit(exp(x)*ln(1+exp(-x)),x,+infinity)
```

$$1$$

From the value of the limit we can deduce that the graph of the function must be approaching 1 as $x \to +\infty$. (Note that at $x = 0$ the value of the function is $\ln(2) \simeq 0.69$.) In other words we expext a horizontal line below $y = 1$, all the way to infinity.

b) The plot of the function is obtained using the function plotfunc of giac; however, from giac the plot is exported as .jpg, .png, .eps, and .pdf on the desktop. From there we insert it back in the TexMacs. Pick the dimensions of the graph carefully so that it fits in the text.

```
> plotfunc(exp(x)*ln(1+exp(-x)),x,0,40,0.01)
```

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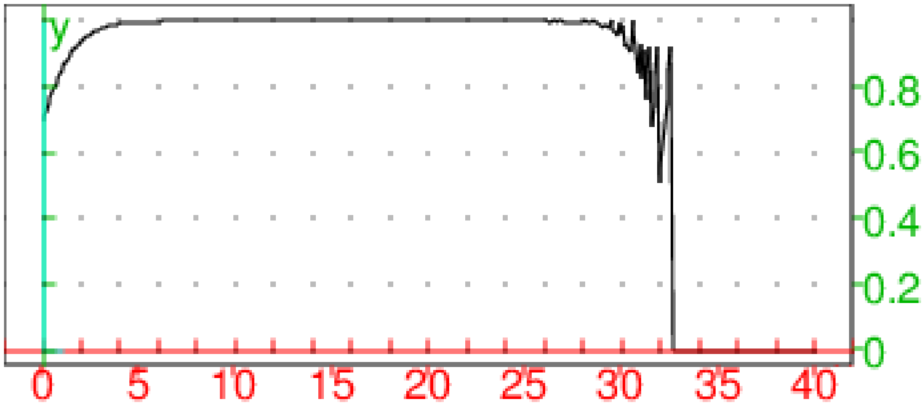

We see that instead oh getting a horizontal line at $y = 1$, all the way to infinity, the function becomes zero at $x \approx 33$. This means that the whole expression becomes 0, which implies that $\ln(1 + \exp(-x))$ becomes $\ln(1 + 0) = \ln(1) = 0$. Recall that the **machine ϵ** is the smallest ϵ such that $1 + \epsilon \neq 1$, which implies that $1 + \frac{\epsilon}{2} = 1$. Therefore, the only way the equation $\ln(1+\exp(-x)) = 0$ holds, is if $\exp(-x) = \frac{\epsilon}{2}$. One must not confuse the ϵ defined in Xcas, which is $\epsilon = 10^{-6}$, with the machine ϵ!

c) So we have to first find that machine ϵ! Once we have it, then the folowing equation will be true:

$$\exp(-x) = \frac{\epsilon}{2} \tag{1}$$

or

$$x = -\ln\left(\frac{\epsilon}{2}\right) \tag{2}$$

and we will be able to get the value of x.

To compute the machine ϵ we run the following little program (found in wikipedia) either in Xcas or as shown below in TexMacs:

```
eps:=1;eps:=0.5*eps; epsp1:=eps+1;
while(epsp1>1)eps:=0.5*eps;epsp1:=eps+1;
```

It computes $\frac{\epsilon}{2}$,

```
> eps:=1;eps:=0.5*eps; epsp1:=eps+1;
  while(epsp1>1){eps:=0.5*eps;epsp1:=eps+1};
```

$$1, 0.500000, 1.500000, 1.000000$$

which is then doubled to obtain the machine ϵ:

```
> eps:=2*eps
```

$$0.000000$$

Indeed, the `eps` so computed satisfies the definition of machine ϵ:

```
> 1 + eps > 1
```

$$true$$

and

```
> 1 + eps/2 > 1
```

$$false$$

The answer is printed as 0 because ϵ is a very small quantity. Therefore, to see its value, we have to multiply ϵ times a very large number. (Start with 10^{20} and work your way down to 10^{14}.)

```
> eps*10^14
```

$$0.710543$$

This means that $\epsilon = 0.7105427 * 10^{-14}$ and from (2) we see:

```
> x=-ln(0.7105427*10^(-14)/2)
```

$$x = 33.271065$$

That is, $x \approx 33$, as deduced from the plot!