Κυματομηχανική

Tutorial Sheet 1
Linear Wave Theory

1. A deep water design wave has a period of T = 15s and a wave amplitude of a = 10m. Calculate the following quantities:
   (i) The phase velocity, c
   (ii) The maximum water particle velocity, u_{max}
   (a) at $y = 0$
   (b) at $y = \eta_{max}$
   (iii) The maximum unsteady water particle acceleration, $\left(\frac{\partial u}{\partial t}\right)_{max}$
   (iv) The maximum horizontal displacement of any fluid particle.

2. If the wave described in Question 1 propagates into a region where the water depth is 30m, calculate the local values of:
   (i) The wave frequency, ω.
   (ii) The wave number, k.
   (iii) The wave height, H.

3. If ak, where a is the wave amplitude and k the wave number, is used as a measure of the wave steepness, calculate its value for the waves described in Questions 1 and 2. Explain:
   (i) Why has it changed?
   (ii) What is the engineering relevance of this result?
   (iii) What is the limiting value of ak, and to what does it correspond?

4. If a deep water wave energy device is able to extract 5% of the transmitted wave energy, calculate the power output per unit width if the typical wave conditions corresponds to a wave period of T = 10s and a wave amplitude of a = 5m.

5. The pressure fluctuations recorded on the sea bed in a water depth of *d* = 10m have a period of *T* = 6s and an amplitude of:

$$\frac{P_{\max} - P_{\min}}{2} = 10KN/m^2$$

   Calculate the maximum horizontal velocity occurring just above the sea bed.

6. Obtain an expression for the horizontal gradient of the wave-induced pressure in finite depth ($\partial p/\partial x$) and show that this exactly balances the product of the density and the unsteady water particle acceleration ($\rho\partial u/\partial t$). Why is this so? What assumptions are necessary for this to be true?

7. Using linear wave theory show that in deep water the convective components of acceleration provide no net contribution to the total horizontal acceleration. Is this result applicable to all water depths?