What is applied mathematics?

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English class for scientists

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What is the math ?

Definitions from encyclopaedias

Speaker has found several definitions :

- Mathematics is a broad-ranging field of study in which the properties and interactions of idealized objects are examined.
- Mathematics is the body of knowledge centered on concepts such as quantity, structure, space, and change, and also the academic discipline that studies them.
- *by Bertrand Russel* : The subject in which we never know what we are talking about nor whether what we are saying is true.

We can conditionally divide mathematics into two parts : **pure** and **applied**

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Pure mathematics

Brief overview

- Definition : Pure mathematics is mathematics motivated entirely for reasons other than application.
- This term comes from the mid-nineteenth century when specialisation and professionalisation started to make a rift more apparent.
- Pure math entered in secondary education in France under the pressure of Bourbaki group
  - formalized teaching style of abstract notions $\longrightarrow$ not accessible to all children
  - no connection with practice $\longrightarrow$ false conslusion : math is « impractical »
  - Result : almost no one wants to study it !

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Purism phenomenon

Godfrey Harold Hardy's : A Mathematician's Apology, 1940

- Hardy considered applied mathematics to be « ugly » and « dull »
  - pure math : « has permanent aesthetic value »
  - applied math : « the dull and elementary parts of mathematics »
- He often compared pure math to painting and poetry
- Hardy considered some physicists, such as Einstein and Dirac, to be among the "real" mathematicians because general relativity and quantum mechanics are "useless"

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Applied mathematics

Definition and main characteristics

Definition

Branch of mathematics that concerns itself with the mathematical techniques typically used in the application of mathematical knowledge to other domains.

- The most important applications in natural scinces (physics) and engineering
- Creation of new areas :
  - **game theory**, which grew out of economic considerations
  - **neural networks**, which arose out of the study of the brain in neuroscience
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Workflow of an applied mathematician

Three important steps of mathematical modeling

1. Define physical problem
2. Use fundamental, phenomenological or empirical laws to construct mathematical model (the system of equations)
   - choose only important parameters
3. Scientific computing : use numerical methods to solve obtained equations

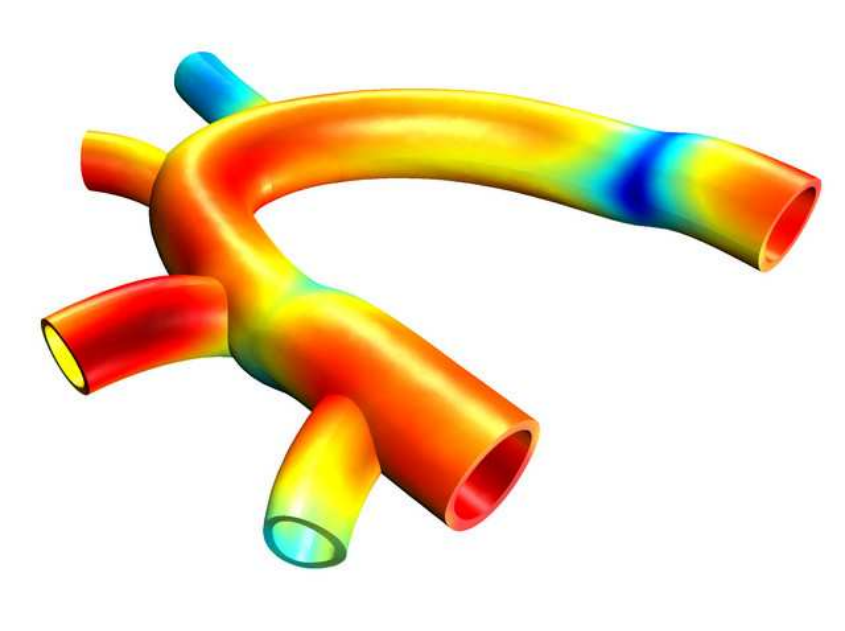
Of course : Analyse critically obtained results !

Outline

Blood vessel

- This model refers to a portion of the vascular system of a young child - the upper part of the aorta artery
- During the flow of blood, pressure is applied to the internal surfaces producing deformation of the vessel walls

- The complete analysis consists of two distinct but coupled procedures :
  - a fluid-dynamics analysis with the calculation of the velocity field and pressure distribution in the blood
  - the mechanical analysis with the deformation of the tissue and artery



Turbofan engine modeling

The modeling of mixing and cooling of the exhaust is important for several reasons :

- It can significantly reduce engine noise levels
- the thermal signature of the exhaust is reduced (military applications)
- the lowering of the temperature helps prevent the overheating of the system

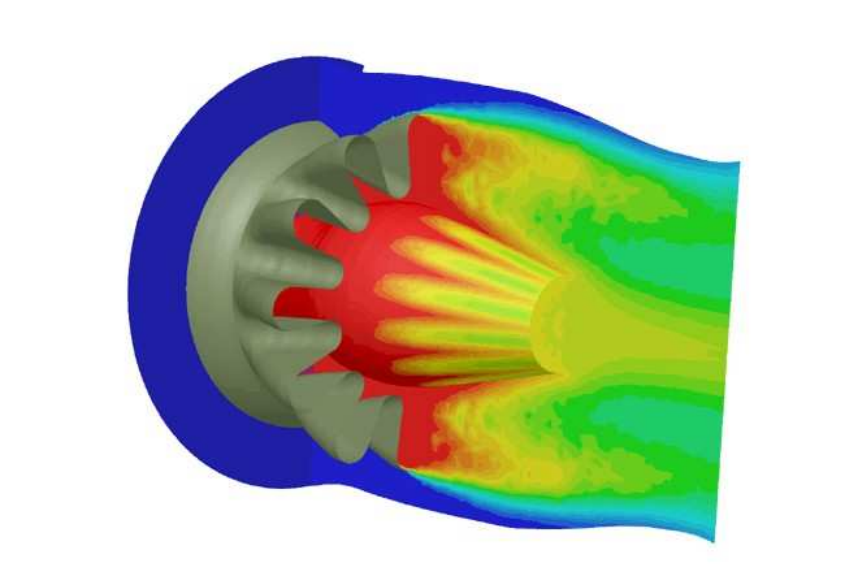


FIG.: Temperature in the mixing zone

Unmanned Aerial Vehicle (UAV) CFD Analysis

CFD analysis is used in the design process to help reduce design cycles

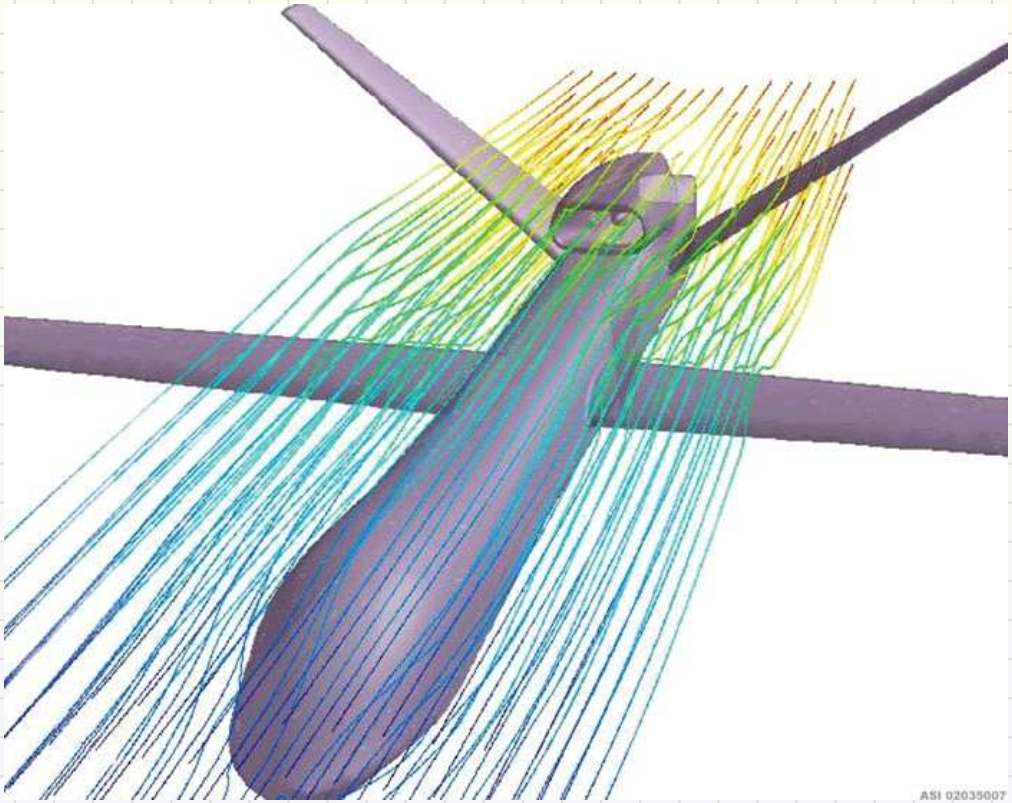
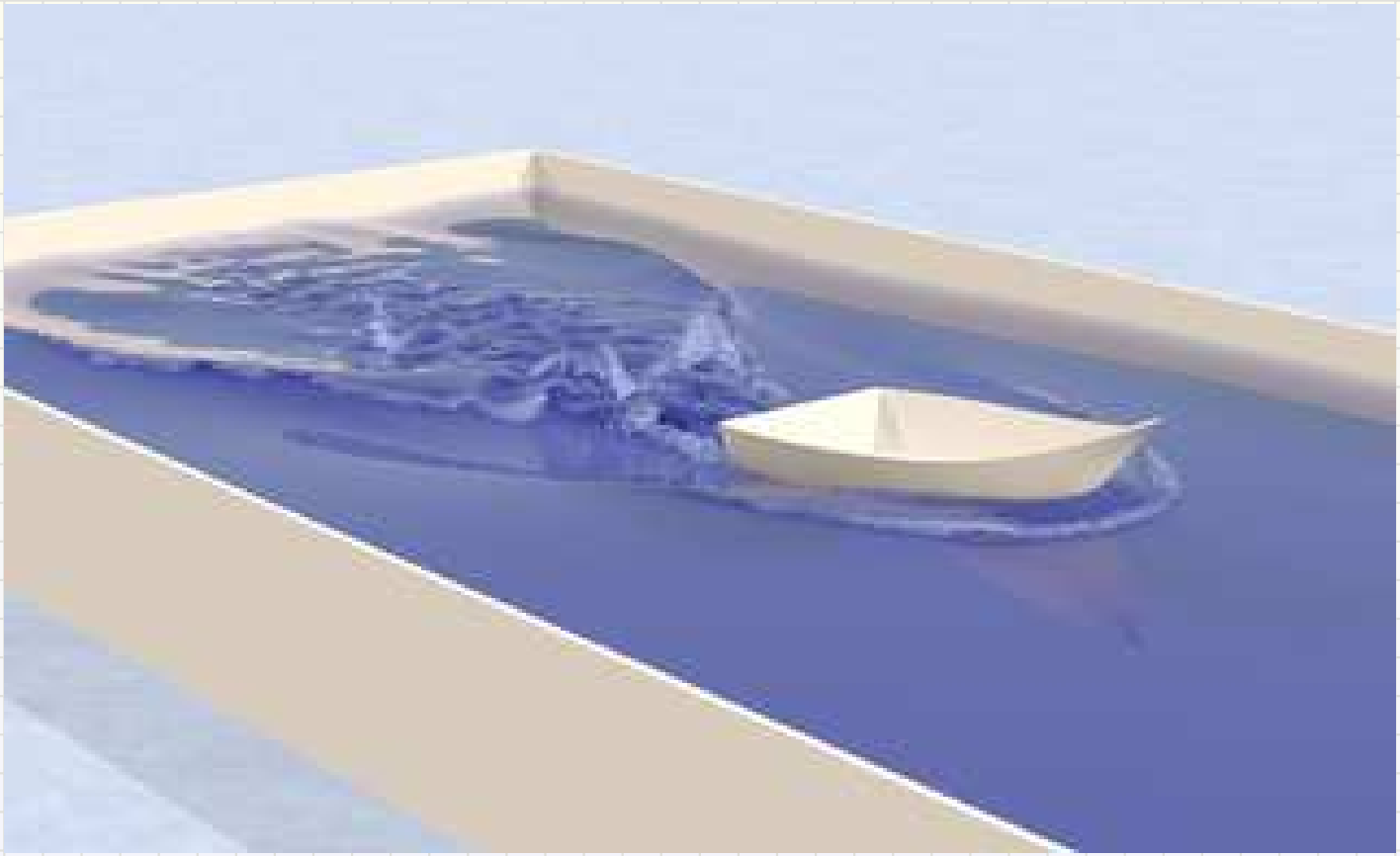


FIG.: Streamlines around the craft

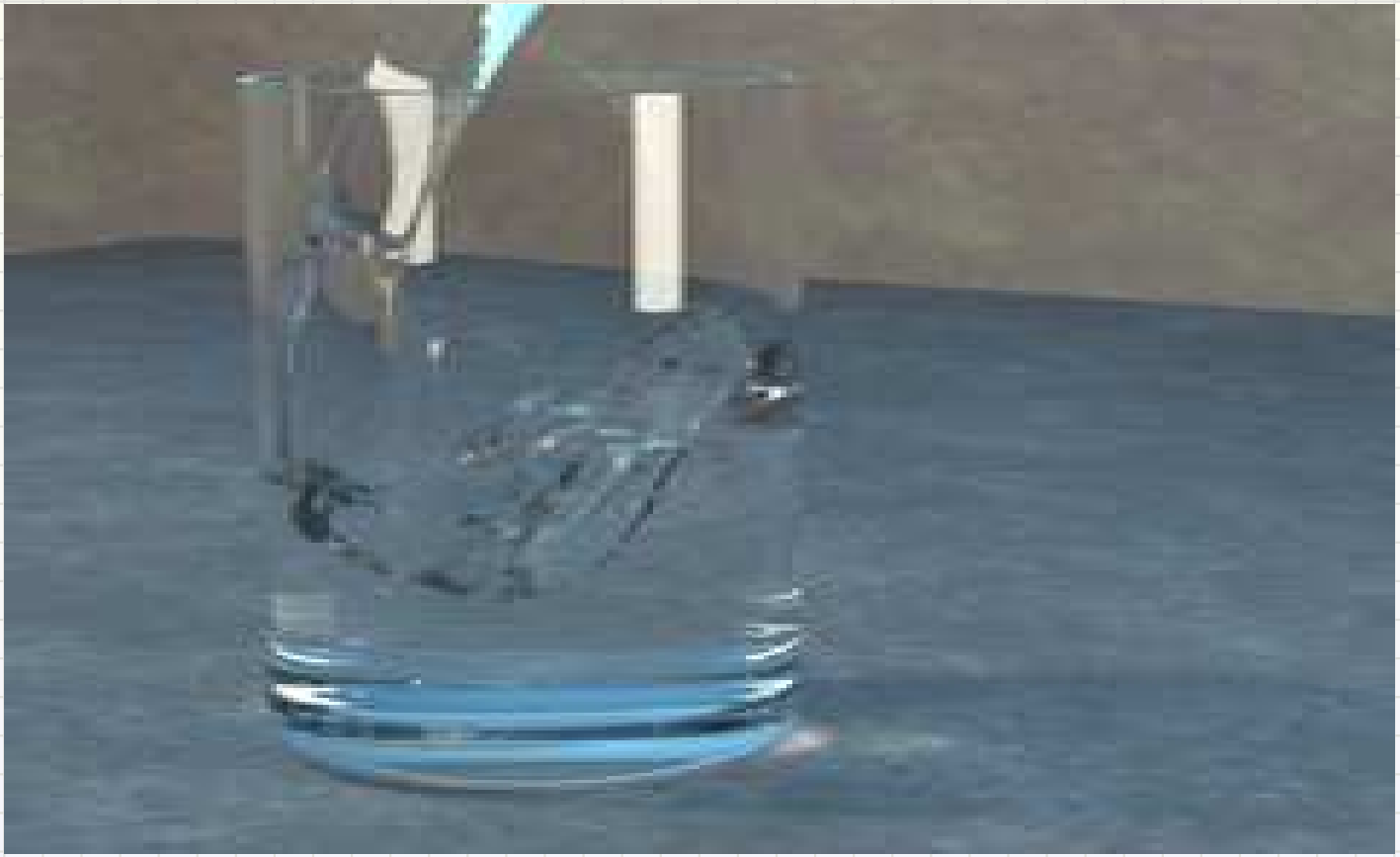
Free surface waves animation

Stanford Computer Science : R. Fedkiw



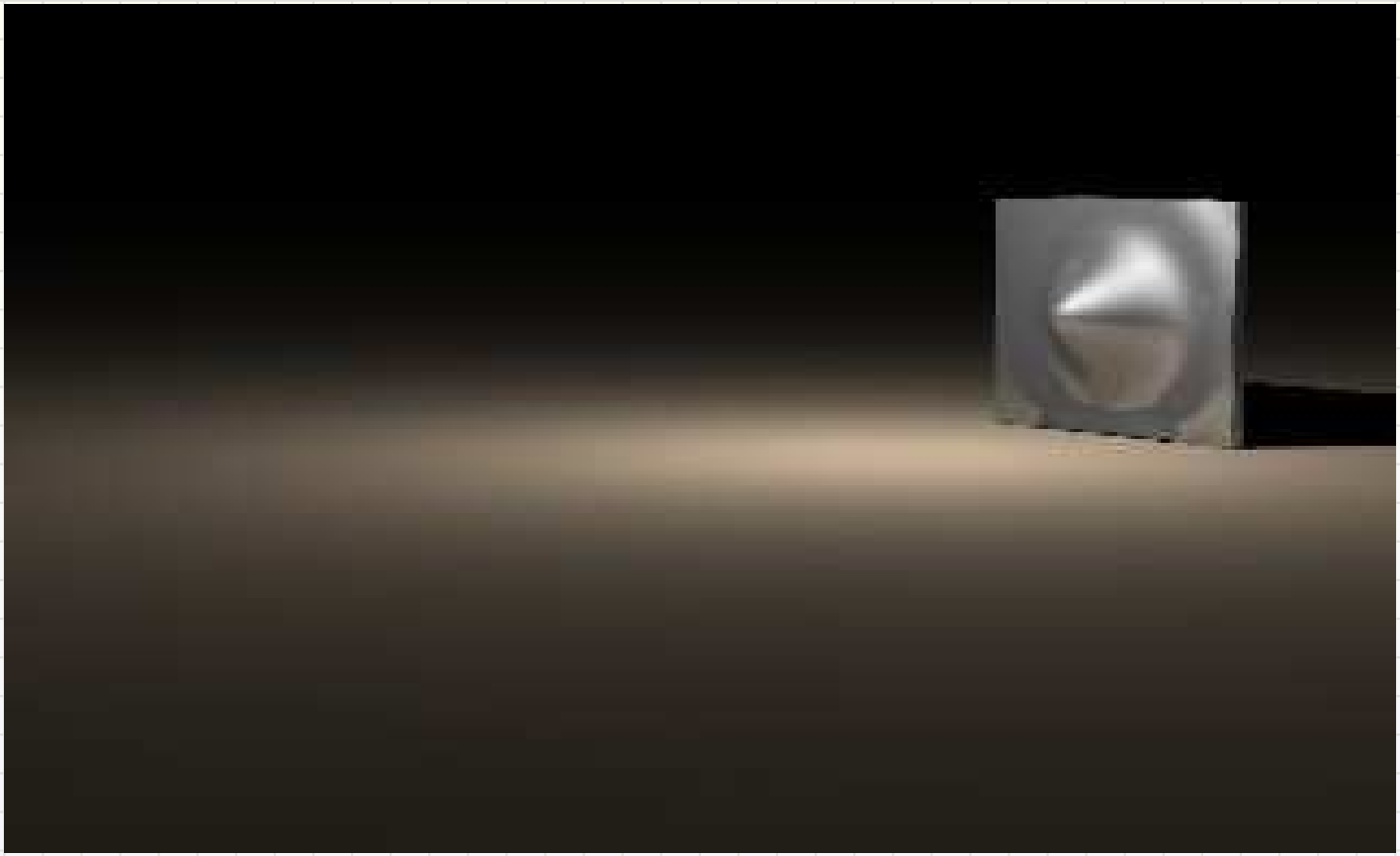
Water animation in the glass

Stanford Computer Science : R. Fedkiw



Example of fracture mechanics

Stanford Computer Science : R. Fedkiw



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Overall conclusions

- Areas of applications are very diversified
- About 80% of our « clients » are military

Nikolai Lobachevsky

There is no branch of mathematics, however abstract, which may not someday be applied to the phenomena of the real world.

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Thank you for your attention !

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