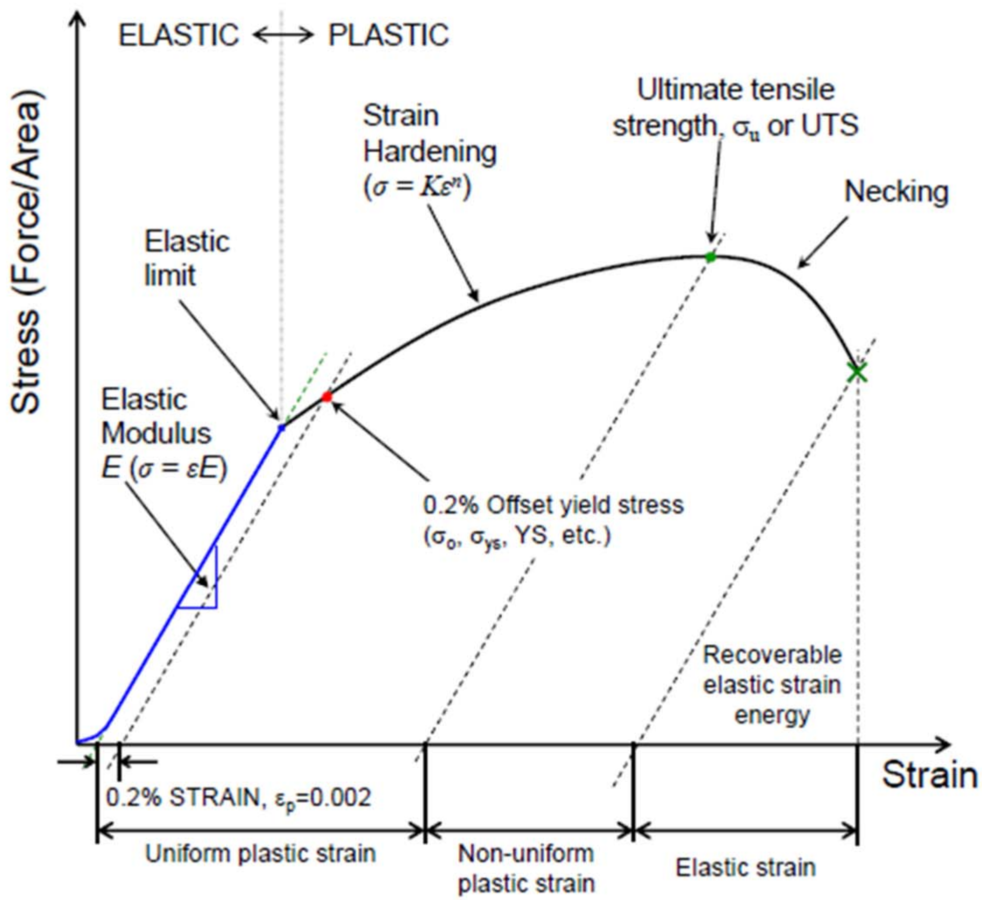
Experimental Strength of Materials

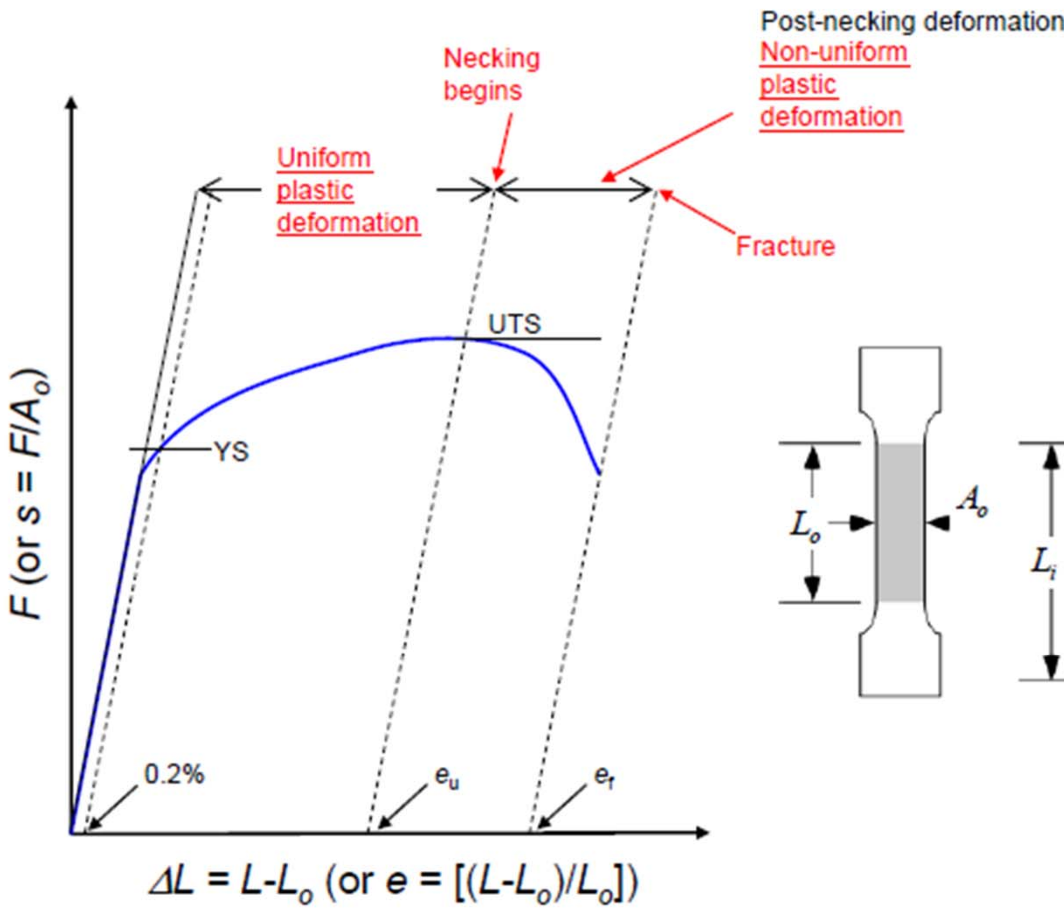
Lecture #2

Panos Tsopelas, PhD

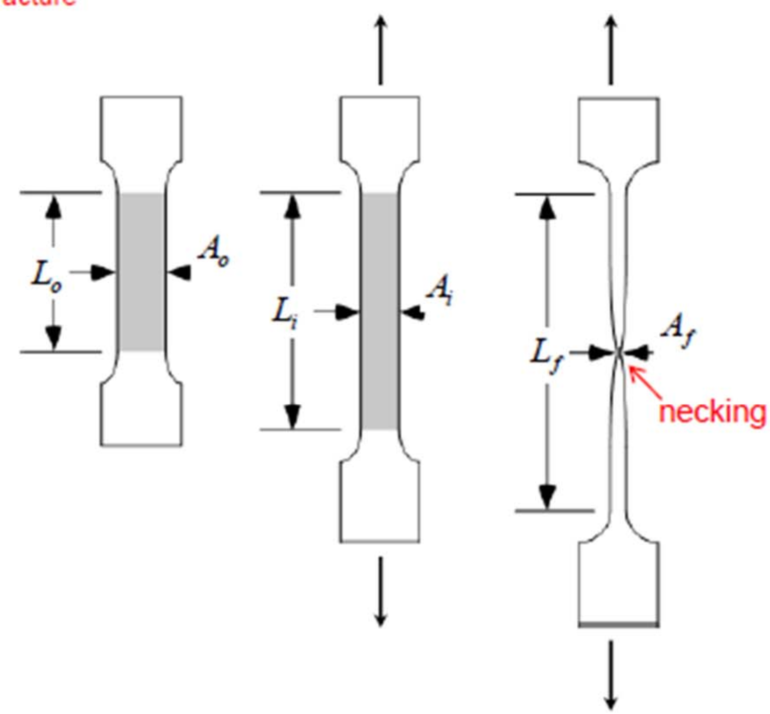
Engineering Stress-Strain Curve in Tension



- Elastic deformation up to elastic limit.
- Plastic deformation after elastic limit.
- Uniform plastic deformation between elastic limit and the UTS.
- Nonuniform plastic deformation after UTS.
- In tension this non-uniform deformation is called necking.



Post-necking deformation
Non-uniform plastic deformation
Necking begins
Uniform plastic deformation
Fracture
UTS
YS
F (or $s = F/A_o$)
0.2%
e_u
e_f
$\Delta L = L - L_o$ (or $e = [(L-L_o)/L_o]$)



L_o
A_o
L_i
A_i
L_f
A_f
necking

Why does necking occur?

- Consider strength increases due to strain hardening vs. reduction in cross-sectional area caused by the Poisson effect.

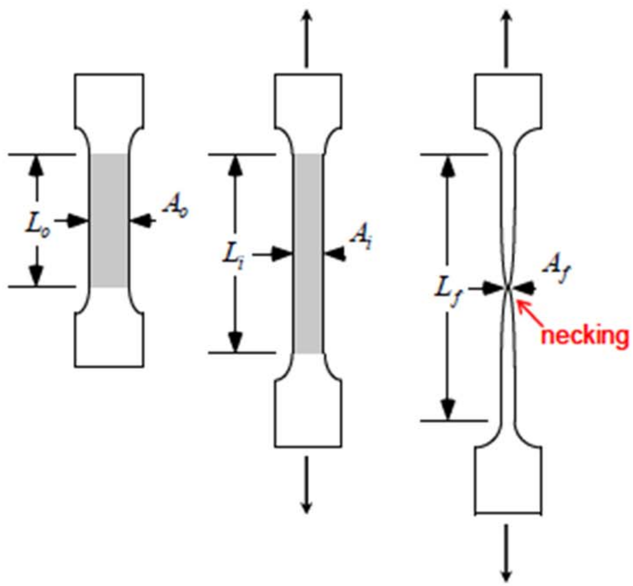
- During plastic deformation, the load carrying capacity of the material increases due to strain-hardening.

- Strain hardening is opposed by the gradual decrease in the cross-sectional area of the specimen as it elongates.

"Why does necking occur?" – cont'd

- At maximum load (i.e., the UTS), stress needed to continue deformation > the reduced cross-section can withstand.

- Leads to localized plastic deformation in reduced section. This is non-uniform flow or "necking."



"The material can't sustain the increase in stress required to continue deforming and work hardening it"

Criteria for Necking – cont'd

- Local ↓ in A (i.e., deformation) causes that region to strain harden locally (relative to the rest of the cross section). The remainder of the cross section then deforms (and strain hardens) sequentially until a uniform cross-section is re-established.

- The rates balance at the UTS [$(dA/d\varepsilon) = (d\sigma/d\varepsilon)$].

- When $(dA/d\varepsilon) > (d\sigma/d\varepsilon)$, deformation becomes unstable. The material cannot strain harden fast enough to inhibit necking.

Criteria for Necking – cont'd

- The criteria for instability is defined by the condition where the slope of the force distance curve equals zero ($dF = 0$):

$$F = \sigma A$$

where

$$F = \text{load},$$

$$\sigma = \text{true stress},$$

$$A = \text{area at max load}$$

NOTE: We are using true stress and strain here rather than engineering

$$\boxed{dF = \sigma dA + A d\sigma = 0} \quad \ldots\ldots\ldots\ldots\ldots (*)$$

Criteria for Necking – cont'd

- Recall that deformation is a constant volume process. Therefore:

$$L_o A_o = LA = \text{constant}$$

$$\frac{dL}{L} = -\frac{dA}{A} = d\varepsilon$$

- If we invoke the instability criteria from the previous page (*), we get:

$$-\frac{dA}{A} = \frac{d\sigma}{\sigma} = d\varepsilon$$

Criteria for Necking – cont'd

- Thus, at the point of tensile instability,

$$\frac{d\sigma}{d\varepsilon} = \sigma$$

When "necking" occurs.

- If we incorporate engineering strain *e*, into the equation presented above, we can develop a more explicit expression:

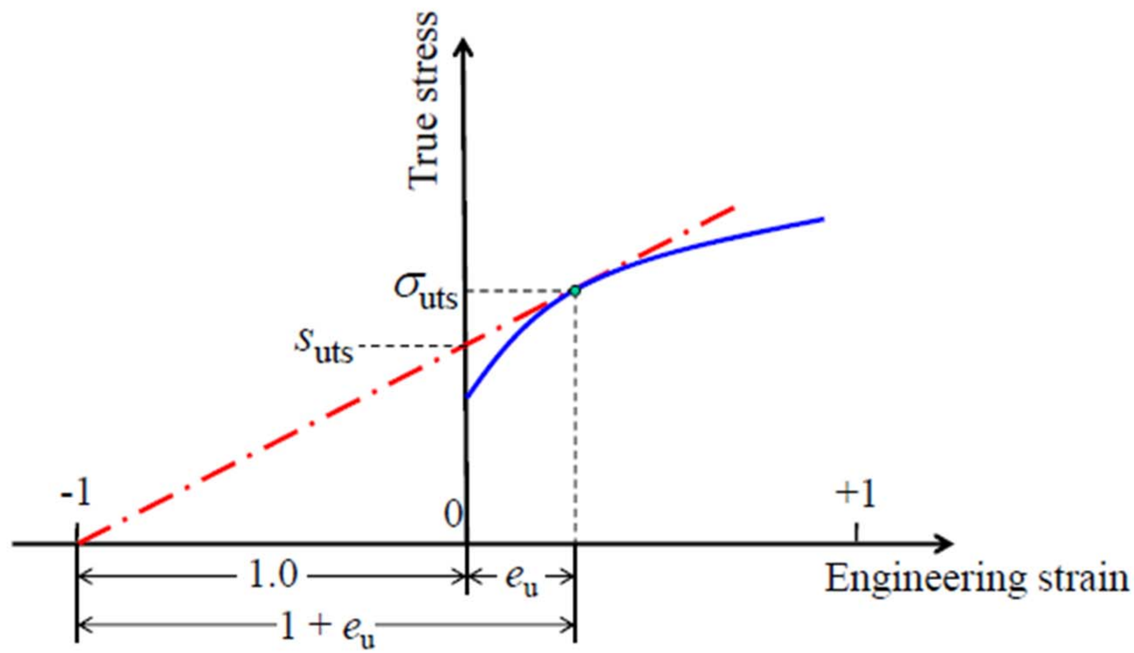
$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{de}\frac{de}{d\varepsilon} = \frac{d\sigma}{de}\frac{dL/L_o}{dL/L} = \frac{d\sigma}{de}\frac{L}{L_o} = \frac{d\sigma}{de}(1+e) = \sigma$$

or

$$\boxed{\frac{d\sigma}{de} = \frac{\sigma}{(1+e)}}$$

- This is known as Considère's construction.

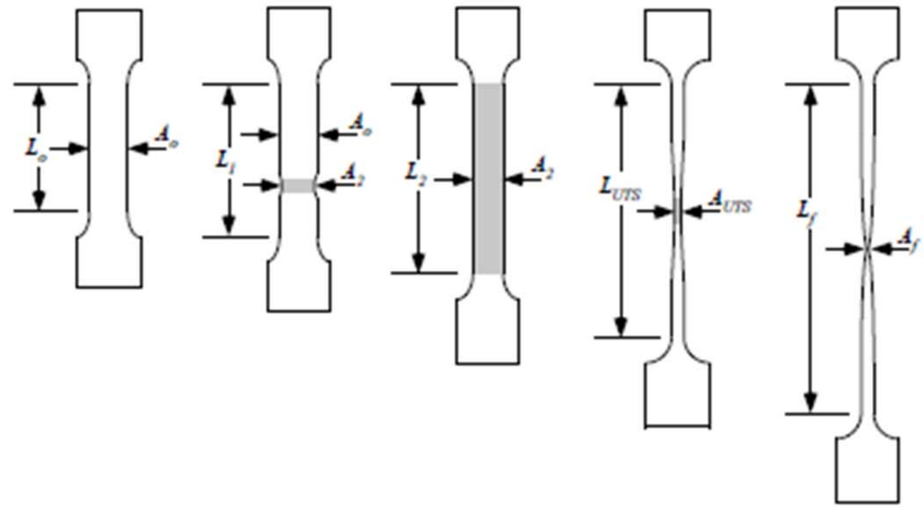
Considère's Construction



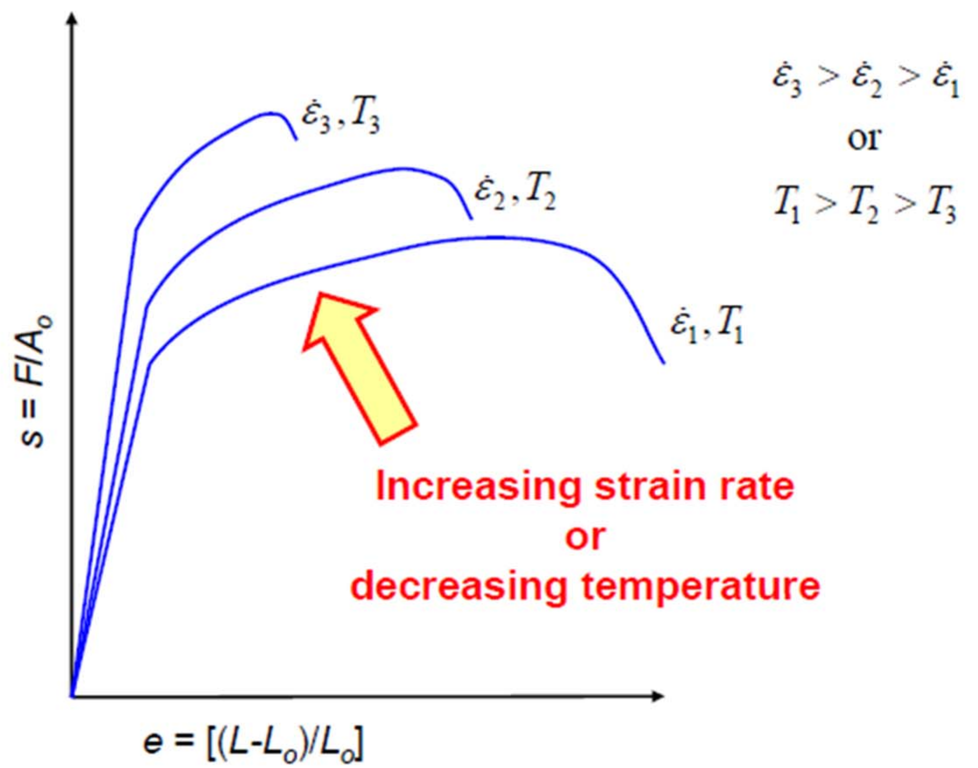
Allows determination of the amount of uniform elongation and the UTS

Process of Necking

(a) During tensile deformation, strain can become localized along the sample length. (b) When strains are less than the UTS, work hardening strengthens the material in the strain localized area relative to the rest of the specimen. (c) The work-hardening rate (WHR) decreases as strain increases. At ε_{UTS} the decrease in cross-sectional area becomes equal to the increase in flow strength due to work hardening. As a result, the localized region (i.e., "neck") becomes permanent. (d) as strain increases, the neck gets bigger until the material fails.



Parameter	Fundamental Definition	Before Necking	After Necking
Engineering stress σ_e	$\sigma_e = s = \frac{F}{A_o}$	$\sigma_e = \frac{F}{A_o}$	$\sigma_e = \frac{F}{A_o}$
True stress σ_t	$\sigma_t = \sigma = \frac{F}{A_i}$	$\sigma_t = \frac{F}{A_i}$	$\sigma_t = \frac{F}{A_{neck}}$
Engineering strain ε_e	$\varepsilon_e = e = \frac{\delta L}{L_o}$	$\varepsilon_e = \frac{\delta L}{L_o}$	$\varepsilon_e = \frac{\delta L}{L_o}$
True strain ε_t	$\varepsilon_t = \ln\frac{A_o}{A_{min}}$	$\varepsilon_t = \ln\frac{L_i}{L_o} = \ln\frac{A_o}{A_i} = \ln(1+\varepsilon_e)$	$\varepsilon_t = \ln\frac{A_o}{A_{neck}}$

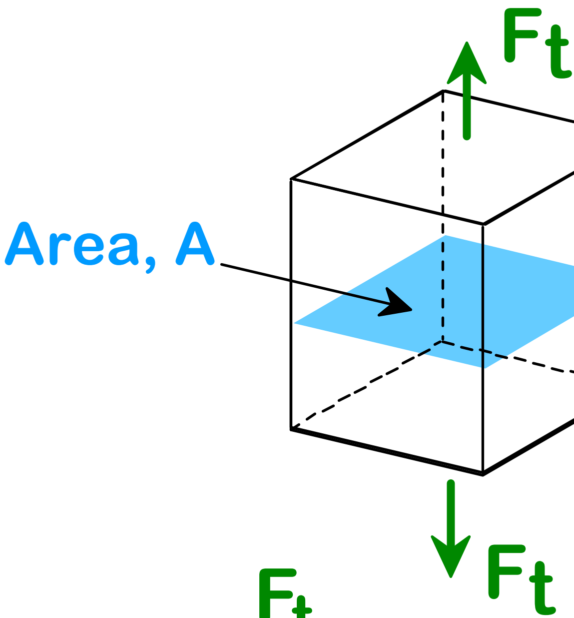


Mechanical properties are sensitive to temperature and strain rate.

HOW AND WHY?

Engineering Stress

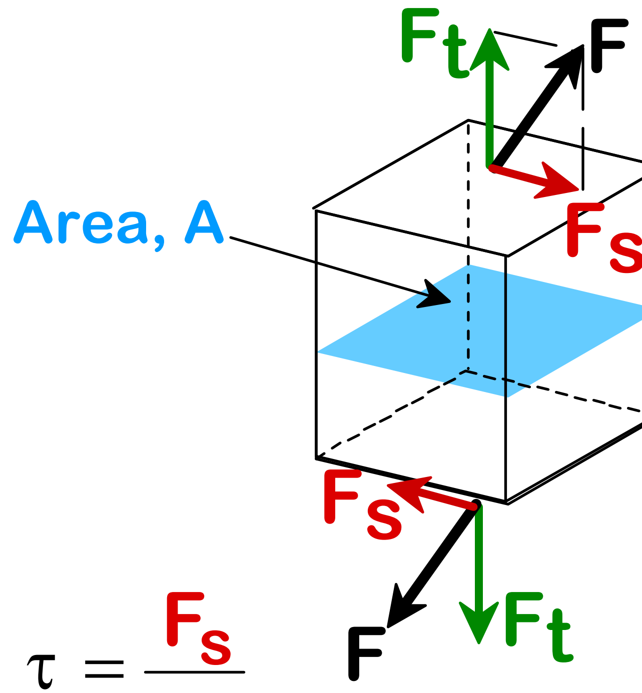
- **Tensile stress, σ:**



$$\sigma = \frac{F_t}{A_o}$$

original area before loading

- **Shear stress, τ:**



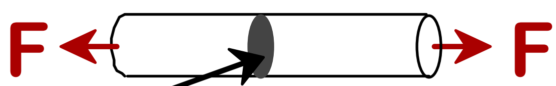
$$\tau = \frac{F_s}{A_o}$$

Stress has units:
N/m^2 (or lb/in^2)

Adapted from Ashby, Eng. Matls 1.

Common States of Stress

- **Simple tension: cable**

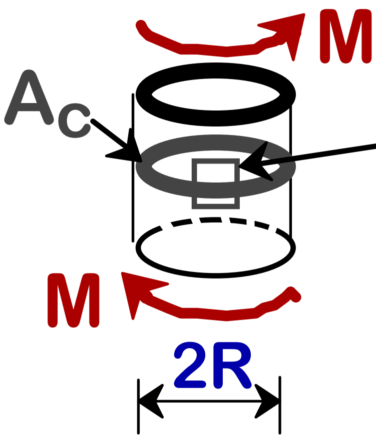


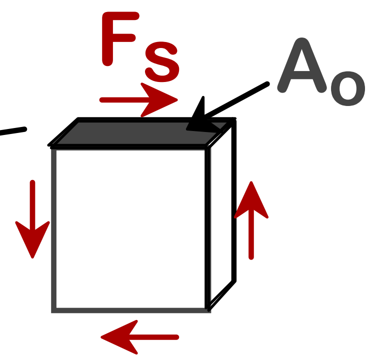
A_o = cross sectional Area (when unloaded)

$$\sigma = \frac{F}{A_o}$$

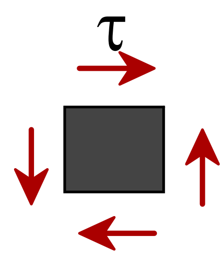
Ski lift (photo courtesy P.M. Anderson)

- **Simple shear: drive shaft**





$$\tau = \frac{F_s}{A_o}$$

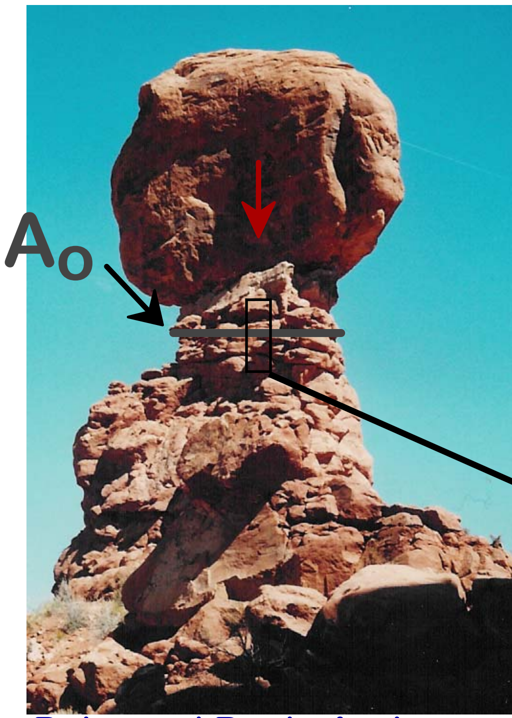


Note: $\tau = M/A_cR$ **here.**

From Callister, Intro to Eng. Matls., 6Ed

Common States of Stress

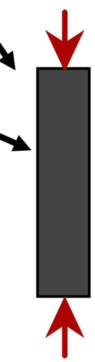
- **Simple compression:**



Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)

Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

$$\sigma = \frac{F}{A_o}$$

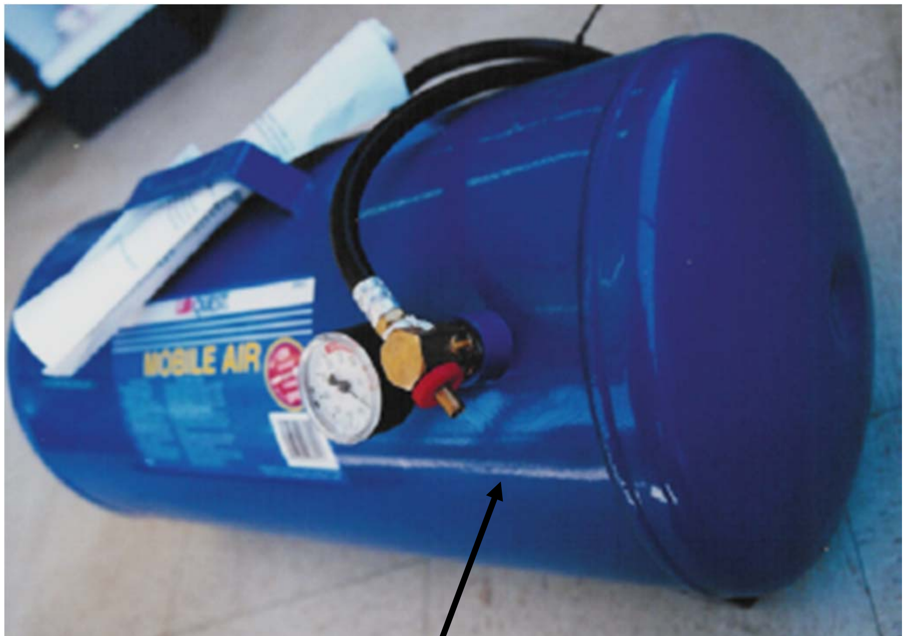


Note: compressive structure member ($\sigma < 0$ here).

From Callister,
Intro to Eng. Matls., 6Ed

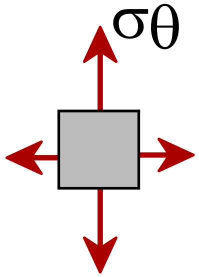
Common States of Stress

- **Bi-axial tension:**



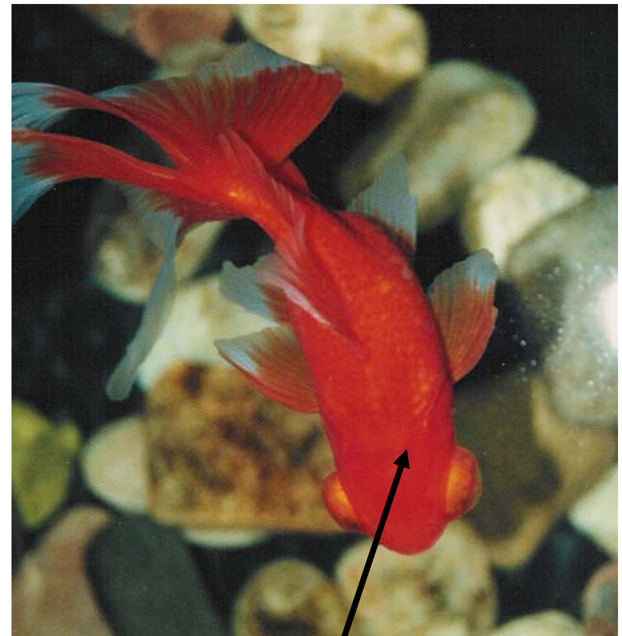
Pressurized tank
(photo courtesy
P.M. Anderson)

$\sigma_\theta > 0$



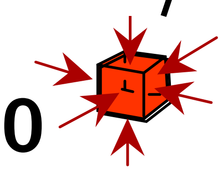
$\sigma_z > 0$

- **Hydrostatic compression:**



Fish under water

(photo courtesy
P.M. Anderson)



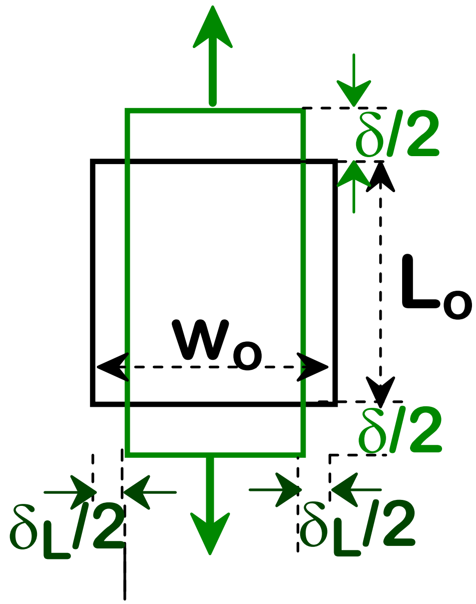
$\sigma_h < 0$

From Callister,
Intro to Eng. Matls., 6Ed

Engineering Strain

- **Tensile strain:**

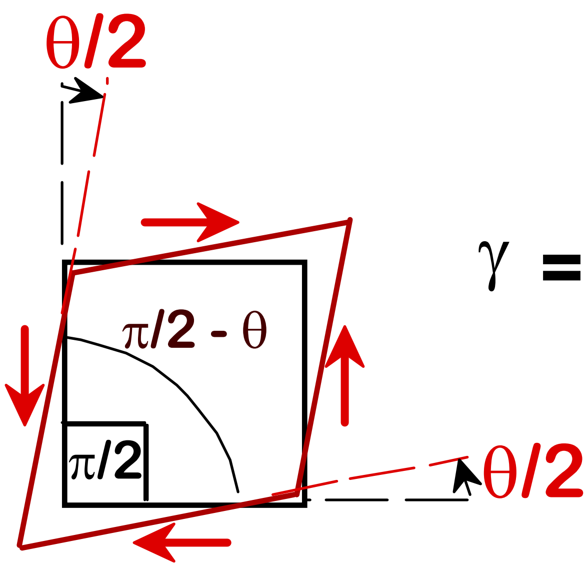
$$\varepsilon = \frac{\delta}{L_o}$$



- **Lateral (width) strain:**

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$

- **Shear strain:**



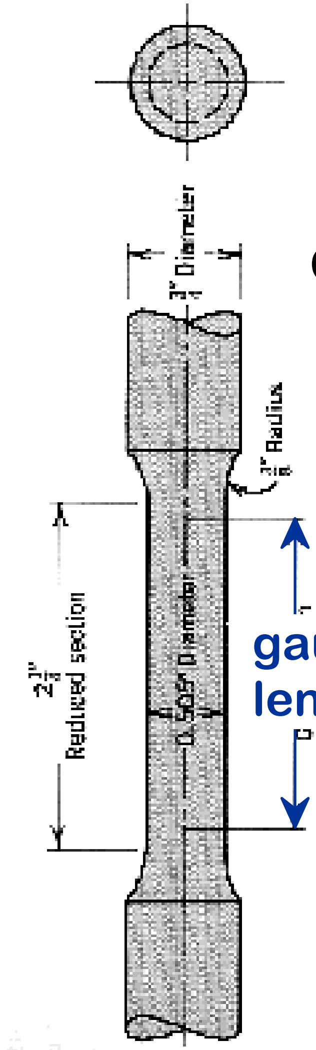
$$\gamma = \tan\theta$$

Strain is always dimensionless.

From Callister,
Intro to Eng. Matls., 6Ed

Strain Testing

• Tensile specimen

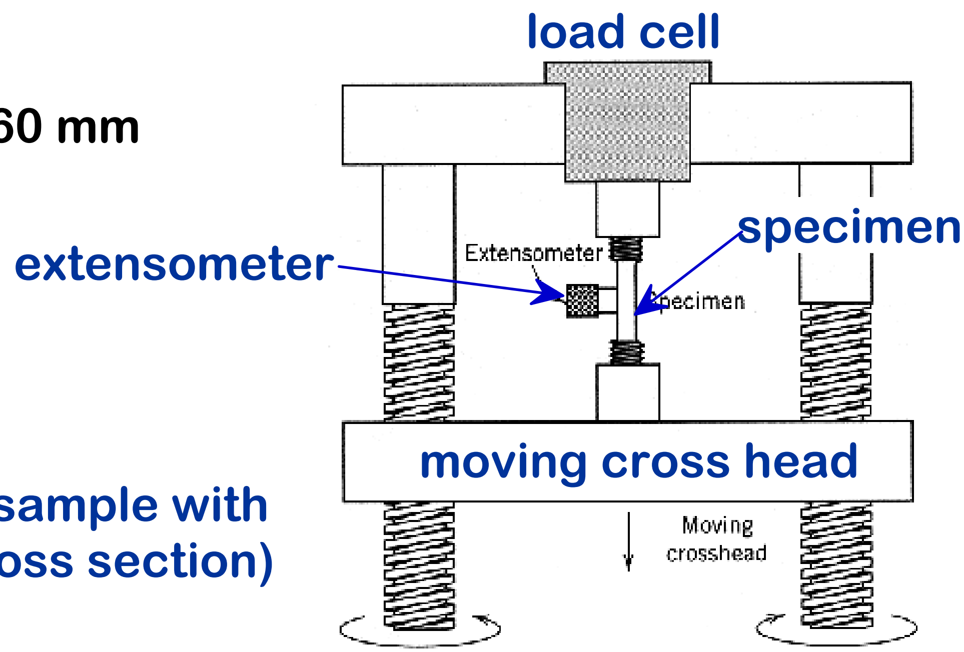


Adapted from Fig. 6.2, *Callister 6e.*

Often 12.8 mm x 60 mm

gauge length = (portion of sample with reduced cross section)

• Tensile test machine



Adapted from Fig. 6.3, *Callister 6e.*

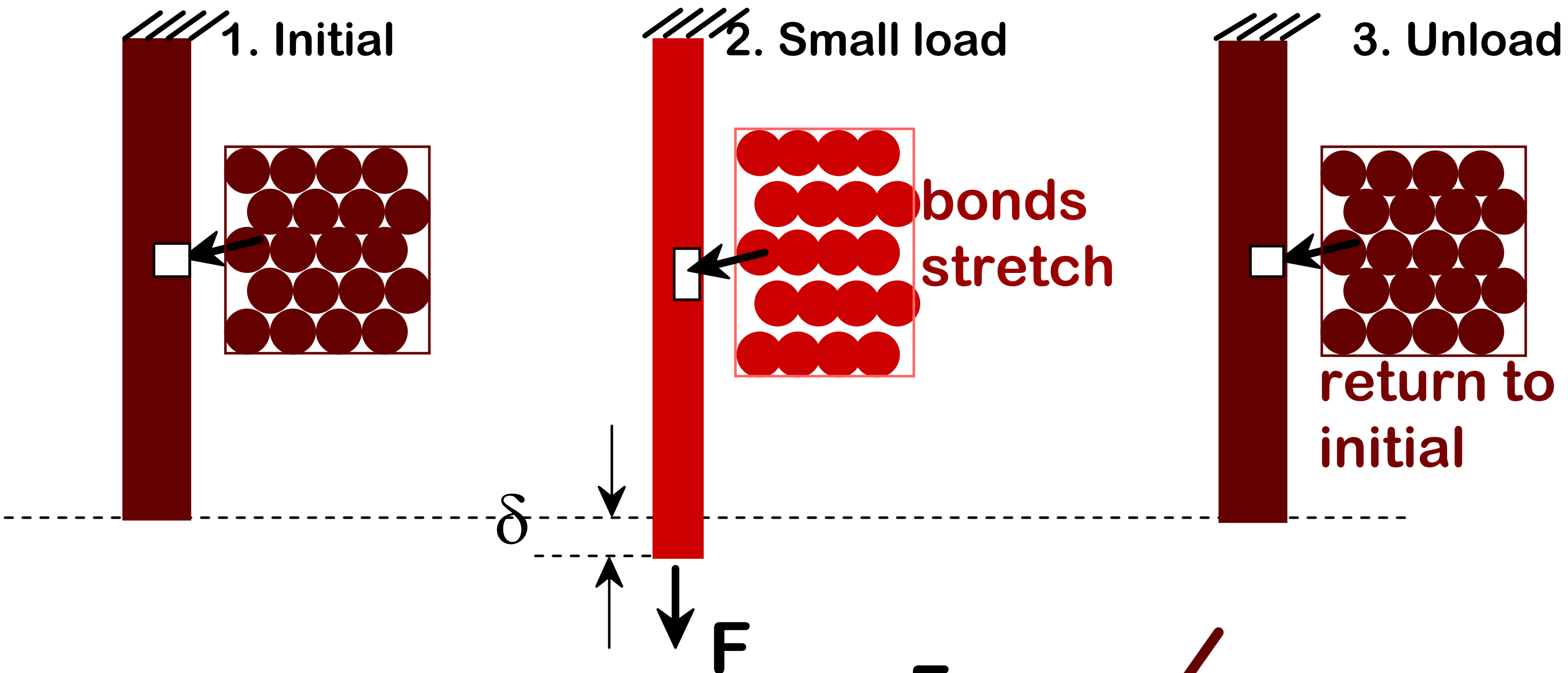
• Other types:

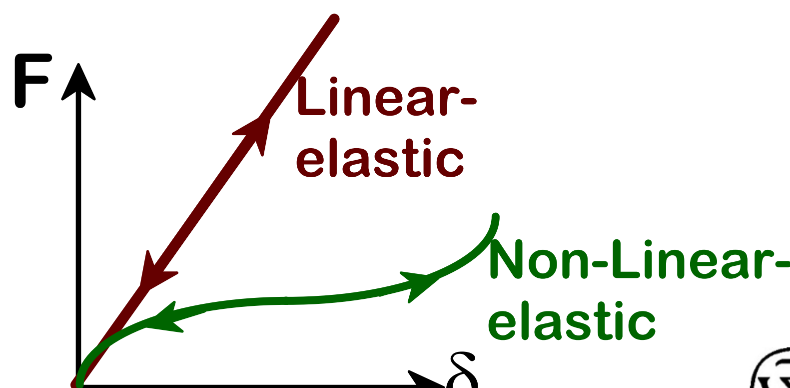
-compression: brittle materials (e.g., concrete)

-torsion: cylindrical tubes, shafts.

From Callister,
Intro to Eng. Matls., 6Ed

Elastic Deformation

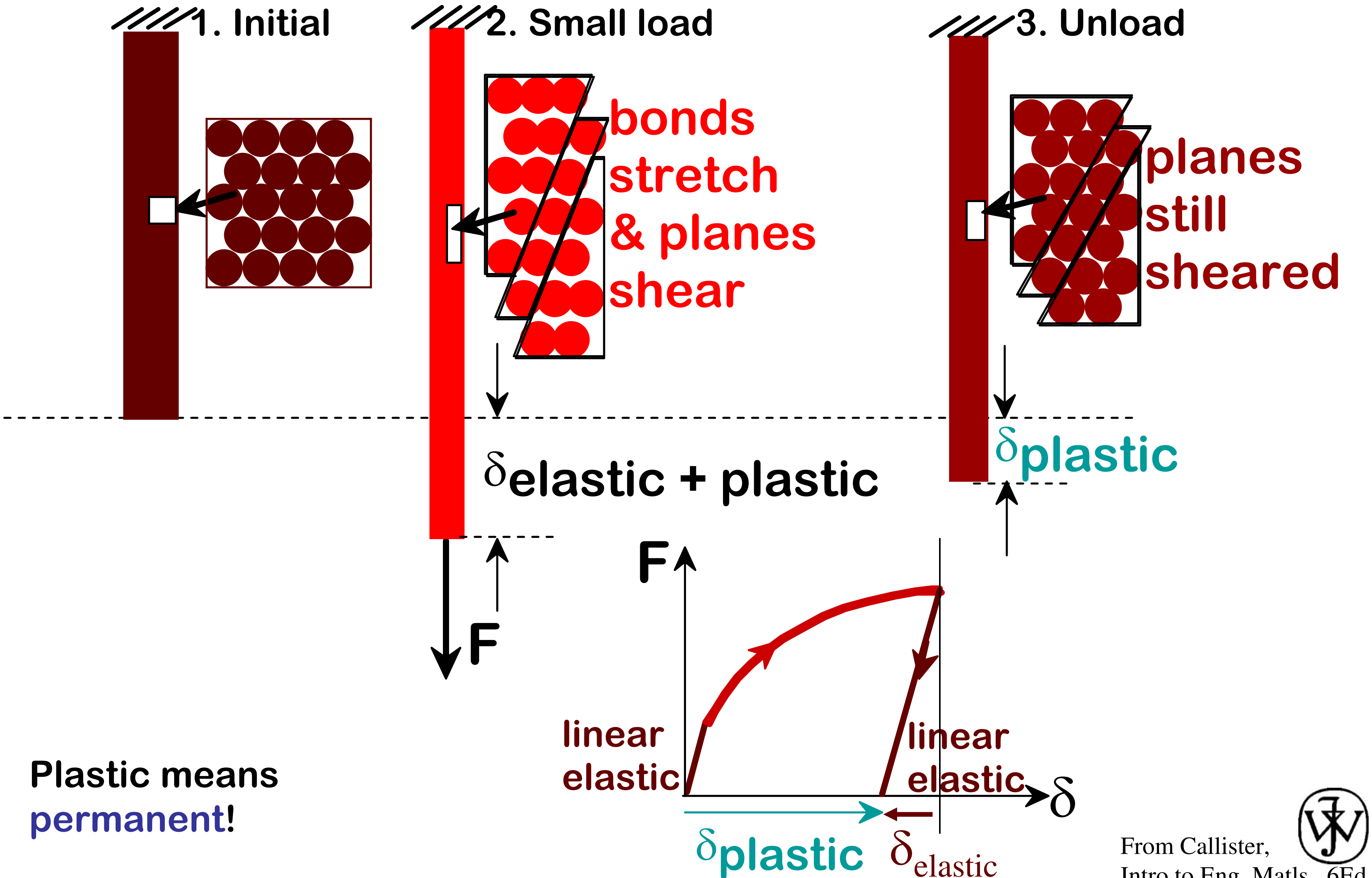




Elastic means reversible!

From Callister, Intro to Eng. Matls., 6Ed

Plastic Deformation of Metals



Plastic means permanent!

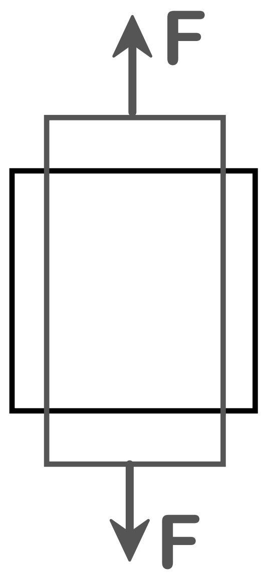
From Callister, Intro to Eng. Matls., 6Ed

Linear Elasticity

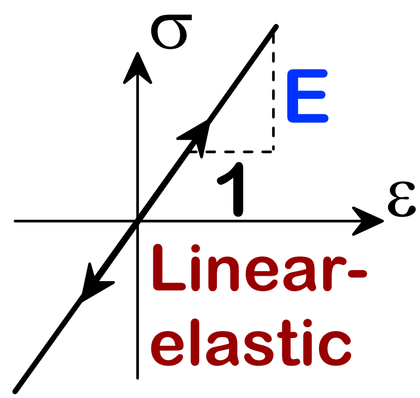
- **Modulus of Elasticity, E:**
  (also known as Young's modulus)

- **Hooke's Law:** $\sigma = E\,\varepsilon$

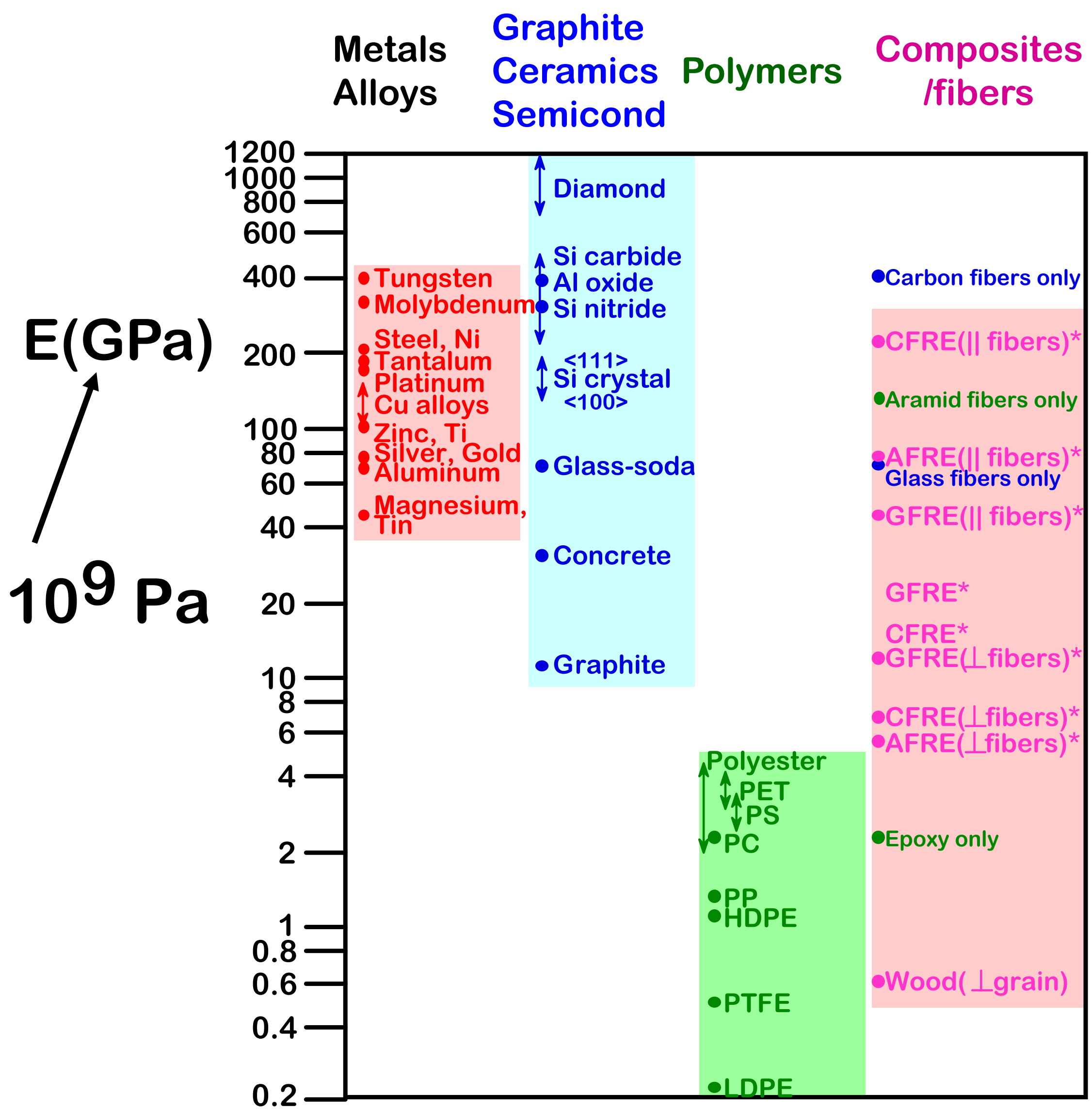
Units:
E: [GPa] or [psi]



simple
tension
test



Young's Modulus, E



$E_{ceramics}$

$> E_{metals}$

$>> E_{polymers}$

Based on data in Table B2, *Callister 6e*.

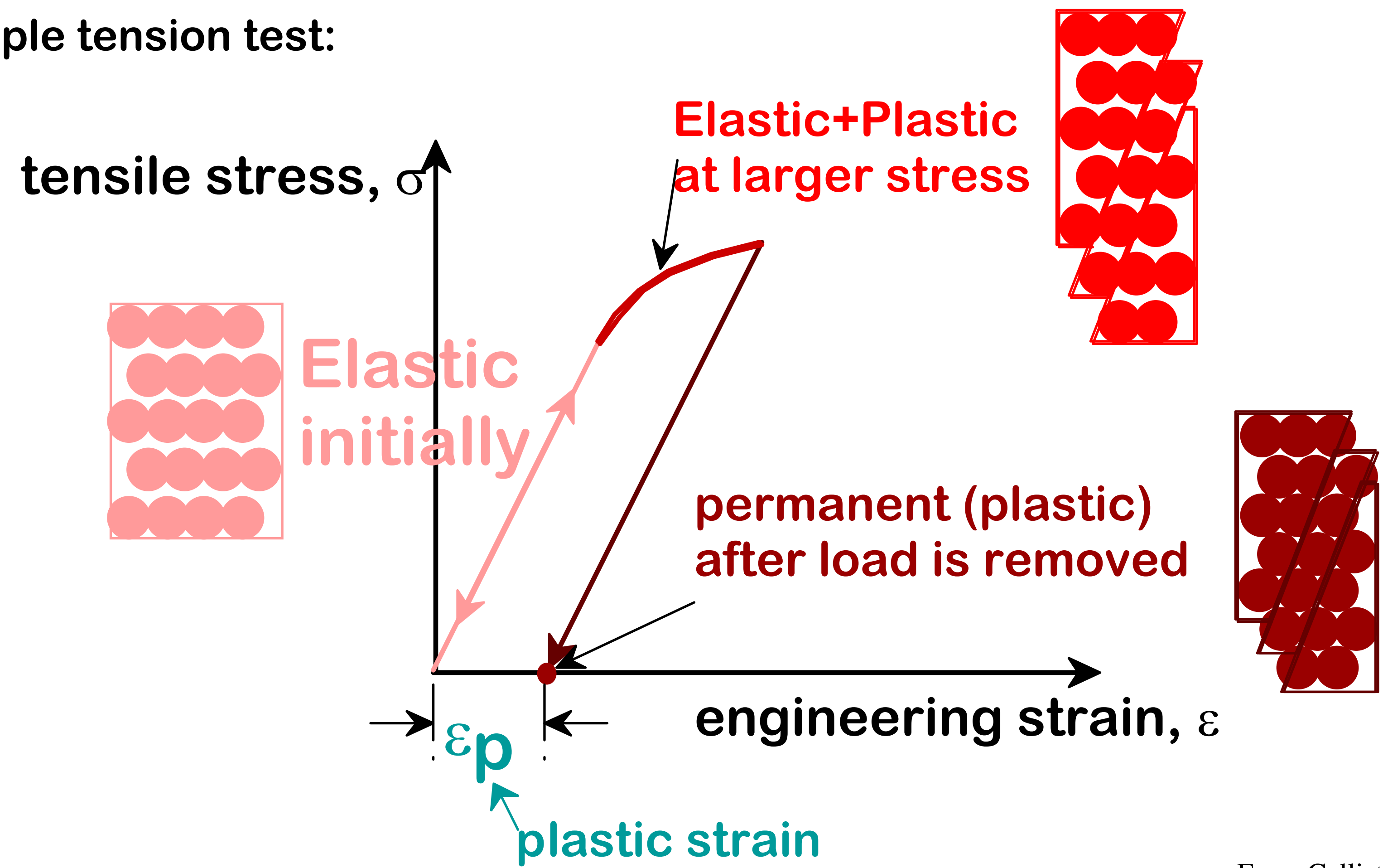
Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

From Callister,
Intro to Eng. Matls., 6Ed

Plastic Deformation

(at lower temperatures, $T < T_{melt}/3$)

• Simple tension test:



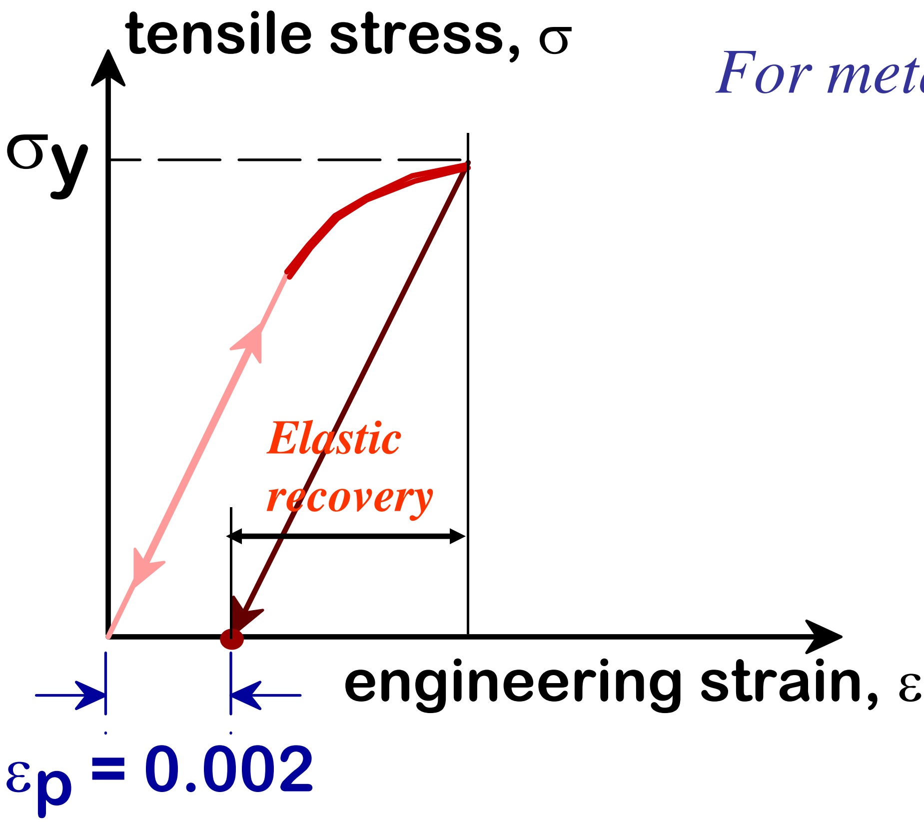
From Callister,
Intro to Eng. Matls., 6Ed

Yield Strength, σ_{YS}

- Stress where *noticeable* plastic deformation occurs.

when εp = 0.002

For metals agreed upon 0.2%

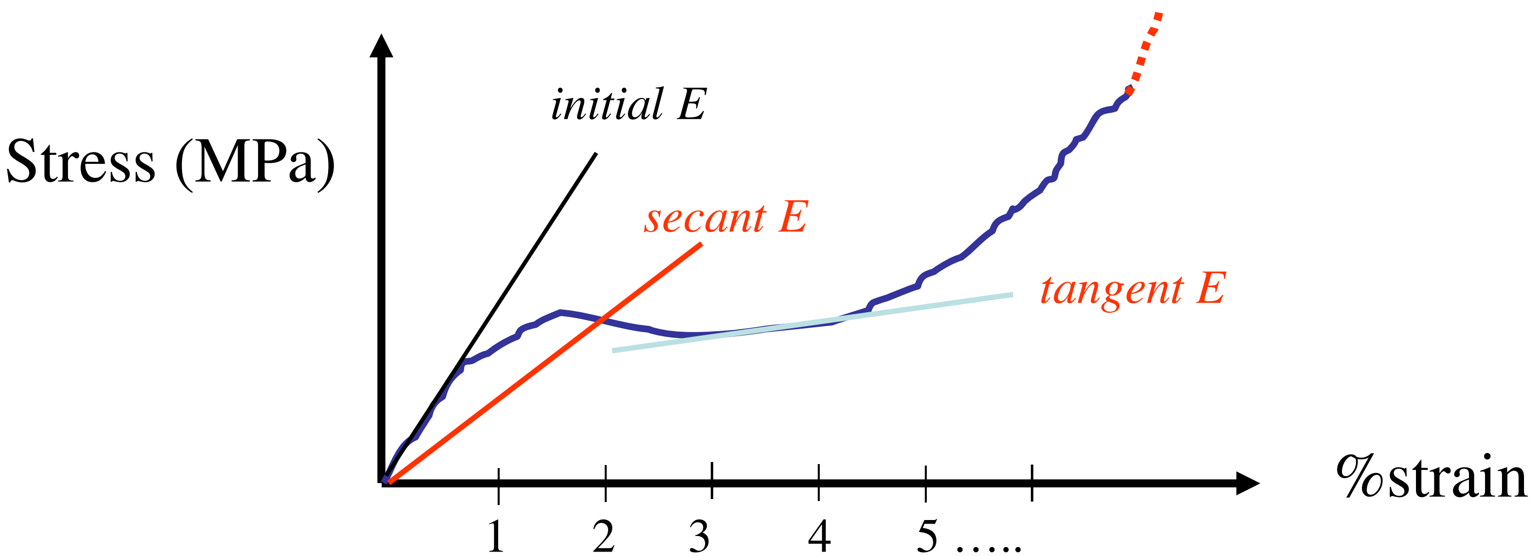


Polymers: Tangent and Secant Modulus

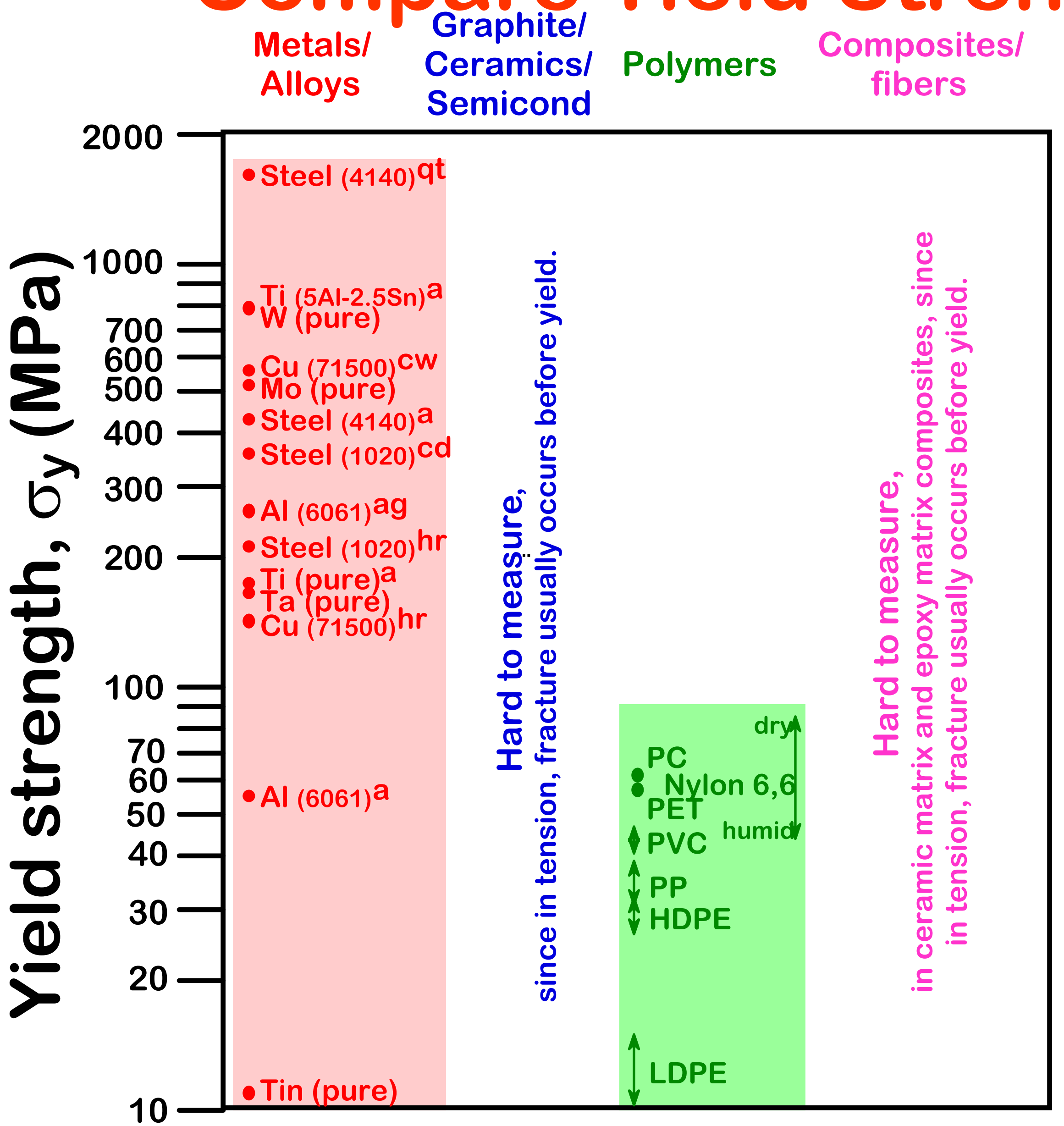
- **Tangent Modulus** is experienced in service.

- **Secant Modulus** is effective modulus at 2% strain.

- **Modulus of polymer changes with *time* and *strain-rate*.**
  - must report **strain-rate** $d\varepsilon/dt$ for polymers.
  - must report **fracture strain** ε_f **before** fracture.



Compare Yield Strength, σ_{YS}



σ_y(ceramics) >> σ_y(metals) >> σ_y(polymers)

Room T values

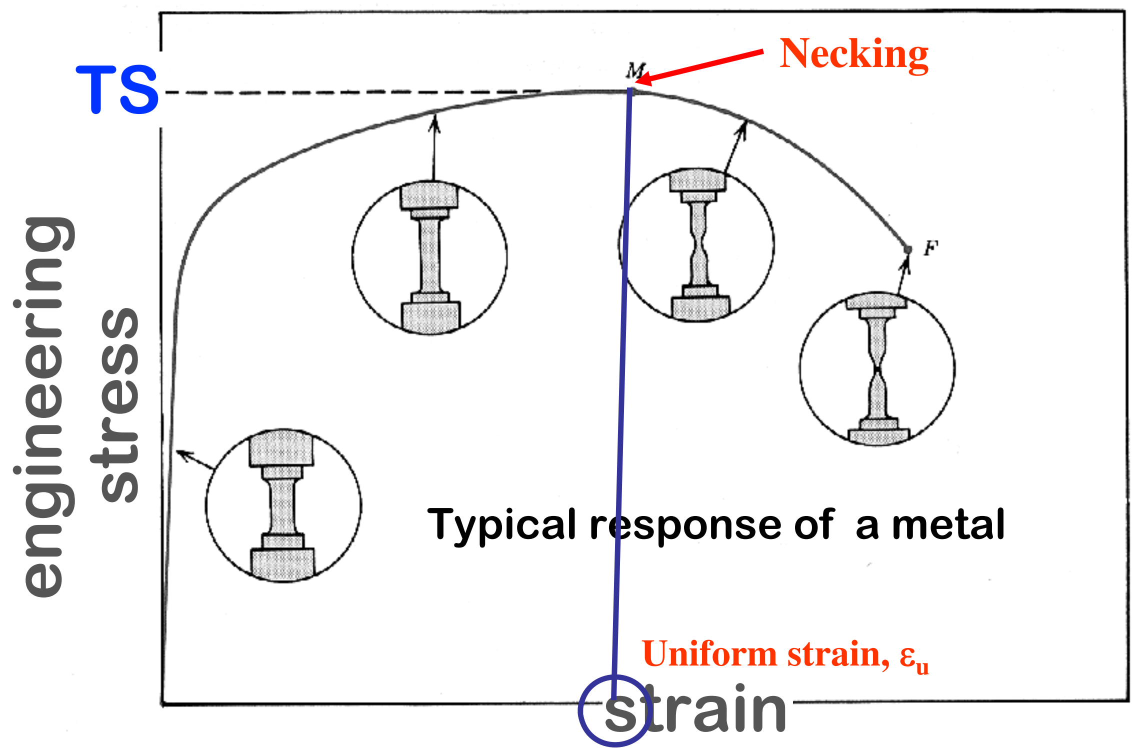
Based on data in Table B4, *Callister 6e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

From Callister, Intro to Eng. Matls., 6Ed

(Ultimate) Tensile Strength, σ_{TS}

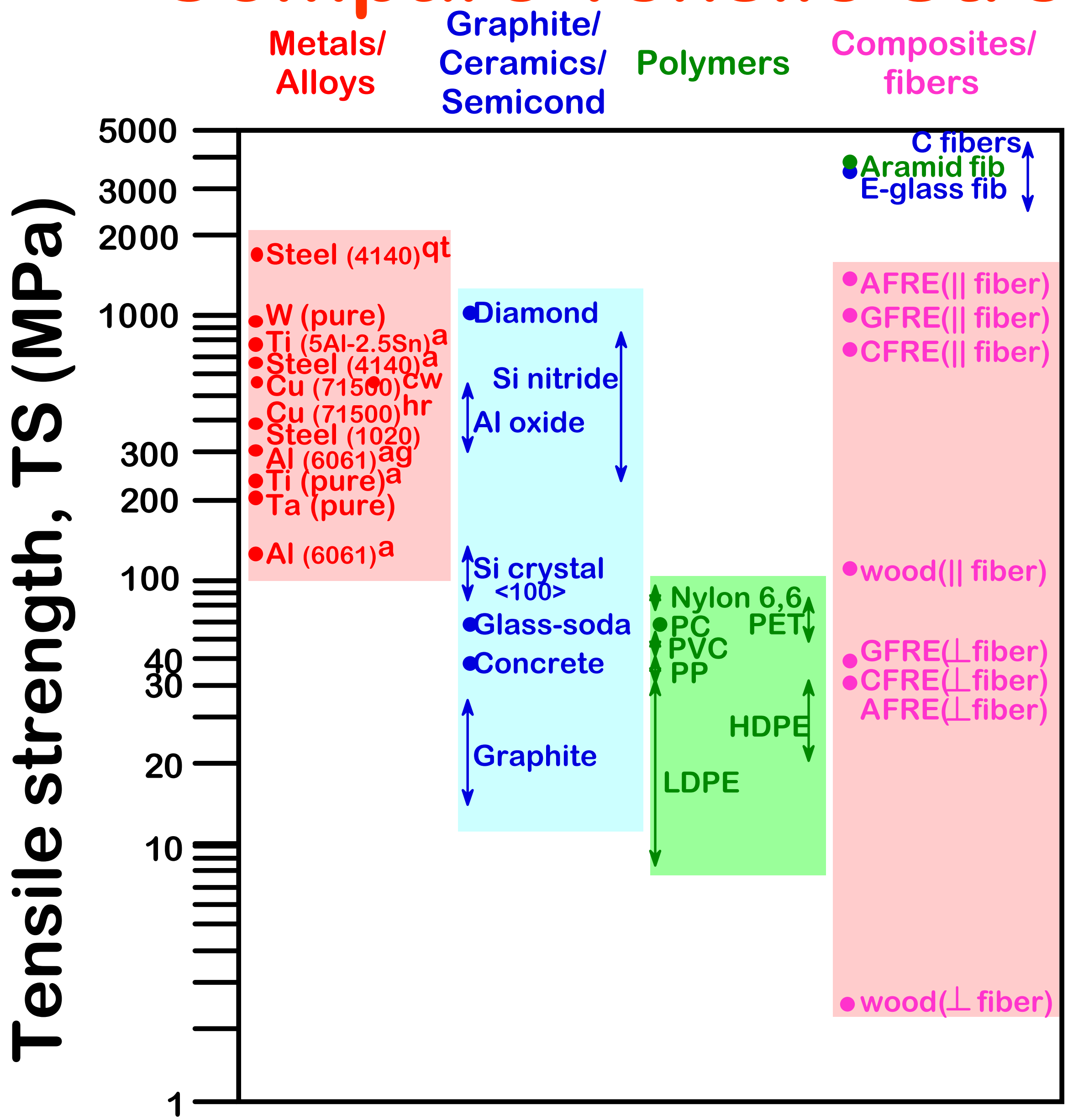
• Maximum possible engineering stress in tension.



Adapted from Fig. 6.11, *Callister 6e.*

- Metals: occurs when necking starts.
- Ceramics: occurs when crack propagation starts.
- Polymers: occurs when polymer backbones are aligned and about to break.

Compare Tensile Strength, σ_{TS}



$TS_{(ceram)}$

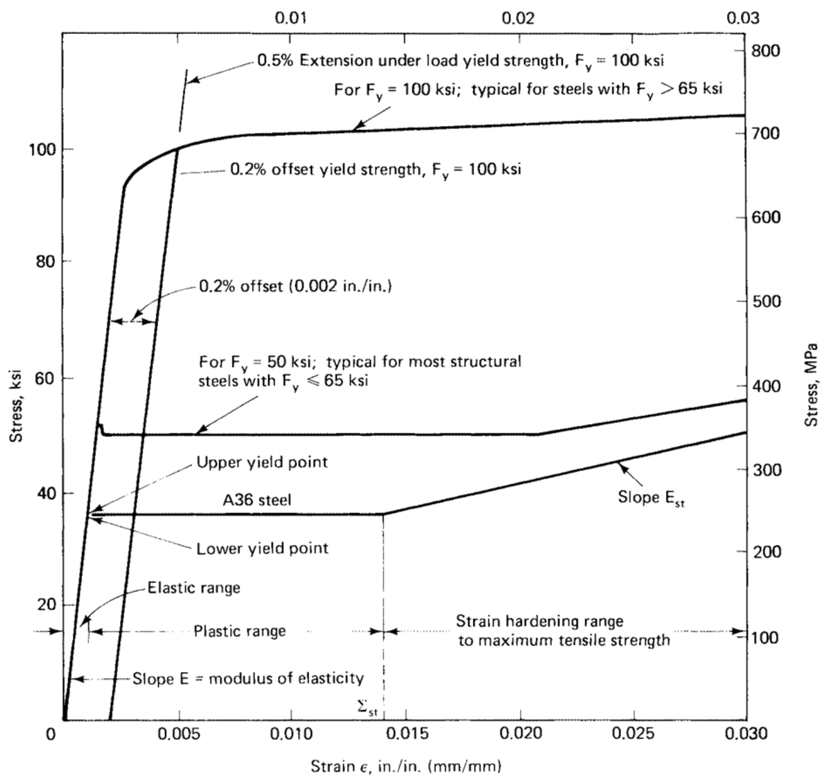
$\sim TS_{(met)}$

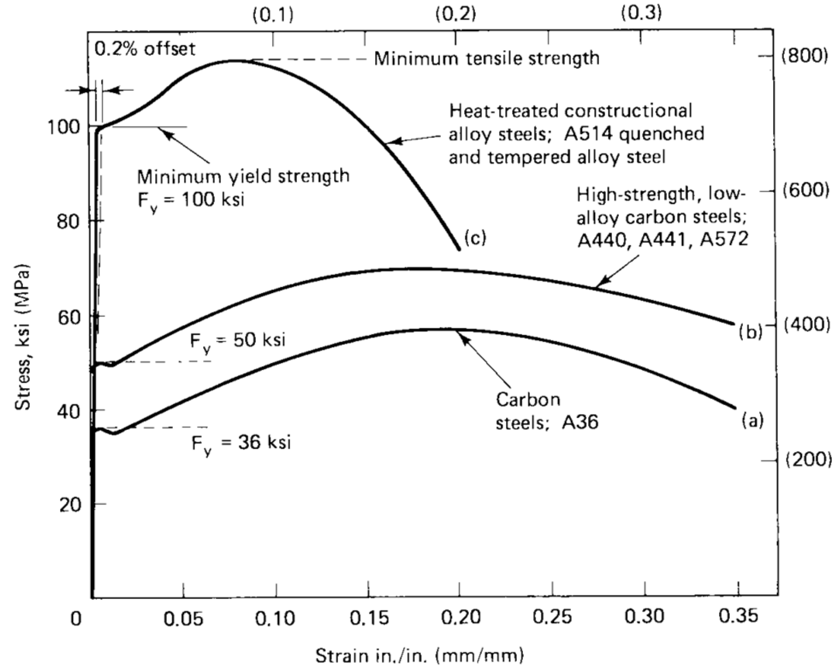
$\sim TS_{(comp)}$

$\gg TS_{(poly)}$

Room T values

Based on data in Table B4, *Callister 6e*.


0.5% Extension under load yield strength, F_y = 100 ksi
For F_y = 100 ksi; typical for steels with F_y > 65 ksi
0.2% offset yield strength, F_y = 100 ksi
0.2% offset (0.002 in./in.)
For F_y = 50 ksi; typical for most structural steels with $F_y \leq$ 65 ksi
Upper yield point
A36 steel
Slope E_{st}
Lower yield point
Elastic range
Plastic range
Strain hardening range to maximum tensile strength
Slope E = modulus of elasticity
Σ_{st}
Stress, ksi
Stress, MPa
Strain ϵ, in./in. (mm/mm)


0.2% offset
Minimum tensile strength
Heat-treated constructional alloy steels; A514 quenched and tempered alloy steel
Minimum yield strength F_y = 100 ksi
(c)
High-strength, low-alloy carbon steels; A440, A441, A572
(b)
F_y = 50 ksi
Carbon steels; A36
(a)
F_y = 36 ksi
Stress, ksi (MPa)
Strain in./in. (mm/mm)

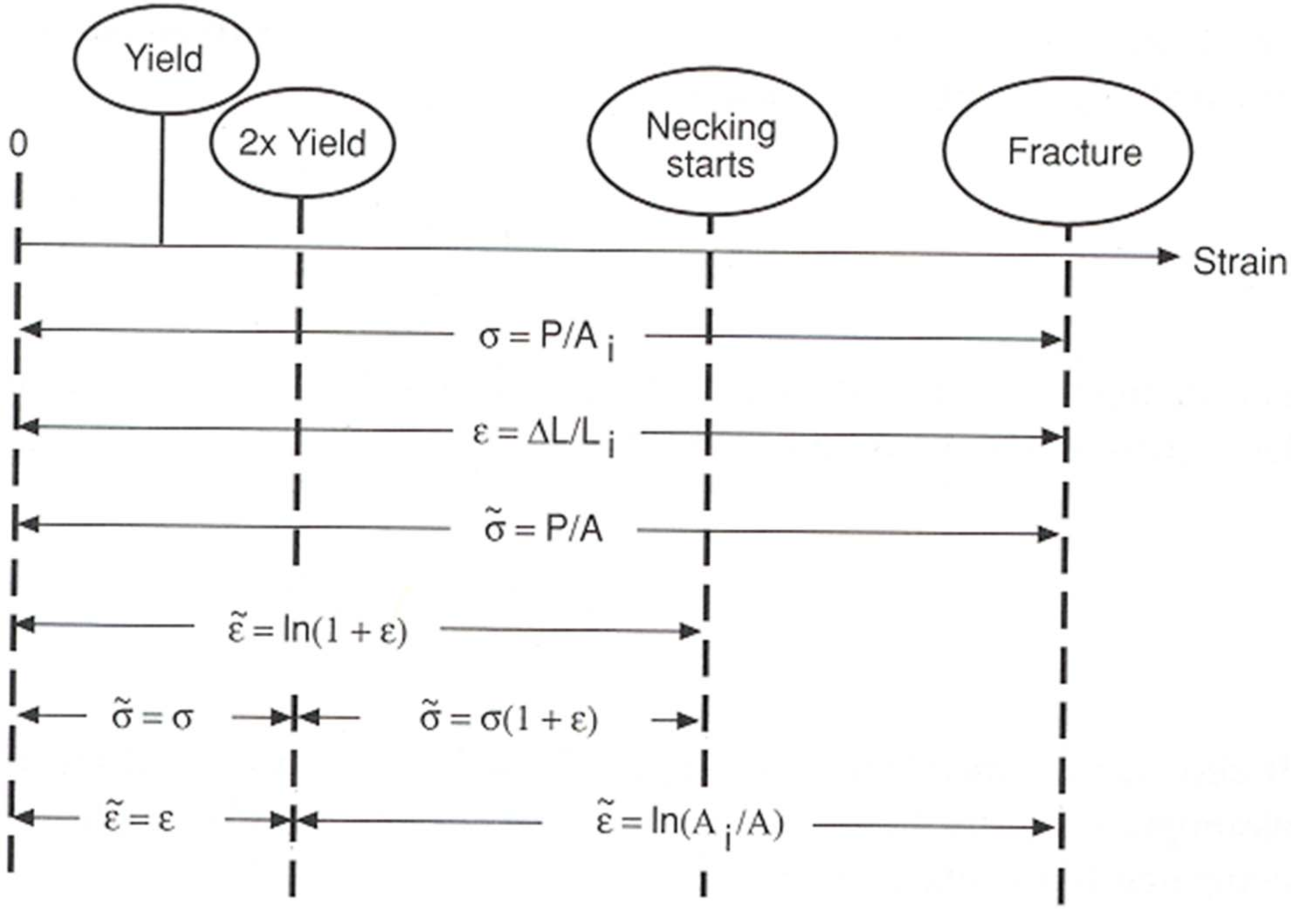
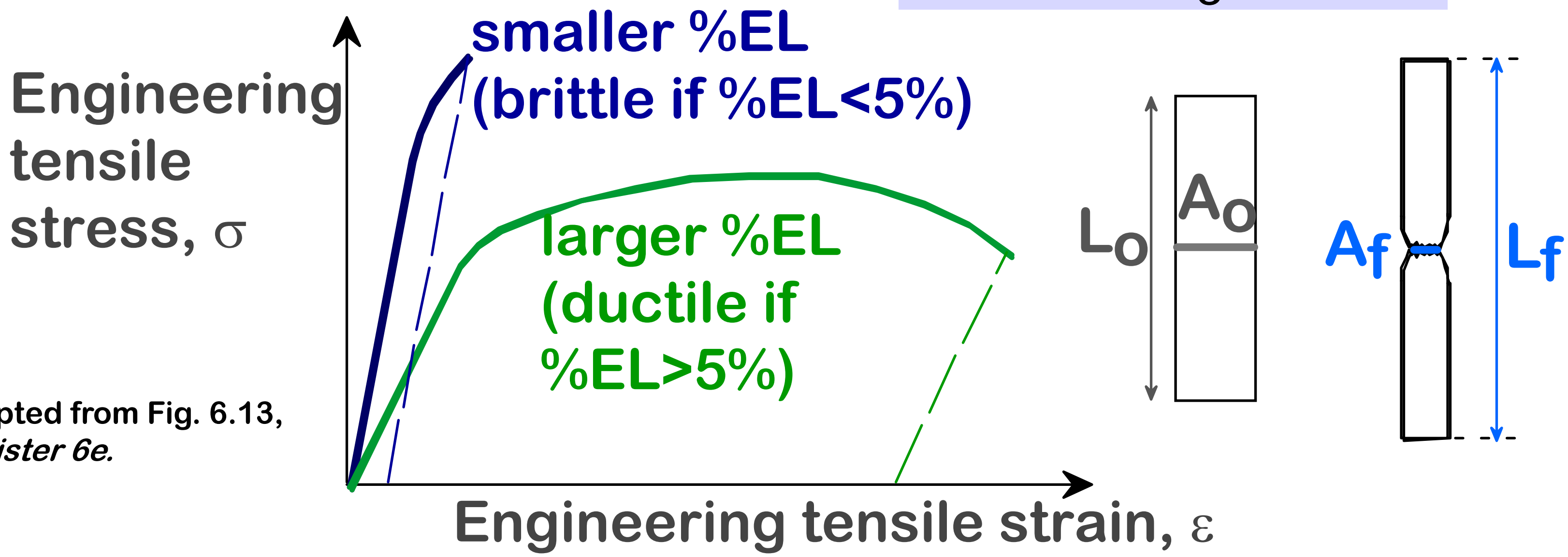


Figure 4.20 Use and limitations of various equations for stresses and strains from a tension test.

Ductility or %EL

- **Plastic tensile strain at failure:**

$$\%EL = \frac{L_f - L_o}{L_o} x100$$



Adapted from Fig. 6.13, *Callister 6e.*

- **Another ductility measure:**

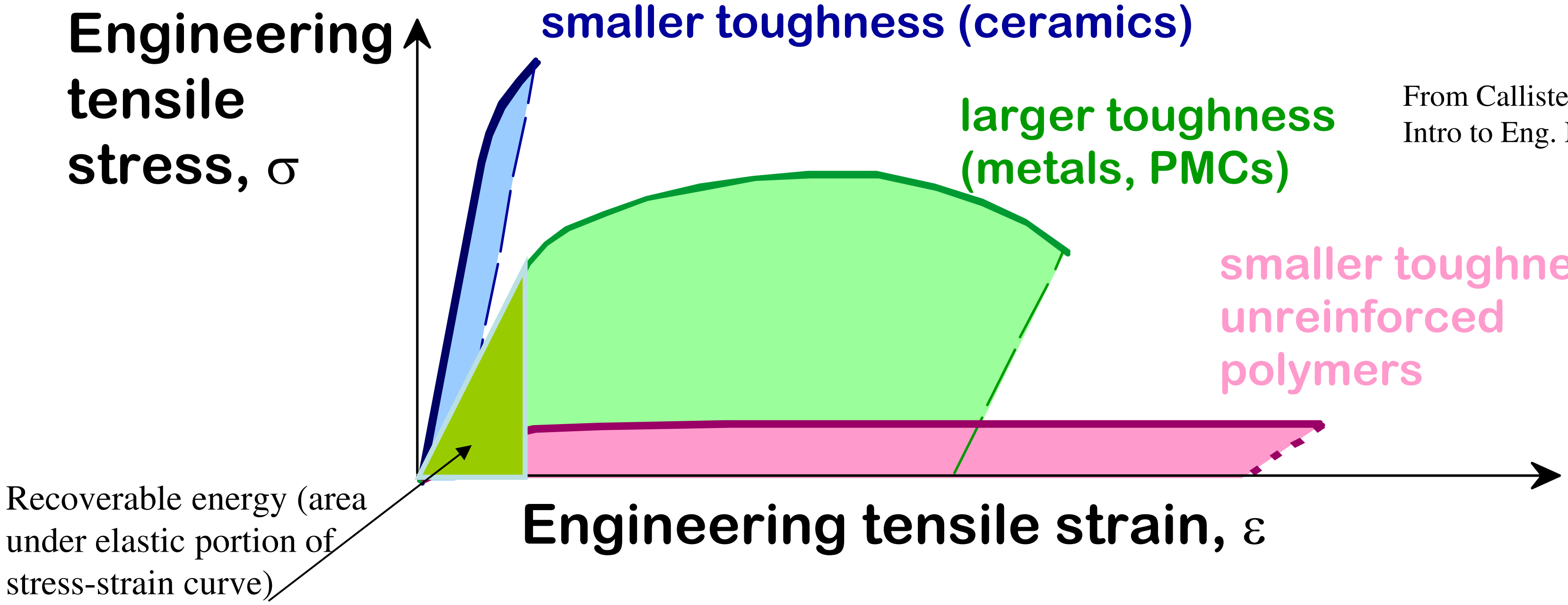
$$\%AR = \frac{A_o - A_f}{A_o} x100$$

- **Note: %AR and %EL are often comparable.**
  - **Reason: crystal slip does not change material volume.**
  - **%AR > %EL possible if internal voids form in neck.**

From Callister, Intro to Eng. Matls., 6Ed

Toughness

- Energy to break a unit volume of material,
  or absorb energy to fracture.
- Approximate as area under the stress-strain curve.

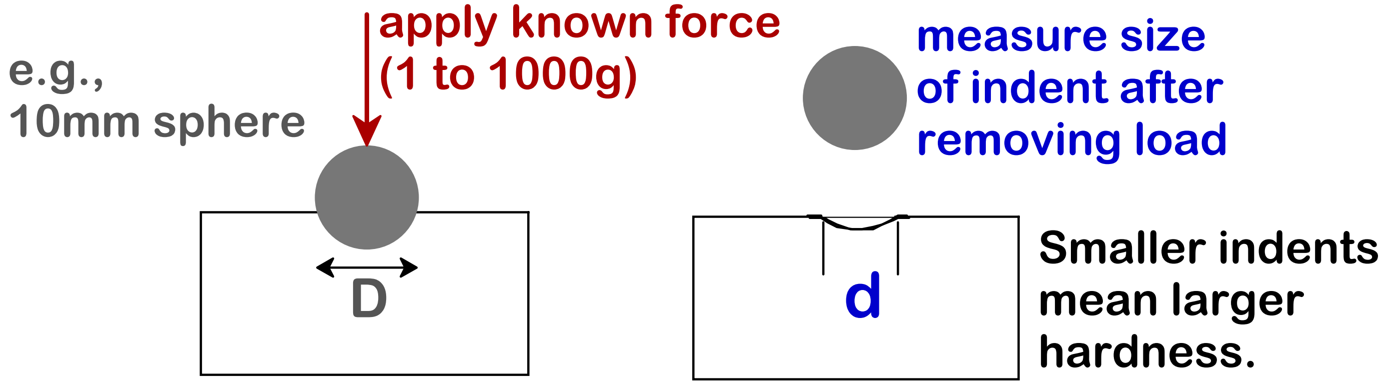


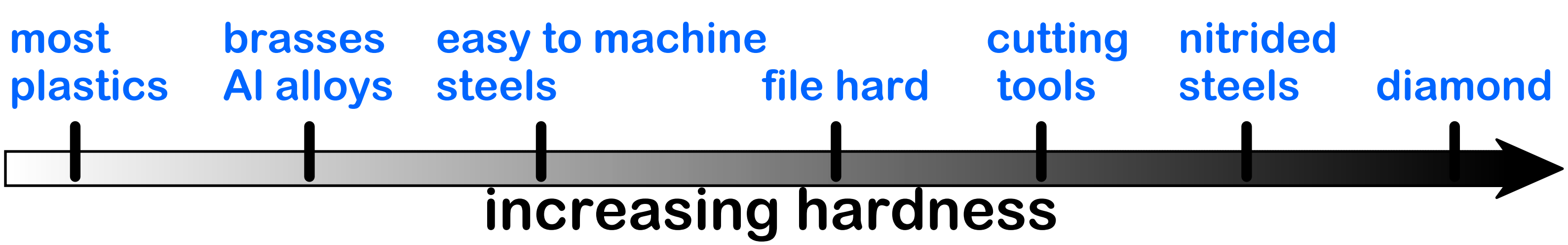
From Callister, Intro to Eng. Matls., 6Ed

Resilience is capacity to absorb energy when deformed *elastically* and *recover* all energy when unloaded ($=\sigma^2_{YS}/2E$).

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  - --resistance to plastic deformation or cracking in compression.
  - --better wear properties.

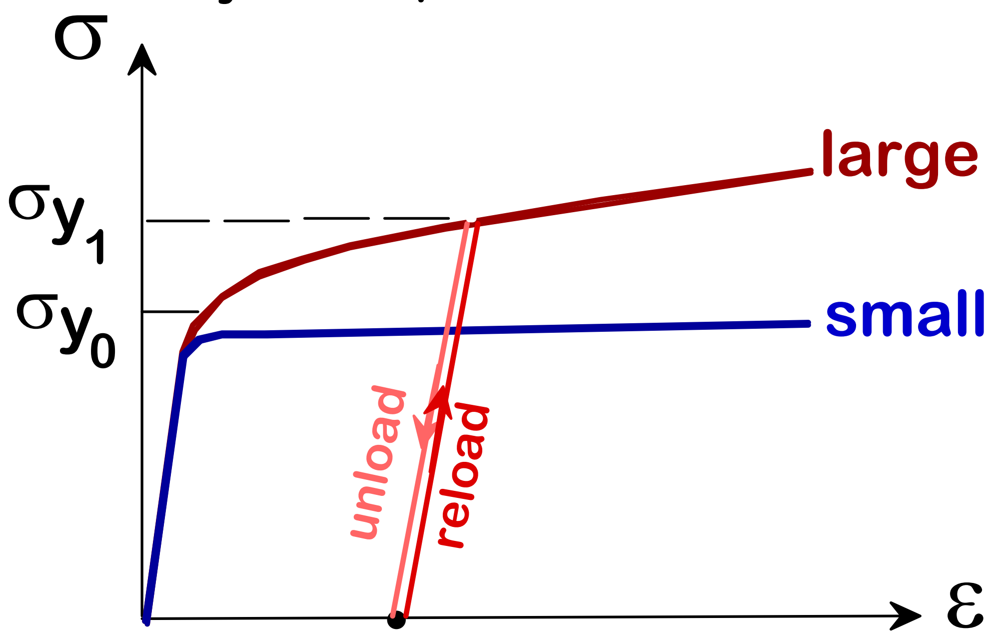




Adapted from Fig. 6.18, *Callister 6e.*

Hardening

- An increase in σy due to plastic deformation.



From Callister, Intro to Eng. Matls., 6Ed

- Curve fit to the stress-strain response after YS:

$$\sigma_T = C(\varepsilon_T)^n$$

hardening exponent:
n=0.15 (some steels)
to n=0.5 (some copper)

"true" stress (F/A)

"true" strain: $\ln(L/L_o)$

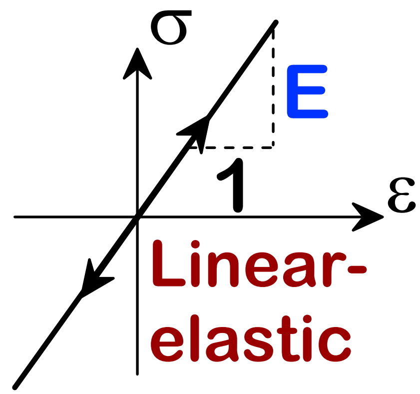
Linear Elasticity: Possion Effect

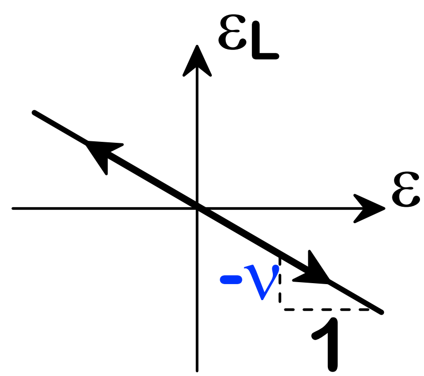
- **Hooke's Law**: $\sigma = E\,\varepsilon$

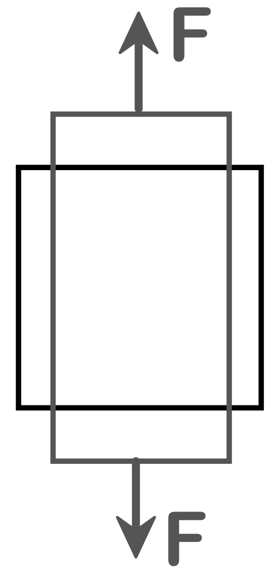
- **Poisson's ratio**, ν:

$$\nu = -\frac{width\ strain}{axial\ strain} = -\frac{\Delta w / w}{\Delta \ell / \ell} = -\frac{\varepsilon_L}{\varepsilon}$$

metals: ν ~ 0.33
ceramics: ~0.25
polymers: ~0.40







simple tension test

Units:
E: [GPa] or [psi]
ν: dimensionless

Why does ν have minus sign?

Poisson Ratio

- *Poisson Ratio has a range* $-1 \le \nu \le 1/2$

Look at extremes

- **No change in aspect ratio:**

$$\Delta w / w = \Delta \ell / \ell$$

$$\nu = -\frac{\Delta w / w}{\Delta \ell / \ell} = -1$$

- **Volume (V = AL) remains constant:** $\Delta V = 0$.

Hence, $\Delta V = (L\,\Delta A + A\,\Delta L) = 0$. **So,** $\Delta A / A = -\Delta L / L$

In terms of width, $A = w^2$, **then** $\Delta A/A = 2\,w\,\Delta w/w^2 = 2\Delta w/w = -\Delta L/L$.

Hence,

$$\nu = -\frac{\Delta w / w}{\Delta \ell / \ell} = -\frac{(-\frac{1}{2}\Delta \ell / \ell)}{\Delta \ell / \ell} = 1/2$$

Incompressible solid.
Water (almost).

Poisson Ratio: materials specific

Metals:	Ir	W	Ni	Cu	Al	Ag	Au	
	0.26	0.29	0.31	0.34	0.34	0.38	0.42	
							generic value ~ 1/3	

Solid Argon: 0.25

Covalent Solids:	Si	Ge	Al_2O_3	TiC	
	0.27	0.28	0.23	0.19	generic value ~ 1/4

Ionic Solids: MgO 0.19

Silica Glass: 0.20

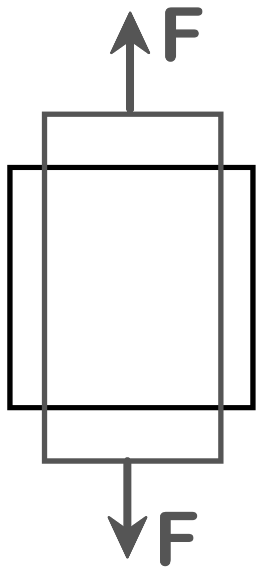
Polymers: Network (Bakelite) 0.49 Chain (PE) 0.40

Elastomer: Hard Rubber (Ebonite) 0.39 (Natural) 0.49

Example: Hooke's Law

• **Hooke's Law**: $\sigma = \mathbf{E}\,\varepsilon$

Copper sample (305 mm long) is pulled in tension with stress of 276 MPa. If deformation is elastic, what is elongation?



simple tension test

For Cu, E = 110 GPa.

$$\sigma = E\varepsilon = E\left(\frac{\Delta\ell}{\ell_0}\right) \Rightarrow \Delta\ell = \frac{\sigma\ell_0}{E}$$

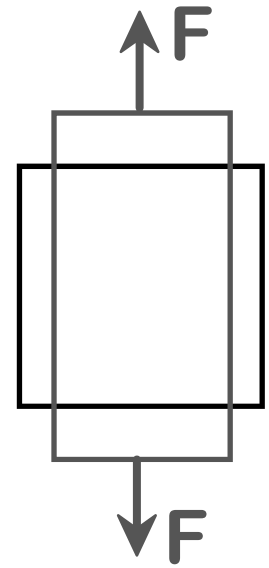
$$\Delta\ell = \frac{(276MPa)(305mm)}{110x10^3 MPa} = 0.77mm$$

Hooke's law involves axial (parallel to applied tensile load) elastic deformation.

Example: Poisson Effect

Tensile stress is applied along cylindrical brass rod (10 mm diameter). Poisson ratio is $\nu = 0.34$ and $E = 97$ GPa.

- Determine load needed for $2.5\text{x}10^{-3}$ mm change in diameter if the deformation is entirely elastic?



simple tension test

Width strain: (note reduction in diameter)

$$\varepsilon_x = \Delta d/d = -(2.5\text{x}10^{-3}\text{ mm})/(10\text{ mm}) = -2.5\text{x}10^{-4}$$

Axial strain: Given Poisson ratio

$$\varepsilon_z = -\varepsilon_x/\nu = -(-2.5\text{x}10^{-4})/0.34 = +7.35\text{x}10^{-4}$$

Axial Stress: $\sigma_z = E\varepsilon_z = (97\text{x}10^3\text{ MPa})(7.35\text{x}10^{-4}) = 71.3$ MPa.

Required Load: $F = \sigma_z A_0 = (71.3\text{ MPa})\pi(5\text{ mm})^2 = 5600$ N.

Complex States of Stress in 3D

- There are 3 *principal* components of stress and strain.
- For linear elastic, isotropic case, use "linear superposition".
- Strain || to load by ***Hooke's Law***: $\varepsilon_i = \sigma_i / E$, i=1,2,3 (or x,y,z).
- Strain ⊥ to load governed by ***Poisson effect***: $\varepsilon_{width} = -\nu \varepsilon_{axial}$.

strain \ stress	σ_1	σ_2	σ_3	Total Strain
ε_1	σ_1/E	$-\nu\sigma_2/E$	$-\nu\sigma_3/E$	→ in x
ε_2	$-\nu\sigma_1/E$	σ_2/E	$-\nu\sigma_3/E$	→ in y
ε_3	$-\nu\sigma_1/E$	$-\nu\sigma_2/E$	σ_3/E	→ in z

In ***x***-direction, the total linear strain is:

$$\varepsilon_1 = \frac{1}{E}\{\sigma_1 - \nu(\sigma_2 + \sigma_3)\} \quad or \quad \frac{1}{E}\{(1+\nu)\sigma_1 - \nu(\sigma_1 + \sigma_2 + \sigma_3)\}$$

Complex State of Stress and Strain in 3-D Solid

- **Hooke's Law and Poisson effect gives total linear strain:**

$$\varepsilon_1 = \frac{1}{E}\{\sigma_1 - \nu(\sigma_2 + \sigma_3)\} \quad or$$

$$\varepsilon_1 = \frac{1}{E}\{(1+\nu)\sigma_1 - \nu\underline{\underline{(\sigma_1 + \sigma_2 + \sigma_3)}}\}$$

Is there something important about Trace of σ (Tr σ)?

- **For uniaxial tension test** $\sigma_1 = \sigma_2 = 0$, so

$\varepsilon_3 = \sigma_3/E$

and $\varepsilon_1 = \varepsilon_2 = -\nu\varepsilon_3$.

Complex State of Stress and Strain in 3-D Solid

- **Hydrostatic Pressure:**

$$P = \sigma_{Hyd} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{Tr\sigma}{3}$$

$$\varepsilon_1 = \frac{1}{E}\{(1+\nu)\sigma_1 - 3\nu P\}$$

- **For volume** **(V=$l_1 l_2 l_3$)** **strain,** **$\Delta V/V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (1-2\nu)\sigma_3/E$**

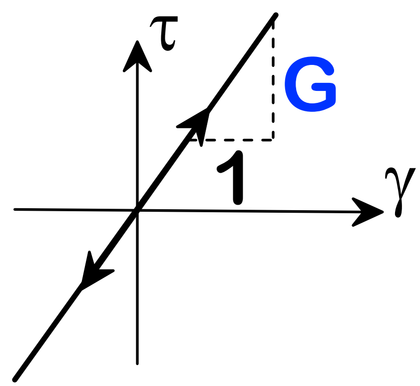
  So $\Delta V/V = 3(1-2\nu)P/E$.

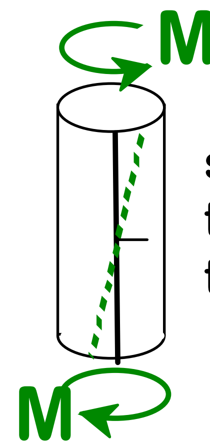
  Bulk Modulus, K: $P = -K\,\Delta V/V$ so $K = 3(1-2\nu)/E$

Other Elastic Properties

- Elastic Shear modulus, G:

$$\tau = G\,\gamma$$

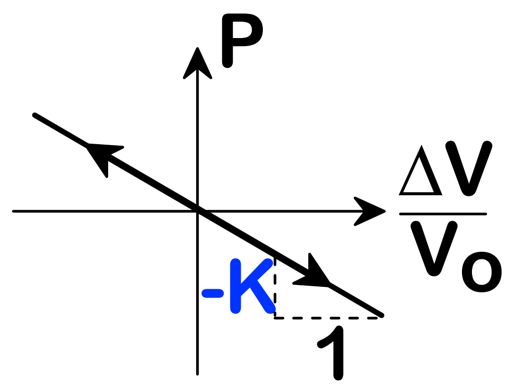


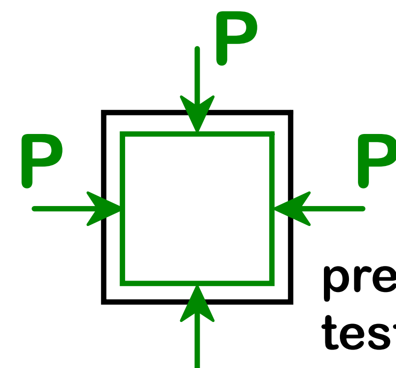


simple torsion test

- Elastic Bulk modulus, K:

$$P = -K\frac{\Delta V}{V_o}$$





pressure test: Init. vol $=V_o$. Vol chg. $= \Delta V$

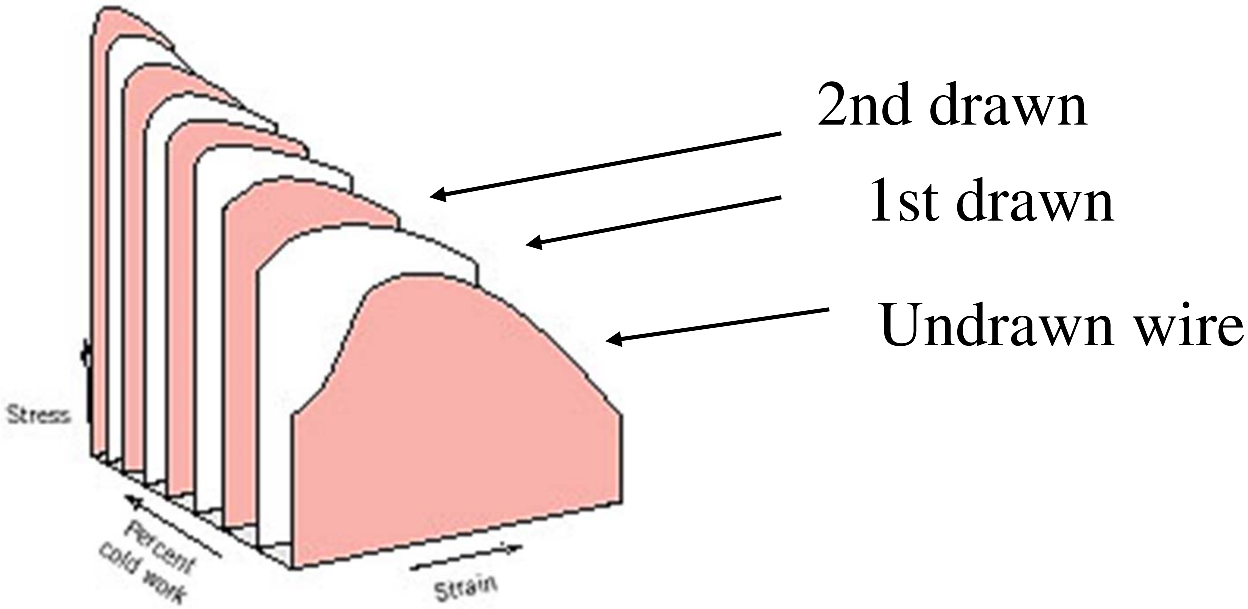
- Special relations for isotropic materials:

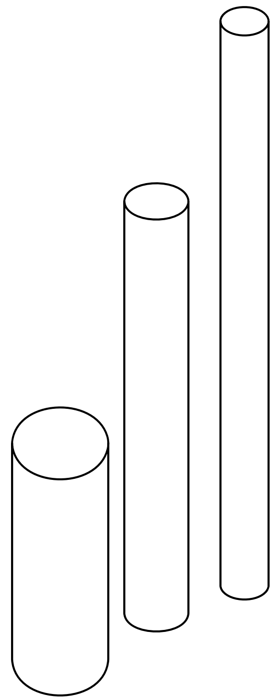
$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

From Callister, Intro to Eng. Matls., 6Ed

Using Work-Hardening

Influence of "cold working" on low-carbon steel.





Processing: Forging, Rolling, Extrusion, Drawing,…

- Each draw of the wire *decreases* ductility, *increases* YS.
- Use drawing to strengthen and thin "aluminum" soda can.

Summary

- Stress and strain: These are size-independent measures of load and displacement, respectively.
- Elastic behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).
- Plastic behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σy.
- Toughness: The energy needed to break a unit volume of material.
- Ductility: The plastic strain at failure.