Experimental Strength of Materials

Lecture #1

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From the notes of Dr. Andrei Reinhorn, UB

Introduction of basic subjects

Learning from Experience

CON-FU-CHI (known as CONFUCIUS in Latin)

“I read → I see
I listen → I understand
I do → I know”

It is only when you experience ‘it’ that you really know ‘it’.

Modern Science and Engineering:

- Theory vs experimentation
- Analytical vs Physical Simulations
- Design vs Practice
- Knowledge-Skill-Methodology

Experimentation –

- quality of data
- reproducibility
- qualifications vs discovery
  - exploration
  - proof of concept
  - qualifications

Modern experimentation

- Combined analytical and physical simulations
- Destructive vs nondestructive testing
- Materials vs components vs structures
- Controlled loading vs random loading
- Electrical vs mechanical measurements

Course Goals

To introduce students to experimental methods, instrumentation, data acquisition, and data processing

The following subjects will be introduced:

Materials testing - steel, timber, plastics, concrete, etc.

Loading Systems - set-ups, loading devices, actuators, control, etc.

Instrumentation - mechanical, electrical, electronic

Data Acquisition - analog and digital

Computerized data processing - numerical and graphical

Resource Materials

1 Dally and Riley, "Experimental Stress Analysis,", McGraw Hill, 1978
2 Harris and Sabnis, "Structural Modeling and Experimental Techniques" CRC Press 1999 (Prof. Harris web page)
3 Nachtigal, C.L., "Instrumentation and Control," Wiley & Sons, 1990
4 Reese and Kawahara, " Handbook of Structural Testing", Prentice Hall / Fairmont Press 1993
5 Malhotra and Carino, "Handbook of Nondestructive Testing of Concrete", CRC Press, 1991
6 Data visualization toolbox from Matlab Link to author of "Visualizing Data", the source for Matlab's toolbox.
7 Instructor's Handouts / Computer Manuals
8 Good strain gage data source from Micro Measurements Group
9 Guide to strain gage installation is also at Micro Measurements Group.
10 Fiber-optic strain gages is at AS-Overlay web site.
13 Data acquisition and analysis on National Instruments website (go to Resource Library).

LAB REPORT ORGANIZATION

1. Summary (executive summary)
   + Information about authors, sponsor, and other participants

2. Scope and general presentation
   - 2.1. Purpose and objectives of testing - general
   - 2.2. Scope of testing

3. Test-set-up overview
   - 3.1. Specimen description–including materials and component properties
   - 3.2. Loading system – description
   - 3.3. Instrumentation set-up and measurement system + calibration procedures
   - 3.4. Data acquisition + schematic information data flow
   - 3.5. Data archiving – structure, model, metadata, curation, transfer

4. Test procedures
   - 4.1. Test schedule & repetitions
   - 4.2. Data monitoring & checking during testing
   - 4.3. Test implementation – notes & metadata

5. Test Results – raw data
   5.1. Data recording and repository inventory
   5.2. Data verification & repository transfer
   5.3. Initial test results

6. Data processing
   6.1. Data checking, verification & recovery
   6.2. Determination of errors & elimination of errors
   6.3. Identifications of material/mechanical parameters and important properties
   6.4. Correction of test results through data processing – procedures

7. Analytical predictions (before modifying analytical models)
   7.1. Calculated model parameters using principles of engineering
   7.2. Calculated response using simplified or sophisticated model
   7.3. Calculated response using identified parameters
   7.4. Comparison of response of experiment analysis with estimated and with measured parameters

8. Discussions and recommendations
   8.1. Discussion of information as obtained from tests
   8.2. Recommendation to reduce gap between computed and tested

Laws in Experimental Studies

1. Murphy’s Law –If something can go wrong it will

2. O’Toole’s Law –Murphy’s Law is too optimistic

3. Reinhorn’s Law –Things are never as bad as they turn out to be

4. Bracci’s Law –Anything can be accomplished with time and ..money

Characteristics of Observations

1. **Qualitative:** Characteristics of behavior identified so that the phenomenon may be accurately described.

   This is described in terms of standardized operations which identify classes of quantities such as length (L), force (F) and time (T).

   Note that L, F, and T are measurable quantities.

2. **Quantitative:** Involves both a number and a standard of comparison. i.e. 3ft , 9lbs , 13.2 minutes These are called UNITS

For example: Velocity has dimensions of LT^{-1}, and units such as mph, ft/sec, and knots

For scientific measurements,

M , L , and T are regarded as basic,

But for engineering purposes,

F , L , and T are more convenient.

Note that they are interrelated through Newton's Second Law of Motion

$$F=ma$$

or

$$F=M(L/T^2)=MLT^{-2}$$

1
Data Reduction

A necessary first step in any engineering situation is an investigation of available data to assess the nature and the degree of the uncertainty. An unorganized list of numbers representing the outcomes of tests is not easily assimilated. There are several methods of organization, presentation, and reduction of observed data which facilitate its interpretation and evaluation.

It should be pointed out explicitly that the treatment of data described in this chapter is in no way dependent on the assumptions that there is randomness involved or that the data constitute a random sample of some mathematical probabilistic model. These are terms which we shall come to know and which some readers may have encountered previously. The methods here are simply convenient ways to reduce raw data to manageable forms.

In studying the following examples and in doing the suggested problems, it is important that the reader appreciate that the data are "real" and that the variability or scatter is representative of the magnitudes of variation to be expected in civil-engineering problems.

1.1 GRAPHICAL DISPLAYS

Histograms A useful first step in the representation of observed data is to reduce it to a type of bar chart. Consider, for example, the data presented in Table 1.1.1. These numbers represent the live loads observed in a New York warehouse. To anticipate the typical, the extreme, and the long-term behavior of structural members and footings in such structures, the engineer must understand the nature of load distribution. Load variability will, for example, influence relative settlements of the column footings. The values vary from 0 to 229.5 pounds per square foot (psf). Let us divide this range into 20-psf intervals, 0 to 19.9, 20.0 to 39.9, etc., and tally the number of occurrences in each interval.

Plotting the frequency of occurrences in each interval as a bar

Table 1.1.1 Floor-load data*

Bay	*Basement*	*1st*	*2d*	*3d*	*4th*	*5th*	*6th*	*7th*	*8th*	*9th*
A	0	7.8	36.2	60.6	64.0	64.2	79.2	88.4	38.0	72.7
B	72.2	72.6	74.4	21.8	17.1	48.5	16.8	105.9	57.2	75.7
C	225.7	42.5	59.8	41.7	39.9	55.5	67.2	122.8	45.2	62.9
D	55.1	55.9	87.7	59.2	63.1	58.8	67.7	90.4	43.3	55.2
E	36.6	26.0	90.5	23.0	43.5	52.1	102.1	71.7	4.1	37.3
F	129.4	66.4	138.7	127.9	90.9	46.9	197.5	151.1	157.3	197.0
G	134.6	73.4	80.9	53.3	80.1	62.9	150.8	102.2	6.4	45.4
H	121.0	106.2	94.4	139.6	152.5	70.2	111.8	174.1	85.4	83.0
I	178.8	30.2	44.1	157.0	105.3	87.0	50.1	198.0	86.7	64.6
J	78.6	37.0	70.7	83.0	179.7	180.2	60.6	212.4	72.2	86.0
K	94.5	24.1	87.3	80.6	74.8	72.4	131.1	116.1	53.6	99.1
L	40.2	23.4	8.4	42.6	43.4	27.4	63.8	18.4	16.2	58.7
M	92.2	49.8	50.9	116.4	122.9	132.3	105.2	160.3	28.7	46.8
N	99.5	106.9	55.9	136.8	110.4	123.5	92.4	160.9	45.4	96.3
O	88.5	48.4	62.3	71.3	133.2	92.1	111.7	67.9	53.1	39.7
P	93.2	55.0	80.8	143.5	122.3	184.2	150.0	57.6	6.8	53.3
Q	96.1	54.8	63.0	228.3	139.3	59.1	112.1	50.9	158.6	139.1
R	213.7	65.7	90.3	198.4	97.5	155.1	163.4	155.3	229.5	75.0
S	137.6	62.5	156.5	154.1	134.3	81.6	194.4	155.1	89.3	73.4
T	79.8	68.7	85.6	141.6	100.7	106.0	131.1	157.4	80.2	65.0
U	78.5	118.2	126.4	33.8	124.6	78.9	146.0	100.3	97.8	75.3
V	24.8	55.6	135.6	56.3	66.9	72.2	105.4	98.9	101.7	58.2

* Observed live loads (in pounds per square foot); bay size: 400 ± 6 ft^2.
Source: J. W. Dunham, G. N. Brekke, and G. N. Thompson [1952], "Live Loads on Floors in Buildings," Building Materials and Structures Report 133, National Bureau of Standards, p. 22.

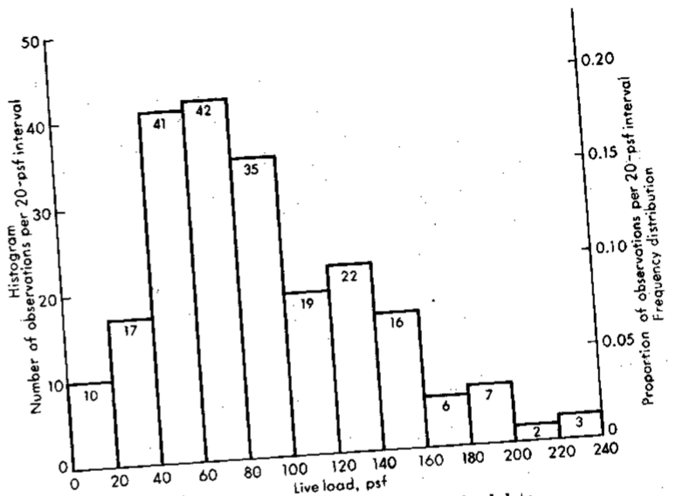


Fig. 1.1.1 Histogram and frequency distribution of floor-load data.

yields a *histogram*, as shown in Fig. 1.1.1. The height, and more usefully, the area, of each bar are proportional to the number of occurrences in that interval. The plot, unlike the array of numbers, gives the investigator an immediate impression of the range of the data, its most frequently occurring values, and the degree to which it is scattered about the central or typical values. We shall learn in Chap. 2 how the engineer can predict analytically from this shape the corresponding curve for the *total* load on a column supporting, say, 20 such bays.

If the scale of the ordinate of the histogram is divided by the total number of data entries, an alternate form, called the *frequency distribution*, results. In Fig. 1.1.1, the numbers on the right-hand scale were obtained by dividing the left-hand scale values by 220, the total number of observations. One can say, for example, that the proportion of loads observed to lie between 120 and 139.9 psf was 0.10. If this scale were divided by the interval length (20 psf), a *frequency density distribution* would result, with ordinate units of "frequency per psf." The area under this histogram would be unity. This form is preferred when different sets of data, perhaps with different interval lengths, are to be compared with one another.

The *cumulative frequency distribution*, another useful graphical representation of data, is obtained from the frequency distribution by calculating the successive partial sums of frequencies up to each interval division point. These points are then plotted and connected by straight lines to form a nondecreasing (or monotonic) function from zero to unity.

In Fig. 1.1.2, the cumulative frequency distribution of the floor-load data, the values of the function at 20, 40, and 60 psf were found by forming the partial sums $0 + 0.0455 = 0.0455$, $0.0455 + 0.0775 = 0.1230$, and $0.1230 + 0.1860 = 0.3090$.† From this plot, one can read that the proportion of the loads observed to be equal to or less than 139.9 psf was 0.847. After a proper balancing of initial costs, consequences of poor performance, and these frequencies, the designer might conclude that a beam supporting one of these bays must be stiff enough to avoid deflections in excess of 1 in. in 99 percent of all bays. Thus the design should be checked for deflections under a load of 220 psf.

Some care should be taken in choosing the width of each interval

† When constructing the cumulative frequency distribution, one can avoid the arbitrariness of the intervals by plotting one point per observation, that is, by plotting i/n versus $x^{(i)}$, where $x^{(i)}$ is the ith in ordered list of data (see Fig. 1.2.1).

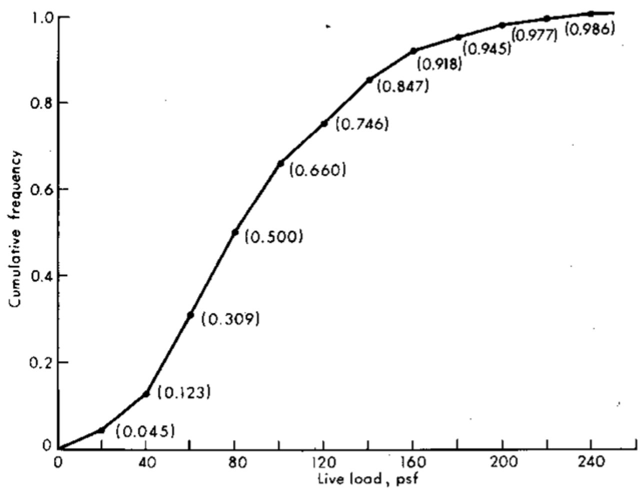
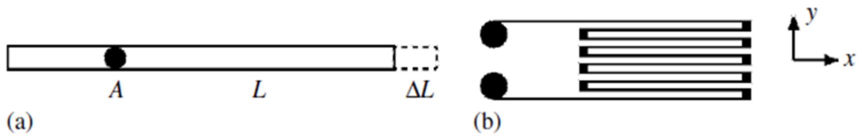


Fig. 1.1.2 Cumulative frequency distribution of floor-load data.

Electrical resistance Strain Gages

- Electrical Resistance of a conductor (wire) changes proportionally to any strain applied (Lord Kelvin 1856)
- By 1930 the first practical application

$$R = \rho \frac{L}{A}$$



- ρ :specific resistivity (inversely proportional to number of mobile electrons per unit volume $\rho = K/(N/V)$)

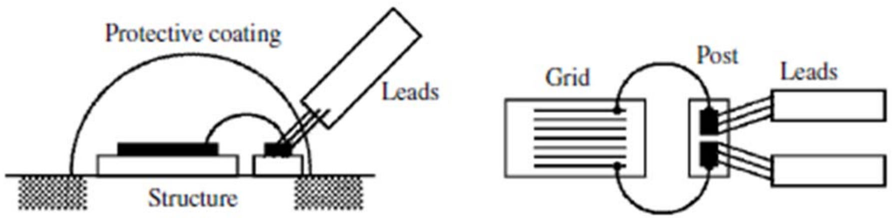
$$R = K\frac{VL}{NA} = K\frac{L^2}{N}$$

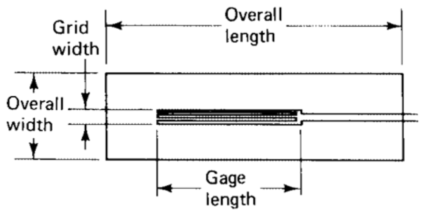
$$\Delta R = K\frac{1}{N}2L\Delta L - K\frac{L^2}{N^2}\Delta N \quad \text{or} \quad \frac{\Delta R}{R} = 2\frac{\Delta L}{L} - \frac{\Delta N}{N}$$

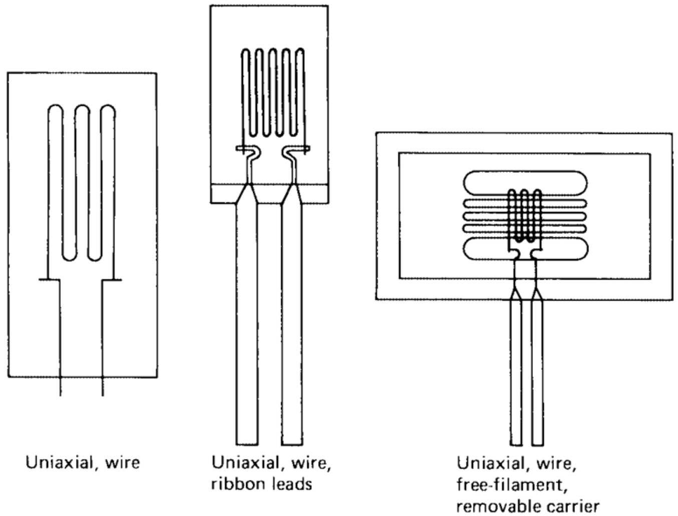
$$\epsilon = \frac{\Delta R}{R} / S_g$$

The core of the gage is the sensing element, which may be of three basic types. The original gages were made by wrapping a very fine wire around a form. These gages have been superseded by foil gages that are formed by photoetching thin sheets (usually less than 0.005 mm (0.0002 in.) thick) of heat-treated metallic alloys to obtain the desired grid patterns and dimensions. Foil gages give excellent flexibility in the design of grid shapes. They also can be obtained in sizes ranging from 0.8 to 152 mm (0.032 to 6 in.). The newer form of gage is of the piezoresistive type, which uses a doped crystal element. These give high signal outputs but are quite inflexible with regard to design and shape.

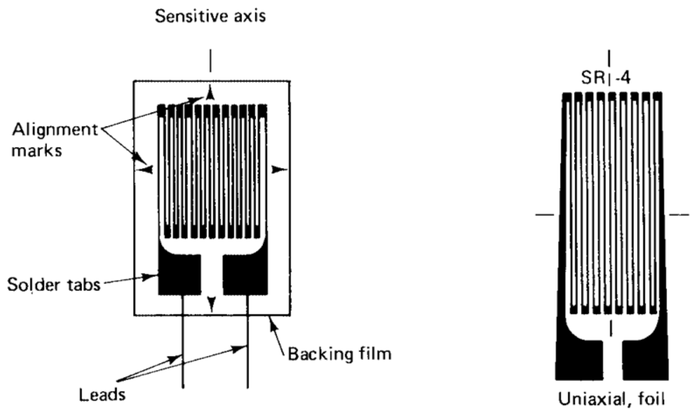
Two of the more popular materials used in strain gages are Constantan, a copper–nickel alloy used primarily in static strain analysis because of its low and controllable temperature coefficient, and Isoelastic, a nickel–iron alloy recommended for dynamic tests where its large temperature coefficient is inconsequential, and advantage can be made of its high gage factor. Silicone is used primarily in semiconductor gages where the specially processed crystals are cut into filaments. They exhibit large resistance changes, but are very temperature sensitive and cannot be self-compensated.




Grid
width
Overall
length
Overall
width
Gage
length


Uniaxial, wire
Uniaxial, wire,
ribbon leads
Uniaxial, wire,
free-filament,
removable carrier

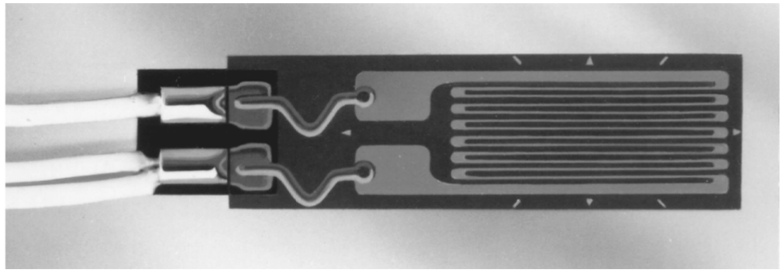
(a)


Sensitive axis
Alignment
marks
Solder tabs
Leads
Backing film
SR-4
Uniaxial, foil

(b)

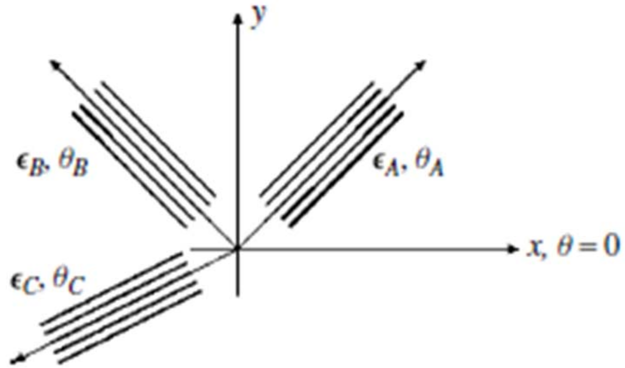
(a)

(b)



(c)

Measure only uniaxial strain



$$\epsilon_{\theta\theta} = \epsilon_{xx} \cos^2\theta + \epsilon_{yy} \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

$$\begin{Bmatrix} \epsilon_A \\ \epsilon_B \\ \epsilon_C \end{Bmatrix} = \begin{bmatrix} \cos^2\theta_A & \sin^2\theta_A & \cos\theta_A \sin\theta_A \\ \cos^2\theta_B & \sin^2\theta_B & \cos\theta_B \sin\theta_B \\ \cos^2\theta_C & \sin^2\theta_C & \cos\theta_C \sin\theta_C \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\epsilon_{\text{data}}\} = [\ C\ ]\{\epsilon_{\text{coord}}\}.$$

Measure R

When a strain gage is on a steel bar experiencing stress of 10ksi (E/300) the change in resistance is given by

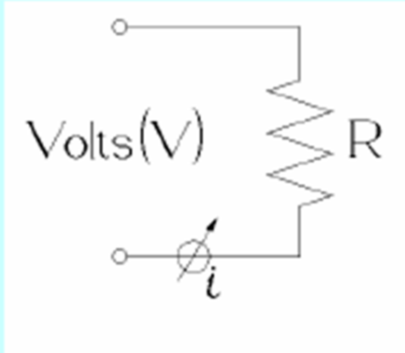
$$\frac{\Delta R}{R} = S_g \epsilon = 2\frac{\sigma}{E} = 2\frac{10000}{30 \times 10^6} = 0.00066\ \Omega/\Omega$$

For 120 Ω gage R+ΔR=120.0007 Ω which is not easily detected by an Ohmmeter

Measurements

1. Direct Resistance (ohms)

$$R_0 = \frac{V}{i_0}$$



$$\Delta R + R_0 = \frac{V}{i_0 + \Delta i}$$

Divide by R_0 (or V/i_0)

$$\frac{\Delta R}{R_0} + 1 = \frac{1}{\frac{\Delta i}{i_0} + 1}$$

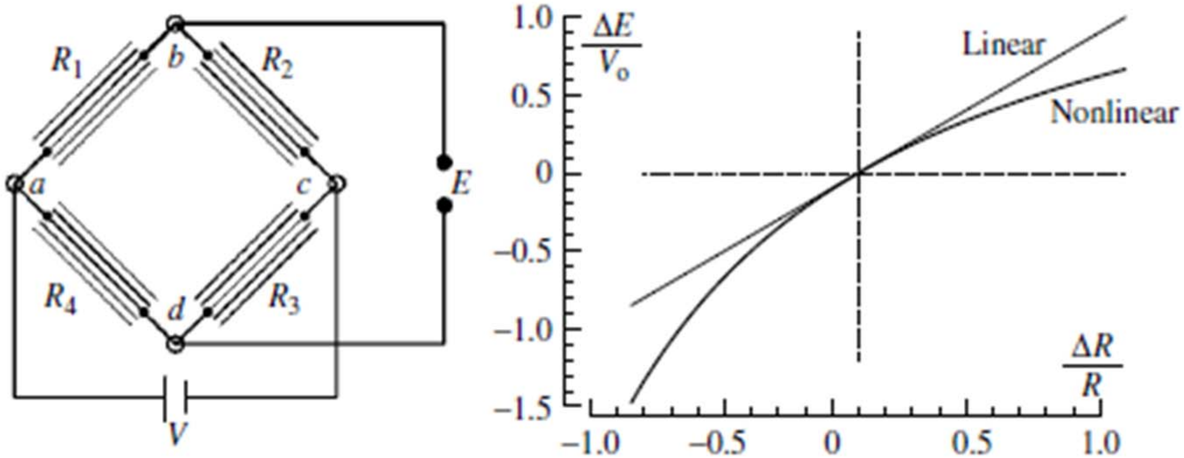
$$\boxed{\Rightarrow \frac{\Delta R}{R_0} = -\frac{\Delta i}{i_0} \cdot \frac{1}{1 + \frac{\Delta i}{i_0}}}$$

This is extremely inaccurate since $\Delta R/R_0$ is very small!

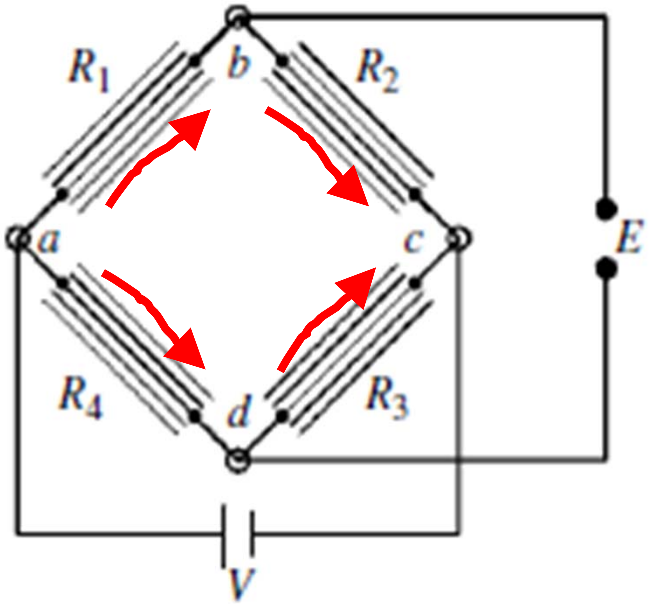
WHEATSTONE BRIDGE

So it required something to measure directly the change of resistance ΔR .

That is a WHEATSTONE BRIDGE



The Wheatstone bridge. (a) Circuit diagram. (b) Nonlinearity of Wheatstone bridge.

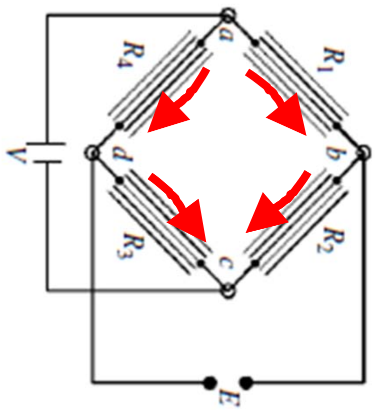


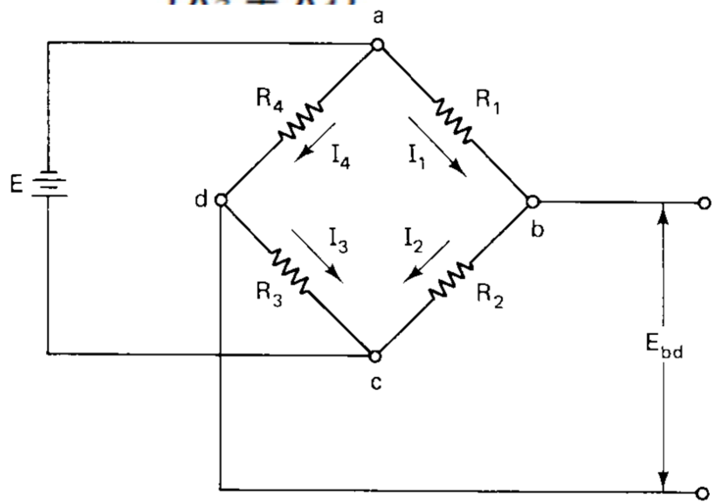
$$I_{abc} = \frac{V}{(R_1 + R_2)}, \qquad I_{adc} = \frac{V}{(R_3 + R_4)}$$

$$V_{ab} = I_{abc} R_1 = \frac{V R_1}{(R_1 + R_2)}, \qquad V_{ad} = I_{adc} R_4 = \frac{V R_4}{(R_3 + R_4)}$$

$$E = V_{ab} - V_{ad} = V \left[\frac{R_1}{(R_1 + R_2)} - \frac{R_4}{(R_3 + R_4)} \right]$$

$$V_{ab} = I_{abc}R_1 = \frac{V R_1}{(R_1 + R_2)}, \qquad V_{ad} = I_{adc}R_4 = \frac{V R_4}{(R_3 + R_4)}$$





$$V_{cb} = I_{abc}R_2 = \frac{V R_2}{R_1 + R_2} \qquad V_{cd} = I_{adc}R_3 = \frac{\quad}{R_3 + R_4}$$

$$\frac{dV_{ab}}{dR_1} = V\frac{R_1}{(R_1 + R_2)^2} \xrightarrow{\text{yields}} dV_{ab} = V\left(\frac{dR_1}{R_1}\right) \frac{\frac{R_2}{R_1}}{\left(1 + \frac{R_2}{R_1}\right)^2}$$

$$\frac{dV_{ab}}{dR_2} = V\frac{R_1}{(R_1 + R_2)^2} \xrightarrow{\text{yields}} dV_{ab} = -V\left(\frac{dR_2}{R_2}\right) \frac{\frac{R_1}{R_2}}{\left(1 + \frac{R_1}{R_2}\right)^2}$$

$$\frac{dV_{cb}}{dR_2} = V\frac{R_2}{R_1 + R_2} \xrightarrow{\text{yields}} dV_{cb} = V\left(\frac{dR_2}{R_2}\right) \frac{\frac{R_1}{R_2}}{\left(1+\frac{R_1}{R_2}\right)^2}$$

For a change in Resistance $dR_1 \neq 0$

$$dV_{ab} = V\left(\frac{dR_1}{R_1}\right) \frac{\frac{R_2}{R_1}}{\left(1+\frac{R_2}{R_1}\right)^2}$$

$$dV \sim \left(\frac{dR_1}{R_1}\right)$$

For a change in Resistance $dR_2 \neq 0$

$$dV_{ab} = -V\left(\frac{dR_2}{R_2}\right) \frac{\frac{R_1}{R_2}}{\left(1+\frac{R_1}{R_2}\right)^2}$$

$$dV \sim -\left(\frac{dR_2}{R_2}\right)$$

For a change in Resistance in all four gages $dR_i \neq 0$

$$dV \sim \left(\frac{dR_1}{R_1} - \frac{dR_2}{R_2} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)$$

$$\frac{dR_1}{R_1} = GF \ \ \varepsilon_1$$

$$\frac{dV}{V} = GF\ (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) \cdot \text{Const}$$

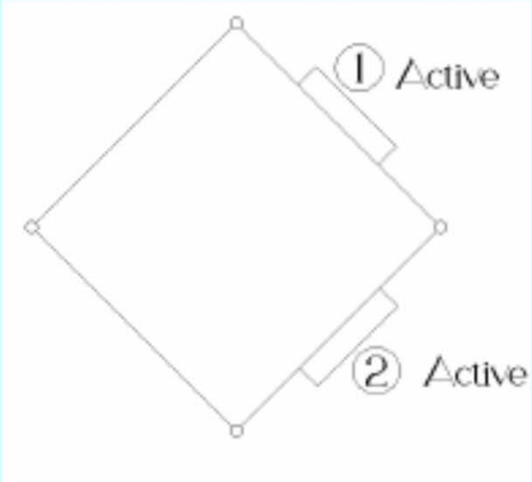
Temperature Compensation

$$\Delta R^{T^\circ} = \alpha_T \Delta T^\circ R \rightarrow \text{change in resistance}$$

This produces a strain reading

$$\varepsilon^{T^\circ} = \frac{\Delta R^{T^\circ} / R}{GF}$$

Apply circuit compensation (half and/or full bridge)



$$\varepsilon_1 = \varepsilon_1 + \varepsilon^{T^\circ}$$

$$\varepsilon_2 = \varepsilon_2 + \varepsilon^{T^\circ}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{GF}{(1+\alpha)^2}\left[\left(\varepsilon_1 + \varepsilon^{T^\circ}\right) - \left(\varepsilon_2 + \varepsilon^{T^\circ}\right)\right]$$

$$\frac{\Delta V}{V} = \frac{GF}{(1+\alpha)^2}\left[\varepsilon_1 + \varepsilon_2\right]$$

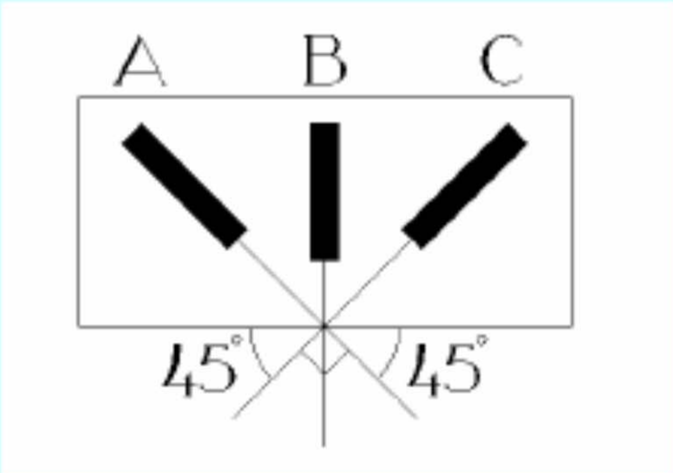
Note: Temperature effects cancel!!

Recommendations:

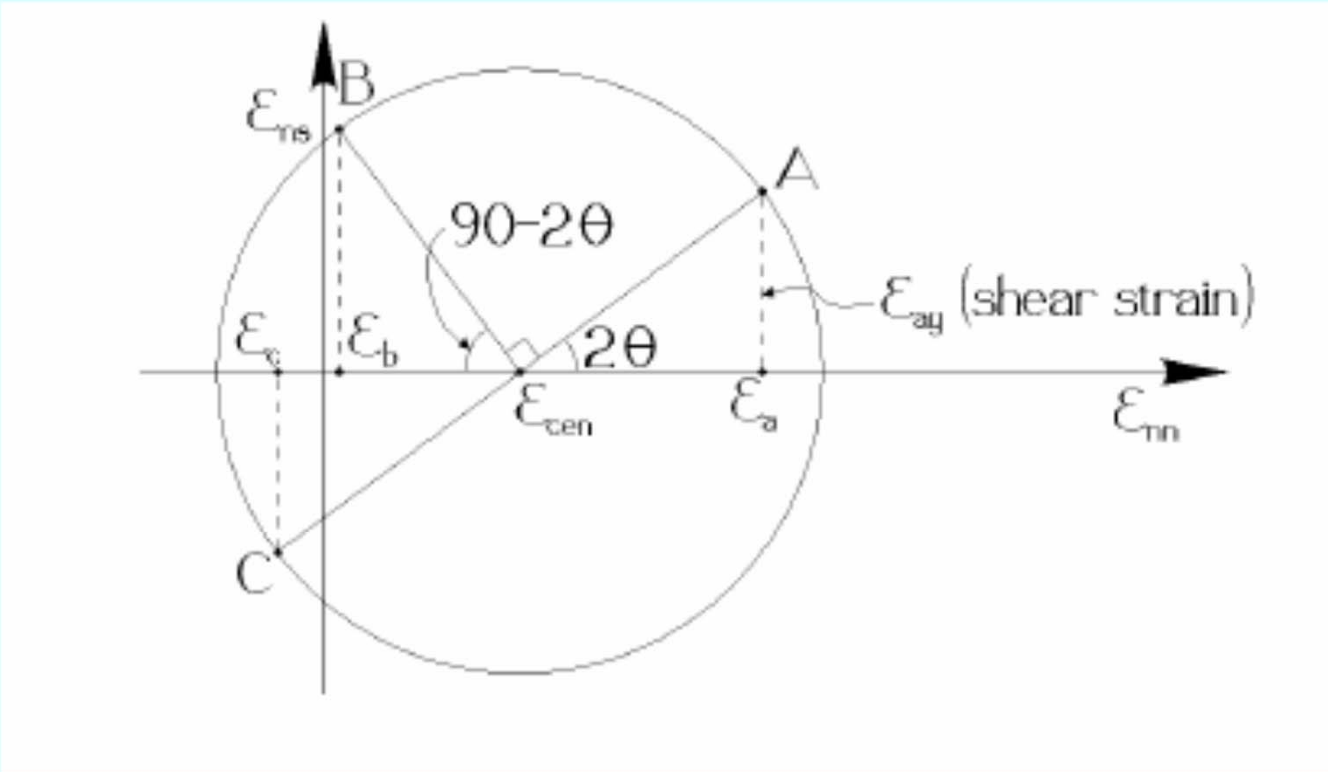
- Use compensated circuits for instruments.
- Use non-compensated circuits for quick measurements of strains. Requires calibration before each test.

Strain Rosettes – Principal Stresses

45° Rosette



- ε_A, ε_B, ε_C are known
- Use Mohr's Circle:



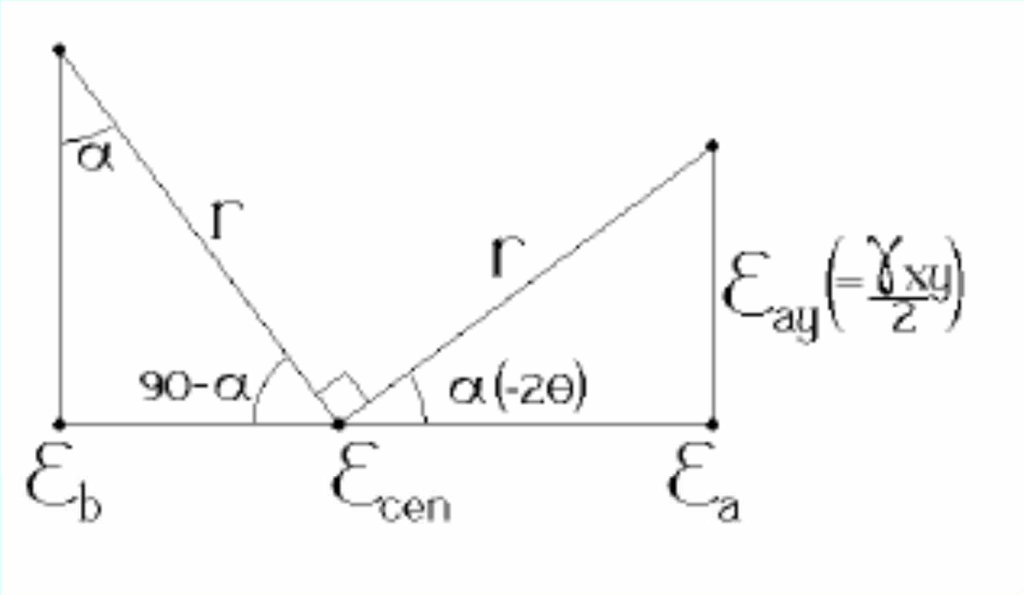
$$\varepsilon_{ay} = \varepsilon_{cen} - \varepsilon_B$$

$$= \frac{\varepsilon_A + \varepsilon_C}{2} - \varepsilon_B$$

(Where ε_B is zero since there is no normal strain.)

$$\varepsilon_{cen} = \frac{\varepsilon_A + \varepsilon_C}{2}$$

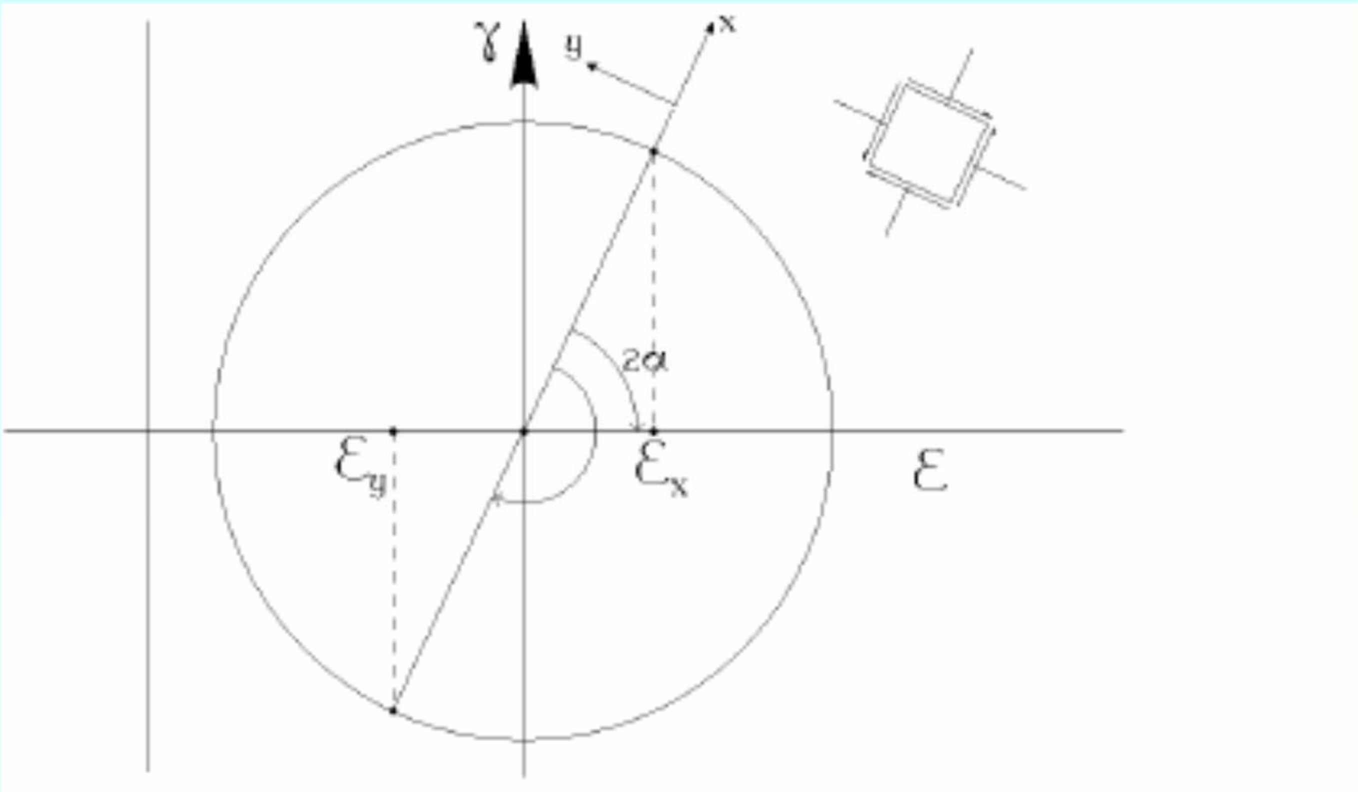
So, from geometry:



$$\sin\alpha = \frac{\varepsilon_{ay}}{r}$$

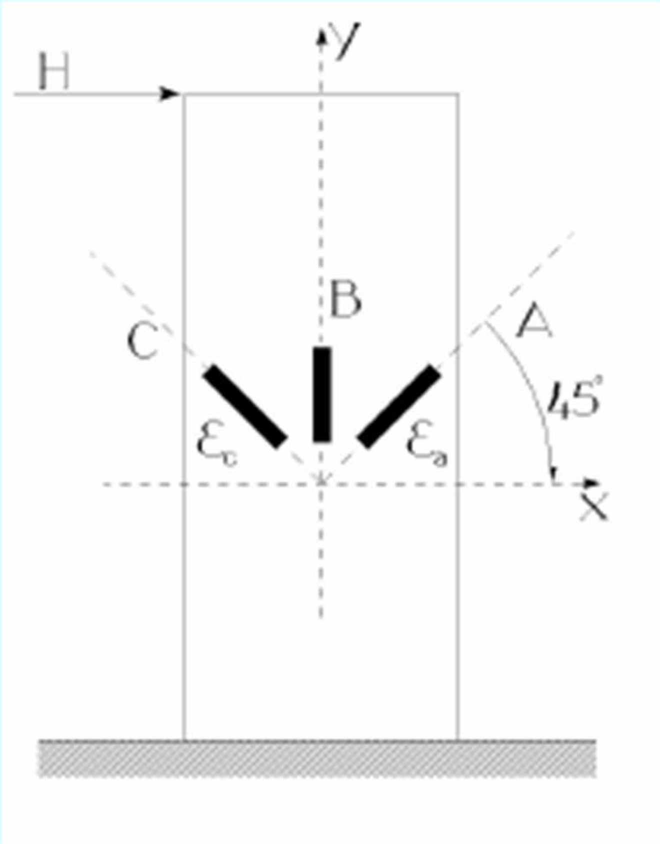
$$\sin\alpha = \frac{\varepsilon_{cen} - \varepsilon_B}{r}$$

$$\Rightarrow \varepsilon_{ay} = \varepsilon_{cen} - \varepsilon_B$$



γ
y
x
2α
ε_y
ε_x
ε

Problem: Determine the arrangement of a rosette to measure shear stress at a point

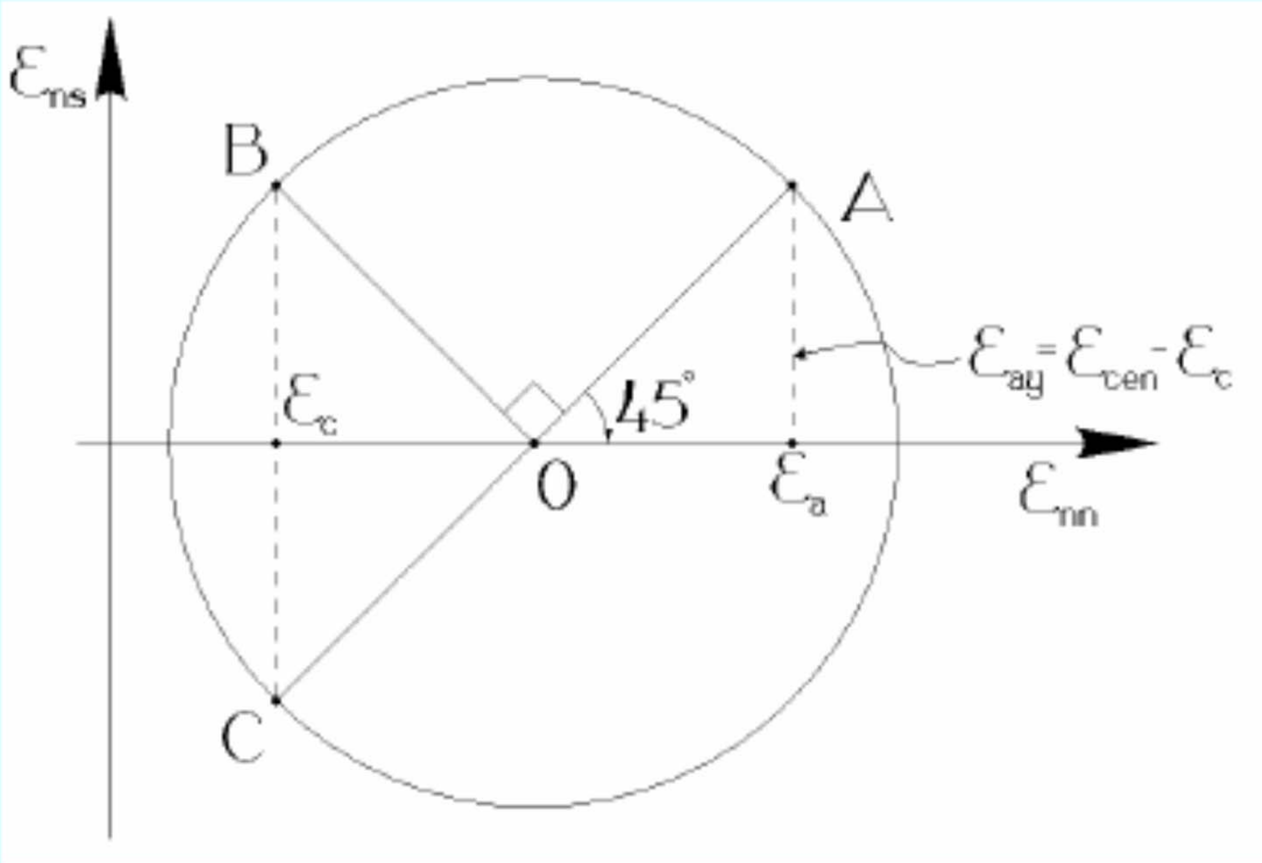


$$\sigma_x = 0$$

$$\sigma_y = 0$$

$$\tau_{xy} = H/A_x$$

where A_x = corresponding shear area

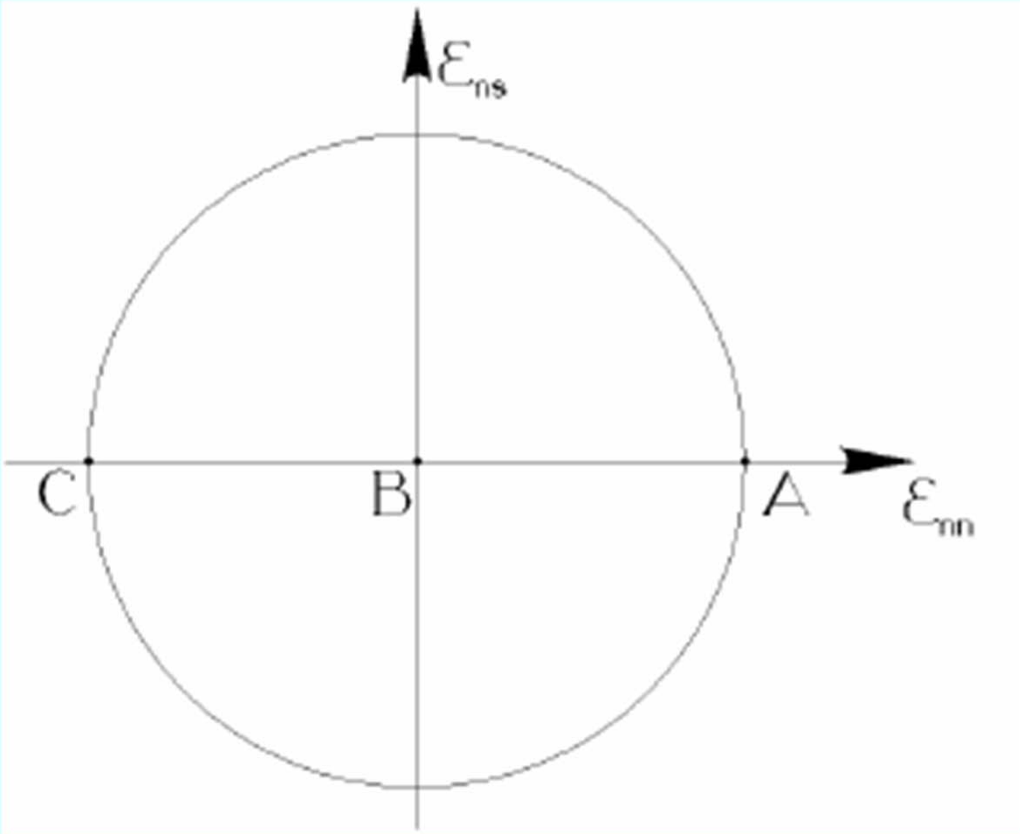


$$= \frac{\varepsilon_A + \varepsilon_C}{2} - \varepsilon_C$$

$$= \frac{\varepsilon_A - \varepsilon_C}{2}$$

Note that this is independent of the magnitude of ε_B, since $\varepsilon_B = \varepsilon_C$;
So it is sufficient to use two gauges to determine ε_{xy}!!

For pure shear behaviour,



$\varepsilon_B = 0,$

$\varepsilon_A = 0,$

$\Rightarrow \varepsilon_{xy} = \varepsilon_A = \varepsilon_C$

Principal Strains

$$\varepsilon_{max,min} = \frac{\varepsilon_A + \varepsilon_C}{2} \pm \sqrt{\frac{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_C)^2}{2}}$$

$$\text{radius} = \sqrt{(\varepsilon_A - \varepsilon_{cen})^2 + (\varepsilon_{cen} - \varepsilon_B)^2}$$

$$= \sqrt{\left(\varepsilon_A - \frac{\varepsilon_A + \varepsilon_C}{2}\right)^2 + \left(\frac{\varepsilon_A + \varepsilon_C}{2} - \varepsilon_B\right)^2}$$

$$= \sqrt{\left(\frac{\varepsilon_A}{2} - \frac{\varepsilon_C}{2}\right)^2 + \left(\frac{\varepsilon_A}{2} + \frac{\varepsilon_C}{2} - \varepsilon_B\right)^2}$$

$$
= \left(\frac{\varepsilon_A^2}{4} - \frac{2\varepsilon_A \varepsilon_C}{4} + \frac{\varepsilon_C^2}{4} + \frac{\varepsilon_A^2}{4} + \frac{\varepsilon_A \varepsilon_C}{4} - \frac{\varepsilon_A \varepsilon_B}{2} \right.
$$

$$
\left. + \frac{\varepsilon_C^2}{4} + \frac{\varepsilon_C \varepsilon_A}{4} - \frac{\varepsilon_C \varepsilon_B}{2} + \varepsilon_B^2 - \frac{\varepsilon_B \varepsilon_A}{2} - \frac{\varepsilon_B \varepsilon_C}{2} \right)^{\frac{1}{2}}
$$

$$
= \left(\frac{2\varepsilon_A^2}{4} + \frac{2\varepsilon_C^2}{4} + \varepsilon_B^2 - \varepsilon_B \varepsilon_A - \varepsilon_C \varepsilon_B \right)^{\frac{1}{2}}
$$

$$
= \left[\frac{1}{2} \left(\varepsilon_A^2 + \varepsilon_C^2 \right) + \varepsilon_B^2 - \varepsilon_B \varepsilon_A - \varepsilon_C \varepsilon_B \right]^{\frac{1}{2}}
$$

$$
= \left[\frac{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_C)^2}{2} \right]^{\frac{1}{2}}
$$

checks!

Principal Direction (Axes)

$$\tan 2\theta = \frac{\varepsilon_{Ay}}{\varepsilon_A - \varepsilon_{cen}} = \frac{\frac{\varepsilon_A + \varepsilon_C}{2} - \varepsilon_B}{\varepsilon_A - \frac{\varepsilon_A + \varepsilon_C}{2}} = \frac{\varepsilon_A + \varepsilon_C - 2\varepsilon_B}{\varepsilon_A - \varepsilon_C}$$

Principal Stresses

Hooke's Law (General Case $\varepsilon_2 = 0$)

$$\sigma_1 = \frac{E}{1 - v^2}\left(E_1 + vE_3\right)$$

$$\sigma_3 = \frac{E}{1 - v^2}\left(E_3 + vE_1\right)$$

$$\sigma_{cen} = \frac{\sigma_A + \sigma_C}{2} = \frac{1}{2}\frac{E}{1-v^2}(1-v)(\varepsilon_A + \varepsilon_C)$$

$$= \frac{E(\varepsilon_A + \varepsilon_C)}{2(1 - v)}$$

$$\sigma_{radius} = \frac{\sigma_1 - \sigma_3}{2} = \frac{1}{2}\frac{E}{1-v^2}(1-v)(\varepsilon_1 - \varepsilon_3)$$

$$= \frac{E(\varepsilon_1 + \varepsilon_3)}{2(1 - v)} = \frac{E(\varepsilon_r)}{1+v}$$

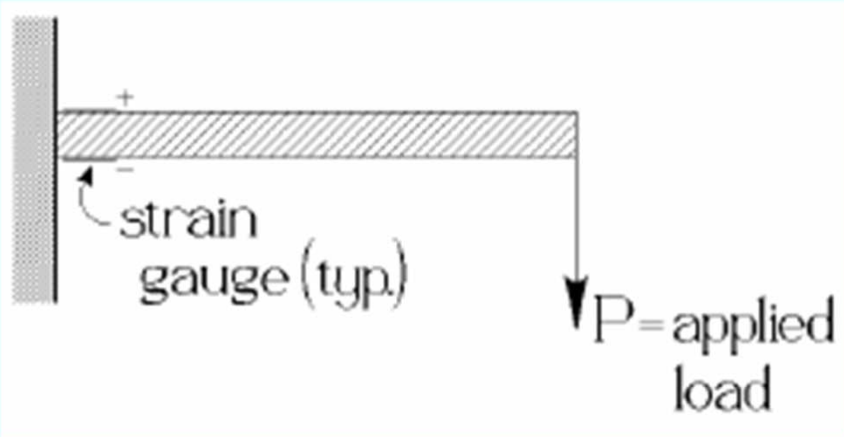
where ε_r = radius normal strain

$$\therefore \sigma_{max,min} = \frac{E(\varepsilon_A + \varepsilon_C)}{2(1-v)} \pm \frac{E}{1+v}\sqrt{\frac{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_C)^2}{2}}$$

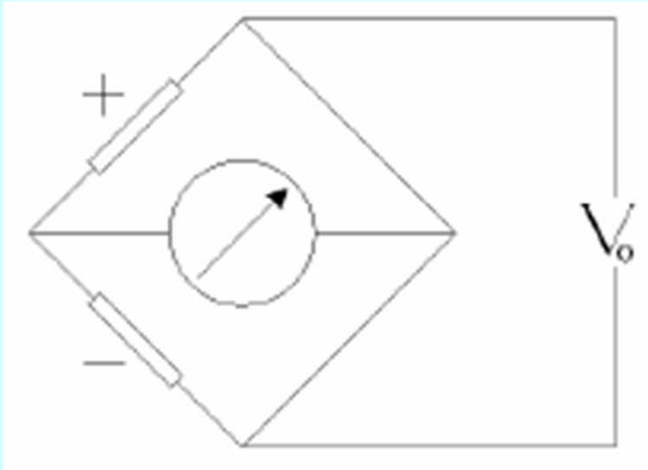
Force Measuring Devices

Moment

i.e.



+ means positive volt reading
- means negative volt reading



Recall:

$$\frac{\Delta V}{V} = \frac{GF}{(1+\alpha)^2}(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4)$$

since $R_1 = R_2$ (assumed), and $\alpha = 1$ (so $(1+\alpha)^2 = 4$)

$$\therefore \frac{\Delta V}{V_0} = \frac{GF}{4}\left(\varepsilon^{+} - \varepsilon^{-}\right)$$

Now, $$\varepsilon^{+} = \frac{\sigma}{E} = \frac{M}{ES} = \frac{PL}{ES} = -\varepsilon^{-}$$

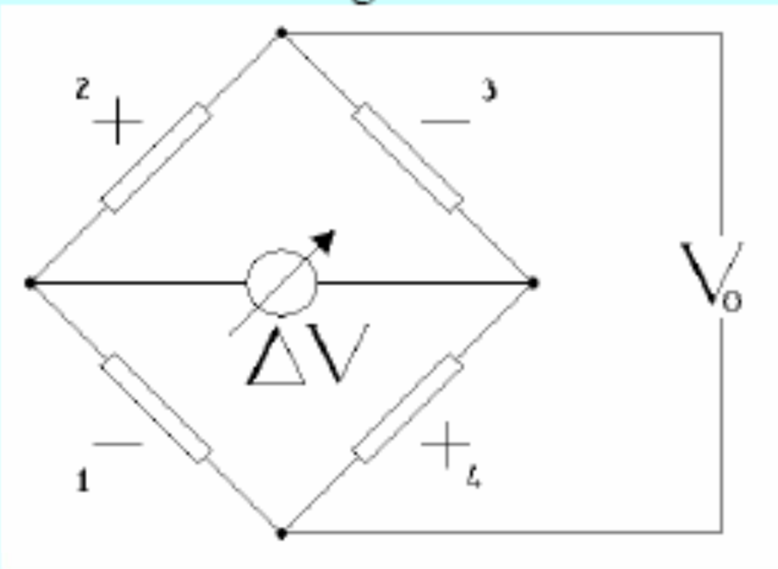
Note: S = Section Modulus = I/Y

$$\therefore \frac{\Delta V}{V_0} = \frac{GF}{4} \cdot 2 \cdot \frac{PL}{ES}$$

or $$\Delta V = \frac{GF \cdot L \cdot V_0}{2E \cdot S} \cdot P = K_0 \cdot P$$

where K_0 = Calibration Factor (Volt/Force)

What if a full bridge circuit is used?



$$\frac{\Delta V}{V} = \frac{GF}{4}\left(\varepsilon^{+} - \varepsilon^{-} + \varepsilon^{+} - \varepsilon^{-}\right)$$

$$= \frac{GF}{4} \cdot 4 \cdot \frac{PL}{E \cdot S} = \frac{GF \cdot L}{E \cdot S} \cdot P$$

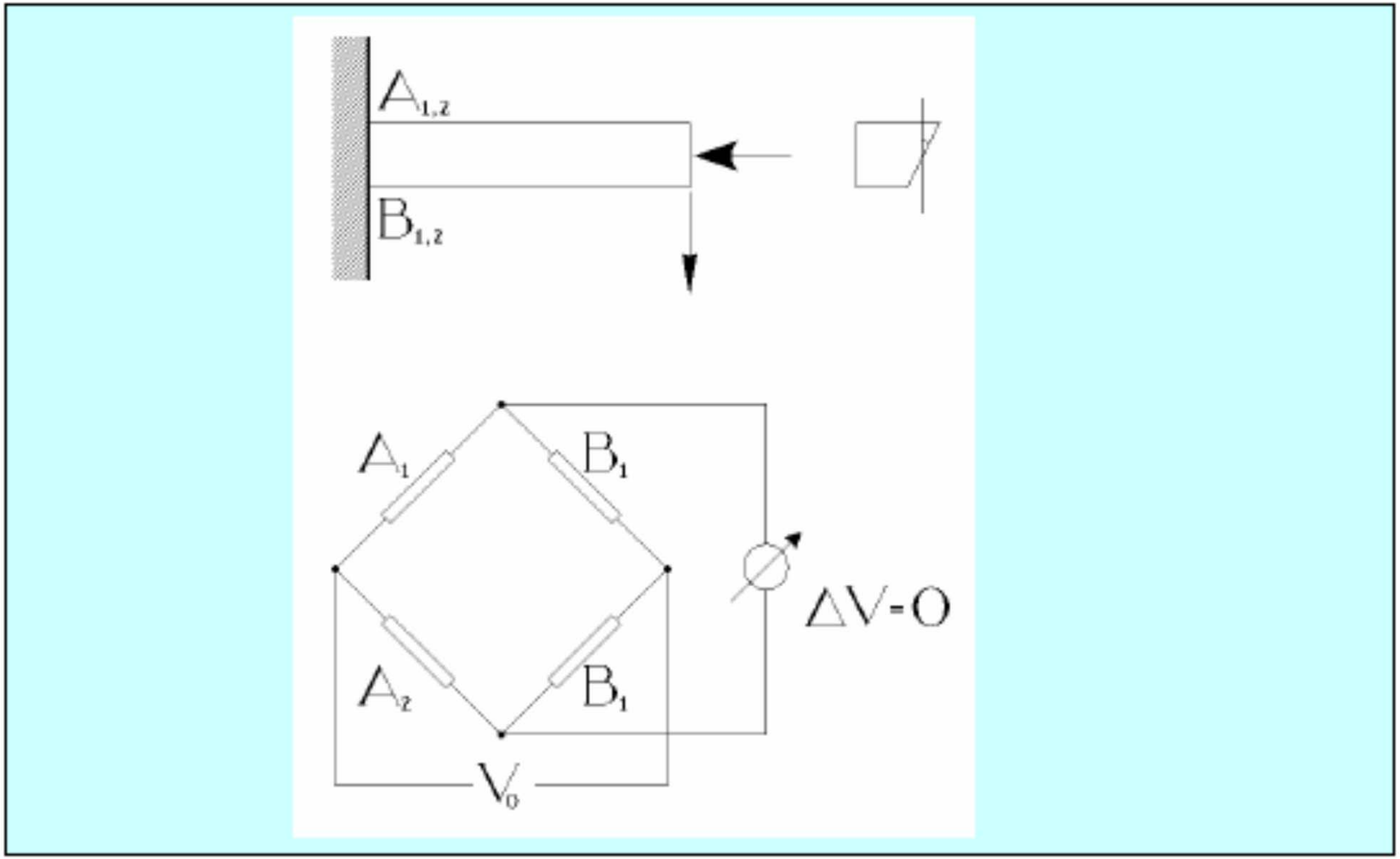
$$\Rightarrow K_0 = \frac{GF \cdot L}{E \cdot S}$$

Note that the calibration factor is doubled (it is twice as sensitive).

What about temperature compensation?

$\varepsilon_T^+ = \varepsilon_T^-$ So the net effect in $\dfrac{\Delta V}{V}$ cancels!!

By using half bridges and full bridges, temperature compensation is automatic.

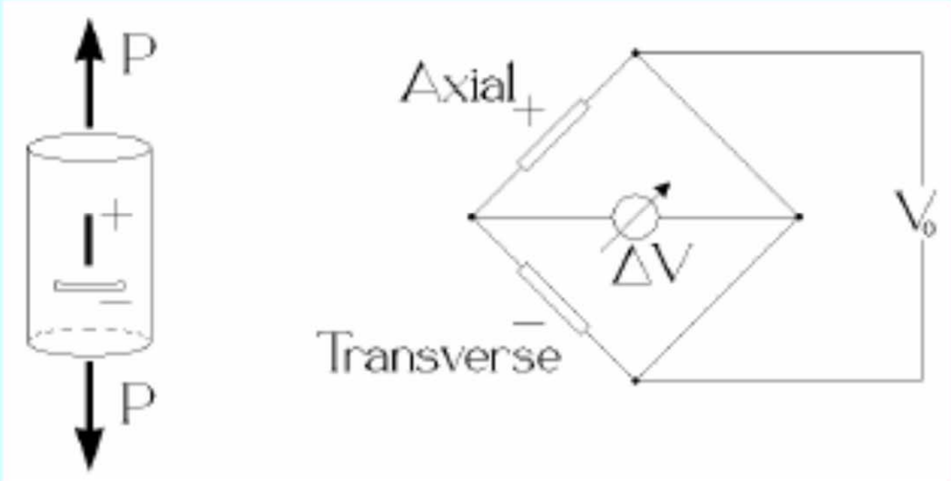

A1,2
B1,2
A1
B1
A2
B1
ΔV=O
V0

Axial Load Measurement

Using one gauge for axial strain measurements;

- Problem of no temperature compensation
- Probably OK for quick measurements

Use a 'dummy' gauge to measure transverse strains.



$$\frac{\Delta V}{V} = \frac{GF}{4} \cdot \left(\varepsilon_{axial} - \varepsilon_{transverse} \right)$$

$$\varepsilon_{axial} = \varepsilon^{+} = \frac{\sigma}{E} = \frac{P}{AE}$$

$$\varepsilon_{transverse} = -v\varepsilon^{+} = \frac{-vP}{AE}$$

$$\therefore \frac{\Delta V}{V} = \frac{GF}{4} \cdot \left(\frac{P}{AE} + \frac{vP}{AE} \right)$$

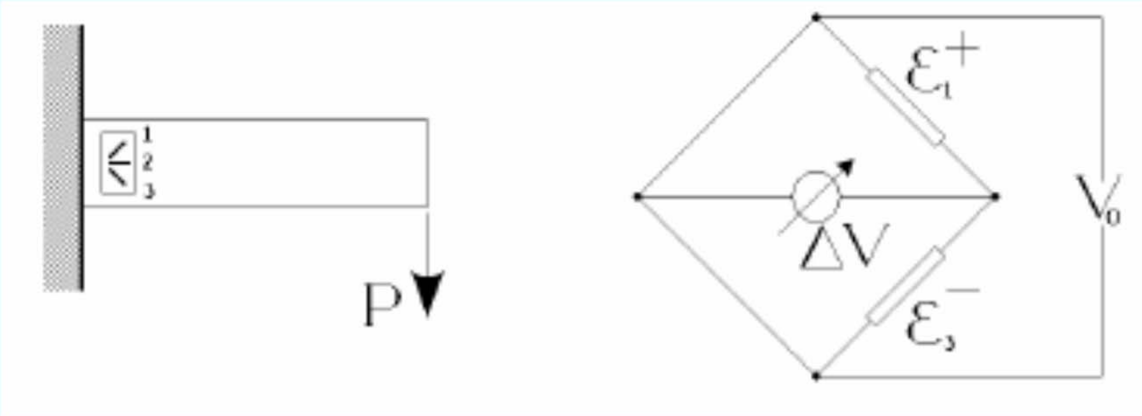
$$\frac{\Delta V}{V} = \frac{GF(1+v)}{4\,AE} \cdot P$$

$$K_0 = \frac{GF \cdot (1+v)}{4 \cdot AE}$$

Note that the transverse gauge created more sensitivity (1+v). Use four gauges (a full bridge) for double sensitivity.

Shear Measurements

Use a rosette with the gauges at 90° apart (1 & 3 in the diagram below).



$$\frac{\Delta V}{V}=\frac{GF}{4}(\varepsilon_1-\varepsilon_3)$$

$$\varepsilon_1=\frac{P}{A_x E}$$

$$\varepsilon_3=-\varepsilon_1$$

Note that for double sensitivity, a full bridge may be used. Shear strains may not be uniform throughout a solid cross section. For this reason, thin wall circular tubes are recommended for accurate shear measurements.

Static Calibration

- Known load applied
- Repeatable values of output
- Errors should be small
- Calibration should be periodically checked
- Environmental conditions should be constant

Procedure for linear (elastic) measuring systems

- Zero output gauge verification
- Sensitivity verification
- Linear and hysteresis verification
- Repeatability verification

Axial Circuit Calibration:

$$\frac{\Delta V}{V_0} = \frac{GF(1+v)}{4AE} \cdot P = K_o \cdot P$$

K_o is the calibration constant of the cell without conditioning

$$\therefore K_o = \frac{\Delta V / V}{P} \qquad \left(\text{units} = \frac{\text{Volts}}{\text{Force}}\right)$$

$$\Delta V_{max} = \frac{GF(1+v)}{4AE} \cdot V_o \cdot P_{max} = K \cdot P_{max}$$

Apply amplifier gain. When amplified, we would want output ΔV_{max} to be 10 volts when the maximum load is applied

$$\therefore \text{Gain} = \frac{10}{\Delta V_{max}} \cdot 1000$$

with ΔV in mV

$$10V = \text{Gain} \ \frac{GF(1+v)}{4AE} \cdot V_o \cdot P_{max} = \text{Gain} \ V_o \ K \cdot P_{max}$$

$$10V / P_{max} = K \ V_o \ \text{Gain}$$

Size of Voltage Output (ΔV)

Strain Output is usually smaller than 2000μSt.

$$\varepsilon_y = \frac{\sigma_y}{E} \cong 2060\mu St \qquad \text{(for steel)}$$

Gauge Factor (GF) for most gauges is typically about 2.0

$$\therefore \frac{\Delta V}{V_0} = \frac{2.0}{4}(2000) = 1000\mu St = 1 \times 10^3$$

or

$$\Delta V_{max} \cong 10^{-3} \times V_0$$

For an excitation voltage of V_0=10V

$\Delta V_{max} = 10^{-2} V = 10mV$

This is extremely small and therefore requires amplification (or gain)!!

Amplification is applied by special conditioners

i.e.

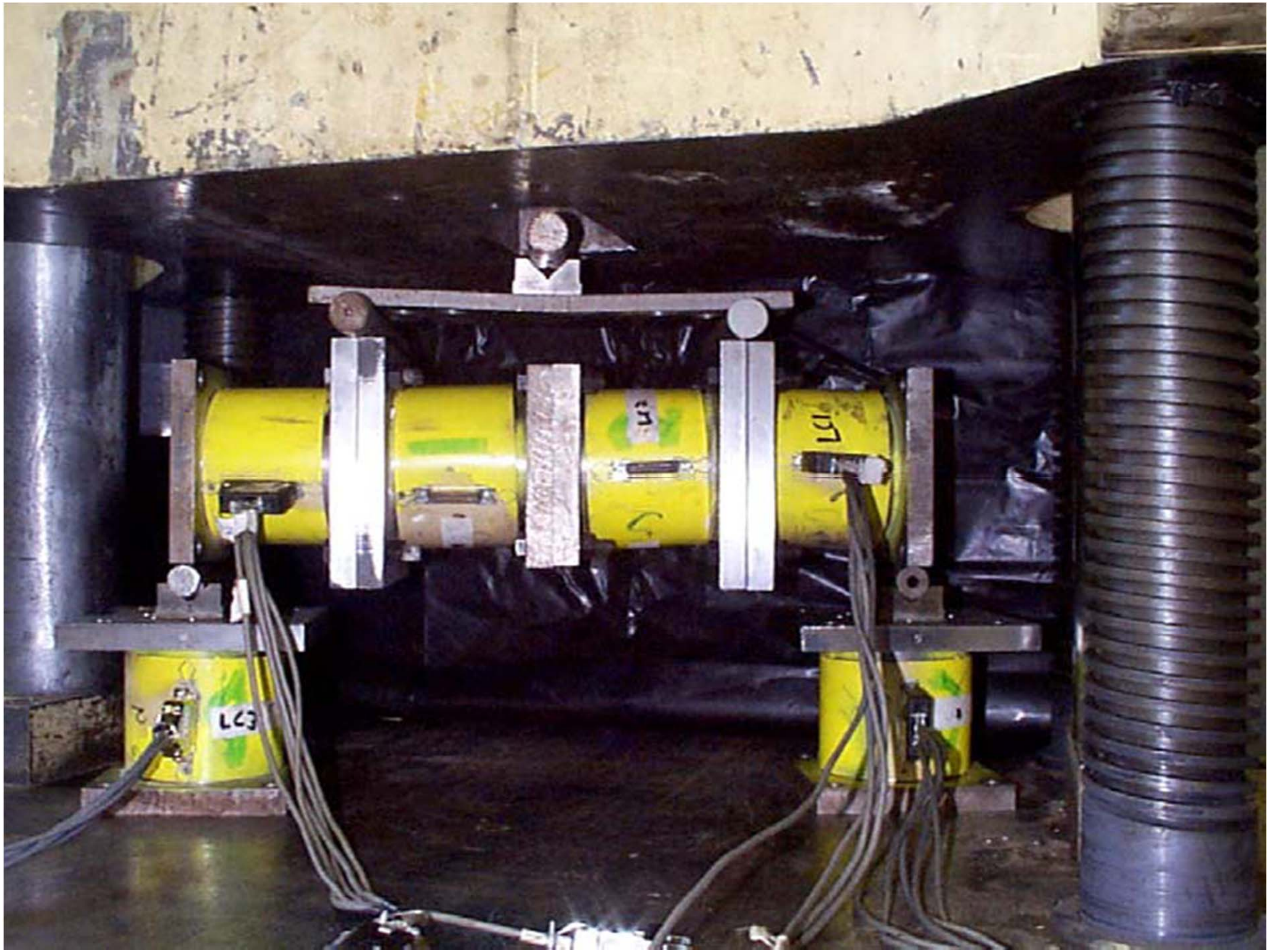
National Instruments SCXI-: 4 Channel Isolation Amplifier
with Excitation Adjustable Gains from 1 to 2000
MicroMeasurements –Vishay - 2100 0r 2300

Example of load cell design:

Program for design:

Example circuits

Calibration: See below


LC1
LC3