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THE CONSUMPTION-INVESTMENT-UNEMPLOYMENT RELATIONSHIP IN SPAIN: AN ANALYSIS WITH REGIONAL DATA

Roberto BANDE
Group of Analysis and Modelling in Economics
University of Santiago de Compostela, **Spain**
roberto.bande@usc.es
Dolores RIVEIRO
Group of Analysis and Modelling in Economics
University of Santiago de Compostela, **Spain**
dolores.riveiro@usc.es

Abstract:

In this paper we analyze the consequences of changes in the consumption patterns on unemployment through an intermediate channel via investment. Specifically, after presenting our theoretical framework, we build a dynamic econometric multi-equational model, in which we estimate a consumption function, an investment function and an unemployment rate equation, using a panel of 17 Spanish regions. This model is characterized by its dynamics and the cross equation relationships. After estimating the model, we run a number of dynamic simulations in order to verify our starting hypothesis, namely that temporary and persistent shocks to consumption have long lasting effects on unemployment, both directly and indirectly, through investment. Our results are especially relevant in the current recessive context of the Spanish economy, which is characterized by severe falls in consumption and unprecedented increases in unemployment.

Keywords: consumption, investment, unemployment, panel data.

JEL Classification: E21, E22, E24

1. Introduction

One of the most prominent and worrying characteristics of the Spanish economy nowadays is the magnitude of the unemployment rate, which after reaching a 25-year low value in the second quarter of 2007 at 7.9%, at of the writing of this paper it has peaked to an unprecedented 25% at the third quarter of 2012. Therefore it is not surprising that the declared objective of the main policies followed since the beginning of the recession is to resume the growth and employment creation path that characterized the 2000's. The contribution of the paper is twofold. Firstly, we show theoretically the existing interrelation between the two major components of aggregate demand, consumption and investment (they usually sum up to a 75% of a country's GDP), and their joint effect on unemployment rate dynamics, as well as the effects of variations in unemployment on consumption. Secondly, we provide empirical evidence supporting this view, based on the estimation of a panel data econometric model, using information gathered for the 17 Spanish regions. The paper is organized as follows. Section 2 provides the motivation for the paper while Section 3 summarizes the theoretical foundations of the relationship between consumption, investment and unemployment. Section 4 presents the econometric model and the main empirical results, while Section 5 summarizes the results of a number of dynamic simulations. Finally, Section 6 concludes.

2. Motivation

While labor economists have focused their attention into a vast number of issues throughout the last decade, more recently their attention has turned back to unemployment and its persistence, precisely when the current recession has brought the unemployment figures back to the high levels of the 80's. In this context, the aim of the paper is to analyze the relationship between consumption, investment and unemployment.

The interest in the analysis of the relationship between aggregate demand variables (consumption and investment) and unemployment is derived from the observed discrepancy between the assumptions and implications of the main macro-labor models on the one hand, and what economic and policy agents (as well as existing data) suggest.

The main social agents (politicians, trade unions, businesses representatives) use to link swings in employment with the evolution of investment.[1] However, such relationship cannot be derived from the main theoretical approaches on the labor markets functioning. The effects of an expansion in the aggregate demand on unemployment would only be observed in the short run, being therefore temporary, given that in the longer run these effects vanish and the unemployment rate would return to its equilibrium value (either the natural rate of unemployment, NRU, or the non - accelerating inflation rate of unemployment, NAIRU), when the economy reaches again the vertical long run Phillips Curve. The existence of hysteresis in the labor market allows for prolonged effects of changes in the aggregate demand on unemployment, such that cyclical variations in the unemployment rate become structural. This goes against the standard NRU models, which rely on the assumption that the cyclical and structural components of unemployment are independent of each other, such that in the absence of errors in expectations, and once that the Walrasian equilibrium conditions are fulfilled; the unemployment rate reaches its natural value, as Friedman (1968) describes.

Most of the theoretical approaches to unemployment, through different analytical perspectives, follow one of these conflicting theories. However, the Chain Reaction Theory (CRT hereafter) asserts that the short, medium and long run are not compartmentalized, but that they interrelated in inter - temporal continuum, through slow and prolonged adjustment processes[2]. Under this view it is shown that the cyclical and structural components of unemployment are interdependent, i.e., temporary and permanent components are interrelated through time, such that the effects of shocks in the labor market (as those steaming from changes in demand) persist in the medium and the long run. In fact, this theory holds that the differentiation between the cyclical and structural component of unemployment is meaningless.

The interaction between dynamics in the labor markets and growing exogenous variables gives rise to the so-called “frictional growth” phenomenon, which precludes the unemployment rate approaching towards the NRU. This different perception on the causes of unemployment and its persistence has its ultimate reflection in the proposed policy measures targeted at reducing the unemployment rate. While the NRU argues in favor of supply side policies (labor market flexibilization, tougher conditions for access to unemployment benefits, etc.) the CRT proposes the use of aggregate demand policies to stimulate economic activity and to reduce unemployment and its persistence (tax cuts on consumption, increased government spending, investment stimuli, etc.)[3]. In this context, in spite of the statements and electoral promises by policymakers, the standard policy rule is to flexibilize the labor market and its institutions, which in many cases has the opposite effect on the final target[4].

The existence of an explicit relationship between unemployment and aggregate demand has not been popular in the macro-labor literature so far. However, a growing number of authors, from very different analytical approaches and for different countries and periods, have found a significant negative relationship between the growth in the capital stock and the equilibrium unemployment rate.

Since the paper by Rowthorn (1999), who takes a CES production function in the context of the Layard, Nickell and Jackman (1991) model, and finds that increases in the capital stock may reduce

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[1]The current President of the Spanish Government, Mariano Rajoy, asserted recently in a partisan meeting in June, 2012, that “Without credit there are no banks, without banks there is no investment, and without investment there is no employment” (see http://noticias.es.msn.com/rajoy-sin-cr%c3%a9dito-no-hay-bancos-y-sin-bancos-no-hay-inversi%c3%b3n-ni-hay-empleo).

[2] The CRT was proposed and developed by Dennis Snower and Marika Karanassou in a series of papers. See Karanassou *et al.* (2010) for a general view on this theory.

[3] The papers by Karanassou *et al.* (2008), Karanassou and Sala (2012) or Bande and Karanassou (2013) emphasize the role of frictional growth in the explanation of unemployment in different economies.

[4] For instance, in the Spanish case, the current government proposed during the electoral campaign of November 2011, to reduce unemployment through an investment boost. Nevertheless, the legislative reforms implemented since it took office consisted in a deep reform of the labour relations framework.

theoretically the equilibrium unemployment, many papers have also shown this type of negative relationship between investment (growth of capital stock) and the equilibrium unemployment rate, both in the medium and long run[5]. This result seems to be robust across different type of modelizations and countries.

Malley and Moutos (2001), using data for OECD countries, find that differences across countries in capital accumulation explain to a greater extent the existing differences in unemployment. Countries with greater capital accumulation (with respect to its trade partners) are those exhibiting lower unemployment rates. Alexio and Pitelis (2003), in an applied exercise for different European countries, also find that one of the potential factors explaining the high and persistent unemployment rates in Europe is an insufficient capital stock growth, as well as an inadequate aggregate demand. They conclude that variations in the components of the demand (and not only in investment) are relevant to explain unemployment fluctuations, a result which is in line to our appraisal.

Kapadia (2005) introduces a production function with capital restrictions, in which installed capacity is determinant. When the capital stock is low, investment has a positive effect on the employment level, and modifies the equilibrium (it reduces the natural rate of unemployment). However, new investment above a given threshold has a neutral effect on (un)employment, it exclusively pushes up real wages. In other words, when the firm has spare capacity, capital accumulation is not able to reduce equilibrium unemployment, and we would be under the standard framework. As long as the firm reaches its potential capacity (which takes place when the capital-labor ratio falls short of the capital restrictions threshold), labor participation in the product, and therefore the wage participation, is reduced, while the capital (and profits) participation rises. This increase in the expected revenues affects investment, and directly reduces the natural rate of unemployment. In this same line, Arestis *et al.* (2007), in a study about the importance of capital stock in the determination of real wages and (un)employment for a panel of nine European countries, confirm their starting hypothesis, namely that capital stock is key in the determination of the wage and unemployment levels in an economy. Moreover, capital scarcity will persistently affect the equilibrium unemployment rate, and for prolonged periods of time. According to these authors, the low capital accumulation rates in the countries under scrutiny led to lower capital stocks and a consequent scarcity. Thus, real wages were too high given the changes in productivity, and due to a limited factor substitution, the predominance of capital-intensive investment brought restrictions in the adjustment between demand and supply of labor.

If we regard the capital stock as a main determinant of the NRU or the NAIRU, we are assuming that there exists a changing factor which modifies continuously the equilibrium unemployment rate. The pace and structure of investment will be influenced by the level of economic activity and other relevant variables, as profitability. Therefore, the variability of the NAIRU will be continuously affected by the path of the aggregate demand.

Even though standard macro models focused on the performance of the labor market do not allow for these types of relationships apart from the short run, the interrelations between consumption, investment and unemployment behind a negatively-sloped Phillips Curve in the medium and long run can be easily justified from standard economic theory.

3. The theoretical relationship between consumption, investment and unemployment

In this Section we provide the theoretical linkages between the three macroeconomic variables considered in the econometric model. Firstly, we consider the employment (and therefore, the unemployment) effects of changes in the aggregate demand. Next, we summarize the relationship between consumption and investment, focusing particularly on the effects of changes of the former on the latter. Finally, we analyze the role played by unemployment on consumption decisions.

[5] The existence of such relationship is known in the literature as the *Modigliani Puzzle*. However, Modigliani himself does not regard it as a puzzle, but the natural expression of the Keynesian paradigm, Modigliani (2000).

According to the Keynesian view, increases in any of the aggregate demand components have positive effects on employment, because prices increase more than nominal wages, lowering thus the real wage. An alternative scenario, in which an increase in aggregate demand triggers, increase in production and employment would be that of sticky prices and wages, not responding thus to the changes in the aggregate demand.

The New Classical Macroeconomics (NCM) school (as the Neoclassical Synthesis had already done) accepts these explanations of the effect of aggregate demand on employment, and therefore on unemployment, but only for the short run. In the long run, when wages and prices are completely flexible and errors in expectations have been corrected, the real effects of changes in demand vanish, and the equilibrium in the labor market returns to the natural rate of unemployment.[6]

The New Keynesian Macroeconomics (NKM) School does not conclude either that changes in aggregate demand should have any significant employment effect in the long run. The equilibrium in the labor market, which corresponds to the NAIRU, is found when the level of employment that makes compatible the wage aspirations of workers in wage bargains and real wages that firms are willing to pay (given their labor costs and the degree of competition in product markets) is reached.

These approaches have, at least, two counterfactual implications. First, the effects of changes in aggregate demand should only be observed in the short run, i.e., they would not be persistent. Secondly, the variations in employment and in real wages should go in opposite directions, i.e., real wage should move counter-cyclically, precisely because it is the fall in real wages which triggers employment growth. However, available data suggest that movements in real wage are, in general, cyclical, and that, as we will show, the effects of changes in the aggregate demand have prolonged effects on employment and unemployment.

Let us use a rather simple framework to show how a change in aggregate demand may persistently affect employment. Assume that the equilibrium in the labor market is determined by the intersection of a labor demand curve, *DL* (which can represent either the marginal product of labor in a perfectly competitive context, or the relationship between real wages and demand for labor by imperfectly competitive firms fixing prices and employment for a given nominal wage), and a labor supply curve, *SL* (which can be the outcome of a leisure-income choice process in a perfectly competitive context, or the relationship between real wages and employment arising from a wage bargaining process between firms and labor unions), as Figure 1 shows. Departing from an initial equilibrium in point A, both the NCM and the NKM claim that given the labor demand curve *DL*, the only way to increase the equilibrium level of employment after an increase in aggregate demand is through a greater supply of labor at a lower real wage, reaching a new equilibrium in quadrant [II] of Figure 1, in A', for instance. This positive effect on employment will disappear when the real wage reverts to its initial value, once that the adjustment in wages and prices is complete, and/or when errors have been revised and expectations are correct.

However, it is possible to find an equilibrium in which the increase in aggregate demand implies a persistent increase in employment, compatible with a rise in real wages, i.e., graphically we would reach equilibrium in quadrant [I] in Figure 1. For this to occur it is necessary that as a result of the increased demand, the labor demand curve *DL* shifts rightwards (reaching an equilibrium in B, for instance), or that the labor supply curve *SL* shifts, assuming an upward sloping demand curve *dl*, reaching a new equilibrium at point C.

Lindbeck and Snower (1994) summarize the different transmission channels which must be open in each case for changes in aggregate demand to have persistent effects on the labor market. These channels make compatible an increase in employment with higher real wages, both in the medium run (when we assume that capacity is fixed, either fully utilized or under excess) and diminishing labor

[6] If rational expectations were considered, the employment effects of expected changes in aggregate demand would be absent even in the short run, given that agents foresee perfectly the forthcoming increase in inflation, and adjust their labour market behavior immediately.

returns, as well as when full flexibility in the productive capacity is assumed (in the long run), with a perfect adjustment of capital stock.

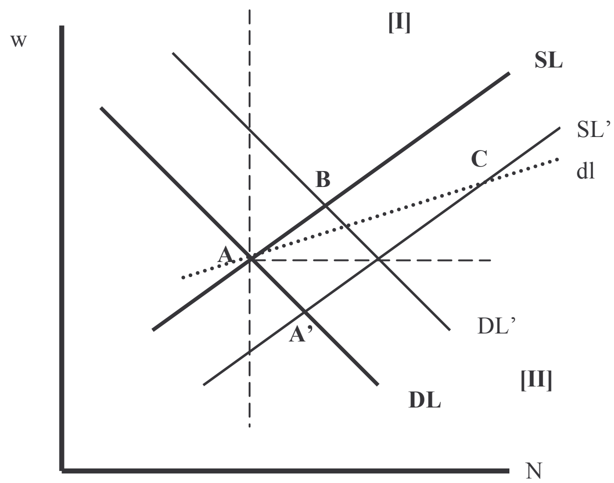

Figure 1. Effects of an increase in aggregate demand on employment

Let us assume that capital stock is given in the medium run and fully utilized. Thus, the production function of each of the *F* firms operating in the product market would be given by:

$$q = q(n), q_n > 0; q_{nn} < 0 \quad (1)$$

where q represents the production of each firm, n denotes their individual employment level and q_i is the partial derivative of q with respect to *i-th* production factor. Profit maximization by firms implies that aggregate employment in the economy, which is equal to the individual level of employment multiplied by the number of firms, is given by the usual negatively-sloped labor demand function:

$$N = FL\left(\frac{w}{1-m}\right); L' < 0 \quad (2)$$

where L is the inverse of the marginal product of labour (MP_N), w is the real wage and m is the Lerner index of monopoly power.

Given this labor demand function, an increase in aggregate demand will lead to greater employment without a fall in the real wage only if the labor demand curve shifts rightwards. Lindbeck and Snower (1994) show that this will happen if and only if as a result of the increase in aggregate demand monopoly power is reduced (due, for instance, to an increase in the total number of firms or an increased product demand price elasticity), or to an increase of the marginal product of labor. This rightward shift of the labor demand curve would also take place whether the capital stock is flexible, in the long run.[7]

If in the short and medium run we consider that there is excess of capacity, the increase in aggregate demand can also lead to a rightward shift of the labor demand curve if it triggers an increase in the marginal product of capital (MP_K). On the other hand, an increase in employment would be followed by an increase in capital utilization, and therefore the slope of the labor demand curve will not only depend on how will the MP_N react to the increased employment (negatively in the case of

[7] Moreover, when capacity is flexible, the rightwards labour demand shift can also happen when, as a consequence of the increased aggregate demand, the user cost of capital falls, being capital and labour complementary, or the user cost of capital increases, being both production factor substitutive, but this effect is not the most relevant.

decreasing returns of labor), but also on how will the MP_K respond: it will increase if capital and labor are complementary. In this case, the production opportunities for each firm would be:

$$q \leq q(n,k), q_n > 0, q_k > 0, q_{nn} < 0, q_{kk} < 0 \quad (1')$$

The slope of the labor demand curve, in the presence of excess of capacity, is given by:

$$\frac{dw}{dN} = F(1-m)(q_m + \bar{h}q_{kn}), \quad (3)$$

where $\bar{h}$ is the upper limit of the *k/n* ratio range chosen by the firms[8].

Thus, in an imperfect competition context, if the degree of complementarity is high enough as to allow for a greater variation of the MP_K than the variation of the MP_N, the slope of the labor demand curve, *dl*, is positive, such that an increase in aggregate demand, which shifts the bargained real wage curve (or the labor supply curve), may increase the employment level without a reduction in real wages.

Following this line of reasoning, an increase in aggregate demand, due for instance to a greater investment in public infrastructures, would trigger in the short run an excess of capacity, which in turn, if labor and capital are complementary enough, would imply an upward sloping labor demand curve. This would increase employment, due to the shift to the right of the *SN* curve, without a reduction in the real wage. In the medium and long run, the labor demand curve *DL* would shift to the right as the marginal product of labor rises.

However, the employment effect would also be present and (as will be shown later) greater if the event that triggers the increase in aggregate demand is greater consumption. In this case, in addition to the direct effect of an increased consumption on employment, and in the absence of barriers to entry, more firms would enter in the market, and monopoly power would be reduced. This would shift in the medium run the labor demand curve to the right. Therefore, we may express the unemployment rate in each period *t* as a negative function of both realized consumption (C_t) and investment, I_t

$$u_t = u(C_t, I_t); u_C < 0, u_I < 0 \quad (4)$$

Consumption and investment are also intertwined. Particularly, the growth in consumption is likely to affect positively investment, which implies a second round effect on employment, with a subsequent further reduction in unemployment, given the increase in the marginal product of labor.

Firm *i*, who produces a good *X*, takes decisions on an investment project. Therefore, it will relate the decision with the cost of undertaking the project, which is normally defined by the user cost of capital, which we can proxy through the real interest rate (*i*) and with the expected return of the project, which will be directly related with the consumption demand that the firm expects for the good *X*. This demand, in turn, is affected by the business cycle, which we can proxy through the income level (or its growth). Therefore, at an aggregate level, investment will depend positively on consumption and income, and negatively on the interest rate:

$$I_t = I(Y_t; C_t; i_t); I_Y > 0, I_C > 0, I_i < 0 \quad (5)$$

Lastly, let us discuss the relationship between consumption and the unemployment rate. Consumption in each period depends negatively con the unemployment rate. According the life cycle

[8] Firms choose, in a first stage, the capital stock level and technology (the *k/n* ratio) which maximize expected profits. In a second stage they fix the employment level, the production level and the price, given the available information on the rest of variables.

hypothesis (Ando and Modigliani, 1954, Modigliani and Brunberg, 1954, Modigliani, 1970) and the permanent income hypothesis (Friedman, 1957), individuals take decisions on consumption trying to maximize lifetime utility, which depends on the lifetime consumption they can afford to with their lifetime income and wealth. Thus, consumption in each period (which is aimed to be kept at a steady lifetime or slightly upward sloping path) depends on lifetime permanent income (or broad income) which includes present and future earnings derived both from labor and from real and financial assets.

The intertemporal utility maximization problem for the consumer, subject to the restrictions imposed by her lifetime expected income and wealth, and under standard assumptions, gives that consumption in each period is a function of present and future labour income, *Y*, and income from accumulated assets, *W*,[9]

$$C_t = f(W_t; \sum_{s=0}^{T} E_t Y_{t+s}) \tag{6}$$

In order to proxy the expected future labor income, no doubt the main determinant for a great part of the workforce is the likelihood of being employed. This likelihood, in turn, may be proxies by the unemployment rate. The greater the unemployment rate, the lower the probability assigned by an individual to the likelihood of being employed in the future, and therefore the lower the future expected labor income, and consequently, current consumption[10]. Thus, we may establish a functional form between aggregate consumption in each period, C_t, and income (Y_t), wealth (W_t) and the unemployment rate (u_t):

$$C_t = C(Y_t; W_t; u_t)\; C_Y > 0, C_W > 0, C_u < 0 \tag{7}$$

4. Econometric results

This section summarizes the specification and estimation of a macro econometric model, consisting in empirical versions of equations (4), (5) and (7), i.e., a multi-equational model, which tries to explain the interrelations between the three variables under scrutiny, consumption, investment and unemployment.

4.1. Data

The data used in our empirical analysis has been gathered from different statistical sources, which are detailed in Table A1 in the Appendix, providing also the corresponding definition for each variable. The reduced time dimension of some of the potentially important variables to explain the evolution of the variables of interest led us to make use of regional data, which allow outweighing the

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[9] The specification of the function depends on the assumptions on the utility function, the interest rate and the inter-temporal discount rate. For an individual living for *T* years, which leaves no debts, and assuming quadratic utility functions, real interest rates and inter-temporal discount rate equal to zero, and strictly positive marginal utility, the consumption function would be of the type (Hall, 1982): $C_t = \frac{1}{T}\left(W + \sum_{t=1}^{T} E_t(Y_t)\right)$

[10] Note that this fall in current consumption implies an increase in current savings for precautionary reasons. There exists a vast literature focused on the effect of uncertainty about the evolution of expected future income on consumption and savings decisions (see *inter alia* the papers by Leland, 1968, Sandmo, 1970 or Drèze and Modigliani, 1972). This literature, however, has not yet got to a consensus as regard as how to measure this uncertainty, both at the micro and the macroeconomic level. Thus, some authors suggest the use of measures based on the volatility of future expected income (see Blanchard and Mankiw, 1988, Hahm, 1999, Hahm and Steigerwald, 1999 or Menegatti, 2007, 2010), while other group of authors base their attention on measures related to the unemployment (Dynarski and Sheffrin, 1987, Carroll, 1991, Malley and Moutos, 1996). More recent papers, as those of Mody *et al*., (2012) or Bande and Riveiro (2012) take into account both type of measures in empirical models of precautionary savings. In any case, given that in the present paper we are interested in the effect of the probability of perceiving future income on current consumption decisions, we assume that this probability can be proxied correctly through the unemployment rate.

limited time dimension of some variables with the cross-section component, maximizing thus the available number of observations. For this reason, and given that there exist data for the 17 Spanish regions for the main macroeconomic figures (consumption, disposable income, gross fixed capital formation, the unemployment rate, etc.) we opted for a panel data approach. The criteria for choosing the data were, firstly, homogeneity, and secondly the time dimension. All of the variables have been deflated, in order to isolate from the effect of inflation on consumption and investment decisions, such that our estimated model is completely real. The sample is initially 1980 to 2007, but for some of the variables (for instance the Madrid stock index, which proxies financial wealth) there is only available data since 1985. In any case, in the estimation of the model we have adjusted the sample size of each equation to data availability.

4.2. Econometric methodology

We construct a structural vector autoregresive distributed lag model (VARDL), with the aim of explaining the dynamics of the three variables under study. Moreover, as we have mentioned, in order to maximize the available statistical information, this model will be estimated as a panel data model, using the breakdown of the 17 Spanish regions (*ComunidadesAutónomas*). The specific functional form of the econometric model is:

$$A_0 y_{it} = \sum_{j=1}^{p} A_j y_{it-j} + \sum_{s=0}^{q} B_{it-s} X_{it-s} + \sum_{r=0}^{k} C_r Z_{t-r} + e_{it} \tag{8}$$

where y_{it} is a (3x1) vector of endogenous variables (consumption, investment and the unemployment rate), X_{it} is a vector of regional exogenous variables, while Z_t is a vector of exogenous national variables (which are common to all regions). Matrices A, B and C are of coefficients to be estimated, while e_{it} is a vector of error terms identically and independently distributed.

The estimation of model (8) is done by steps. Firstly, the dynamic structure of each equation is identified following the "general to specific" approach, i.e., we start with a high number of lags of each endogenous and exogenous variable, and then we reduce the model to a more parsimonious representation following the standard statistical information criteria, as the Akaike Information Criteria (AIC). This type of modelization implies a certain level of discretion in the selection of variables to include in each equation, as well as the initial number of lags to include. Vector Autoregresions (VAR's), on the contrary, imply a minimum degree of discretion, given that the main decision in the modelization process is the ordering of the variables in the VAR, decision that conditions enormously the empirical results. For this reason, the econometric literature has developed the so-called Structural Vector Autoregresion (SVAR), in which the atheoretical identification of the equations in the VAR is replaced by the imposition of an economic structure in the error terms. While the main advantage of the SVAR is the opportunity to conduct structural analysis, through inspection of the impulse-response functions, their main disadvantage is the individual equations have no economic interpretation, and are largely ignored.

VARDL models overcome these limitations of the SVAR models, given that the estimated coefficients in each of the equations can be directly interpreted as elasticities, which allow assessing the degree of plausibility and economic intuition of individual results in each equation. Moreover, this technique allows for the construction of impulse-response functions, whose shape is not dependent on the ordering of the variables within the model. For all these reasons we decided to construct our econometric model in terms of a VARDL[11].

As regards the type of panel data model we chose for our econometric exercise, we first must take into account the properties of the series as regards stationarity. The use of dynamic panel data models in the context of time series has generated an important debate in the literature. Banerjee (1999), Baltagi and Kao (2000) or Smith and Fuertes (2011) provide a good approximation to such debate.

[11] For a detailed account of ARDL for the analysis of long run relationships see Pesaran *et al.* (1996) and Pesaran and Shin (1999).

Whether the involved variables in the analysis are stationary or not conditions the type of econometric modelization to follow next (see Smith and Fuertes, 2011). Thus, if the variables are non-stationary (i.e., *I(1)*) we should first test for panel cointegration, and construct an error correction model if such cointegration exist, or estimate the model in first differences otherwise. If the variables are stationary, then we can proceed with the standard techniques for stationary panel data models (Baltagi, 2008).

Therefore, our second modeling stage is to test for unit roots in the variables of our model, both regional and aggregate. For the latter we have chosen the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test[12], even though results with alternative unit root tests, as the ADF, are similar to those reported[13]. Table 1 summarizes these results, from which we observe that for all of the national variables (financial wealth, GDP volatility, social security benefits, real short run interest rates and public debt/GDP ratio) we cannot reject the null hypothesis of stationary at the conventional statistical confidence levels.

Table 1. Unit root tests. National variables

Variable	KPSS
FW_t	0.053
$YVOL_t$	0.080
B_t	0.150
$RSHIR_t$	0.140
$DEBT_t$	0.160

Notes: The critical value for the test at the 5% is 0.14
Source: Author's own elaboration.

As regards the panel unit roots tests, among the different available options in the literature, we opted for the Maddala-Wu (1999) test, based on an exactly non-parametric test based on Fisher (1932). Specifically, the test statistic is $\lambda = -2\sum_{i=1}^{N} \ln p_i$ *which* is distributed as a $\chi^2(2N)$, where p_i is the p-value of the ADF unit root test for each *i-th* cross section unit, *i=1,...,N*. This decision is based on the interesting characteristics of the test (see Maddala-Wu, 1999).

Table 2 summarizes the results of the test for the regional variables included in the model. Note that the null hypothesis is non-stationarity, and therefore the value of the statistic for each variable is greater than the critical value for a $\chi^2(34)$, which is approximately 48. On the light of these results we may conclude with sufficient statistical confidence that the regional variables involved in our model are panel-stationary, and therefore we may use standard stationary panel techniques.

Table 2. Panel unit root test

Variable	Fisher Statistic
C_{it}	62.17
Yd_{it}	52.28
NFW_{it}	114.63
U_{it}	47.65
Y_{it}	59.59
I_{it}	59.54

Source: Author's own elaboration.

[12] See Kwiatkowski *et al*. (1992).
[13] We do not show these alternative tests for brevity, but they are readily available from the authors upon request.

Given these results we thus construct a panel VARDL model with regional fixed effects, through the following specification:

$$A_0 y_{it} = \sum_{j=1}^{p} A_j y_{it-j} + \sum_{s=0}^{q} B_{it-s} X_{it-s} + \sum_{r=0}^{k} C_r Z_{t-r} + e_{it} \qquad (9)$$
$$e_{it} = \mu_i + v_{it}, i = 1,.., N, t = 1,...T$$

In other words, we assume that the error term e_{it} follows a "one-way" error component, also known as a "fixed effects model", in which v_{it}~$iid(0,\sigma^2)$ with $Cov(e_{it},e_{jt})=0$ for $i \neq j$. The vector of scalars μ_t represents the specific regional effects, which we assume are constant through time. In other words, in this model we assume that regions exhibit a similar behavior as regards the slope coefficients for the different variables, and that they only differ in the intercept (Baltagi, 2008).

4.3. Econometric results

Once we have obtained the preferred dynamic specification for each equation in the model (estimated by OLS, see results in the Appendix, Table A2), we estimated the whole panel as a system, by Three Stages Least Squares, in order to take into account the potential endogeneity of some regressors and the cross-equation correlation. Table 3 summarizes the results of this estimation, which in general are good, being all of the variables statistically significant and all of the coefficients show the expected signs.

Column (1) in Table 3 shows the estimated consumption function. We observe that, in addition to a great level of inertia (value of the autoregressive coefficient of 0.87) disposable income affects consumption decisions with a high degree of persistence. Financial and nonfinancial wealth has the expected positive effects (greater in the case of non - financial wealth, which is reasonable in a period in which housing prices experienced a larger increase in returns than average financial assets). Lastly, the unemployment rate has a dampening effect on consumption, in line with our theoretical discussion in Section 2: an increase in the unemployment rate implies a decrease in expected future labor income, which in turn should be translated into reductions in current consumption. The inclusion of the public debt stock (as a % of GDP) in this equation has the purpose to test the Ricardian equivalence hypothesis, by which current tax cuts should be followed by current consumption falls, given that families anticipate the future tax increase to compensate current public deficit, and its corresponding increase in debt, rising their current savings to face such increase. The negative and statistically significant coefficient allows validating partially this hypothesis, even though with a limited impact, in line with previous studies for similar countries (see, for instance, Loayza *et al.*, 2000).

The results of the investment equation estimation are summarized in column (2) of Table 3. Investment shows a lower degree of inertia, depending negatively on current income, consumption (greater sales incentive firms to engage into investment projects), and negatively on short run interest rates (which proxy financial costs of the investment projects). We tried several alternative specifications for the investment function, including proxies for the Tobin's *q*, or real long run interest rates, but none of them provided better statistical results. Lastly, the degree of macroeconomic uncertainty (proxies by the estimated volatility of aggregate GDP growth rate on the sample period, obtained from the estimation of a GARCH (1,1) model, see Table A1) affects negatively investment, being one of the variables (together with consumption) which exerts a greater contemporaneous impact on capital formation.

The estimation of the unemployment rate equation is summarized in column (3) of Table 3, and show results in line with previous literature. Firstly, unemployment exhibits a high degree of persistence[14]. Secondly, the aggregate demand variables show the expected signs (consumption and investment reduce the unemployment rate in the short run), while the aggregate supply variable (social

[14] Note, however, that the unit root tests rejected the hypothesis that this series was *I(1)*, and therefore the hypothesis of pure hysteresis in the regional Spanish unemployment.

benefits) exerts the expected positive effect (increases in benefits tend to raise the reservation wage of workers, and therefore increases the rate of unemployment).

The model in Table 3 provides an excellent fit of the endogenous variables of the model, especially the unemployment rate[15]. Figure 2 depicts the actual and fitted values by the model. Note that the degree of fit to the actual values is remarkable, which indicates, on the one hand, that the dynamic specification of the model is adequate, and on the other hand, that the selected exogenous variables within each equation reflect well the underlying forces behind unemployment rate swings through time[16].

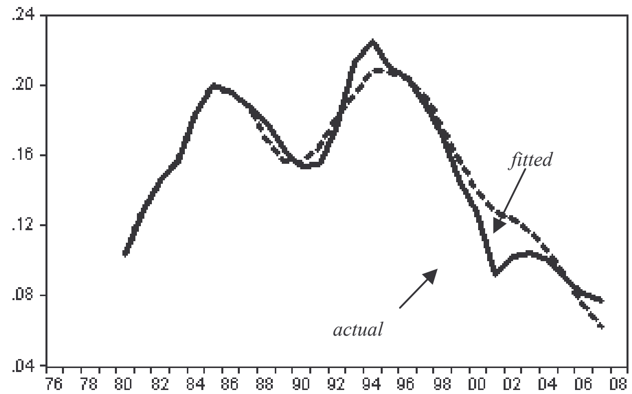

Figure 2. Actual and fitted unemployment rates.

Table 3. Model estimation. 3SLS

(1) *Consumption*	Coef.	*p*-value	(2) *Investment*	Coef.	*p*-value	(3) *Unemployment*	Coef.	*p*-value
c_{it-1}	0.878 (0.02)	0.00	I_{it-1}	0.701 (0.03)	0.00	u_{it-1}	0.78 (0.02)	0.00
Yd_{it}	0.214 (0.03)	0.00	y_{it}	0.404 (0.07)	0.00	I_{it}	-0.02 (0.01)	0.08
Yd_{it-1}	-0.17 (0.03)	0.00	Δc_{it}	2.06 (0.07)	0.00	c_{it}	-0.593 (0.08)	0.00
ΔNFW_{it}	0.034 (0.008)	0.00	$RSHIR_t$	-0.234 (0.15)	0.09	c_{it-1}	0.555 (0.07)	0.00
Δu_{it}	-0.508 (0.09)	0.00	$\Delta YVOL_t$	-0.63 (0.43)	0.01	b_t	0.029 (0.01)	0.09
FW_t	0.022 (0.003)	0.00						
$DEBT_t$	-0.059 (0.01)	0.00						
R^2	0.99		R^2	0.99		R^2	0.93	

[15] The aggregate unemployment rate is computed as the average of the regional unemployment rates.

[16] Note that to obtain the fitted unemployment rate we first had to solve the model formed by the three estimated equations, allowing that the endogenous and exogenous variables take their initial values. We next solve dynamically the model, which implies that the model takes the actual values of the exogenous variables and computes the corresponding values for the endogenous variables, which in turn feed the model in the next period computation.

	(1) Consumption			(2) Investment			(3) Unemployment	
	Coef.	*p*-value		Coef.	*p*-value		Coef.	*p*-value
SER	0.01		SER	0.06		SER	0.01	
DW	1.93		DW	1.88		DW	1.88	
Obs	374		Obs	374		Obs	374	

Notes. Standard errors in parentheses. SER is the standard error of regression, while DW refers to the Durbin-Watson statistic.

Source: Author's own calculations.

5. Unemployment effects of consumption and investment shocks

With the model summarized in Table 3 we next run a number of dynamic simulations which aim at verifying our starting hypothesis, namely that changes in consumption patterns exert a direct effect (via aggregate demand) as well as an indirect effect (through investment) on unemployment. To this end we firstly compute the impulse-response functions associated with the model in Table 3, which inform us about the impact of innovations in the system on the endogenous variables. The IRF are computed by allowing the model to stabilize at its long run steady state, such that all exogenous variables are constant. Next, we impose a shock on each one of the equations and compute the response of the endogenous variables of the system to such shock (specifically, we consider the response to a one-off shock, an AR(0.4) and a AR(0.8) shocks)[17]. The results of the calculations are depicted in Figures 3 and 4.

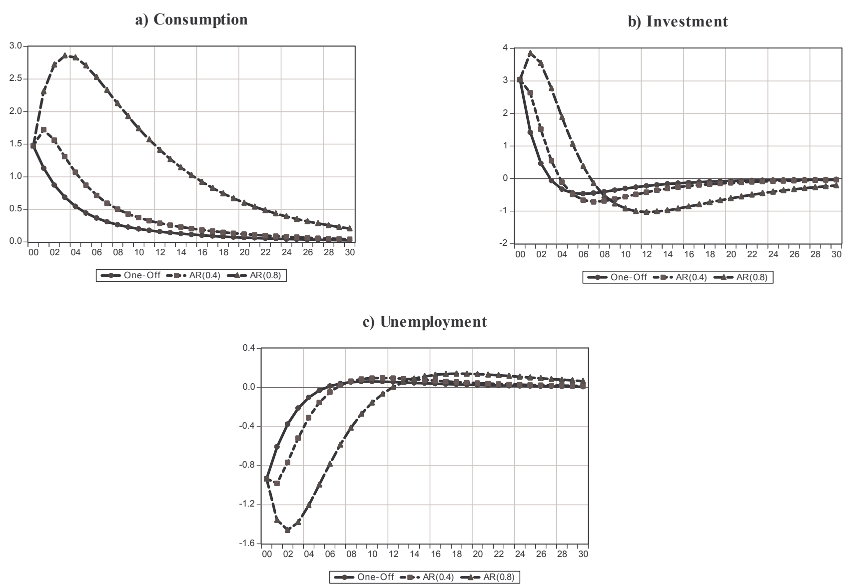

Figure 3. Impulse - Response functions to consumption shocks

[17] A transitory (or one-off) shock is defined as a unitary exogenous change in the inspected equation (consumption, investment or unemployment), with a one year duration. In other words, the shock is present in period *t=0*, and disapears in period *t=1*. An AR(0.4) shock, in turn, is more persistent, since it takes value 1 in period *t=0* and values $\varepsilon_t=0.4\varepsilon_{t-1}$ for *t=1,2…T*. Finally, the AR(0.8) shock takes value 0 for *t=0* and $\varepsilon_t=0.8_{t-1}$

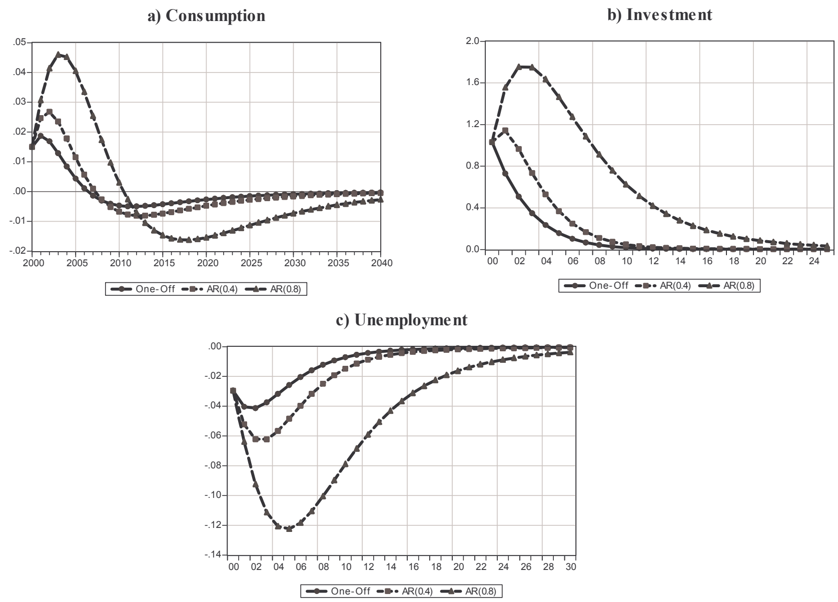

Figure 4. Impulse-Response functions to investment shocks

From the information in these graphs we may point that the first, shocks on consumption (regardless of the degree of persistence of the shock) exert an important effect on the unemployment rate, affecting also investment. Thus, when we consider a temporary one-off shock on consumption, there exists an important contemporaneous impact on investment (which is increased by more than a unit), which would indicate an accelerator effect. The unemployment rate, in turn, is reduced by 0.9 points contemporaneously, with a prolonged effect through 6 periods before reaching is initial value. Therefore, this IRF allows validating the hypothesis that changes in consumption affect unemployment through first and second round effects. Obviously, the more persistent is the shock on consumption, the greater the unemployment effects (see panel c) on Figure 3.

If we consider shocks on investment, the effects are similar; however the contemporaneous impact on the unemployment rate is lower than that of consumption shocks. In any case, an exogenous increase in investment is followed by further increases in consumption through the reductions in unemployment, with prolonged effects on the system.

A synthetic way of summarizing the information provided by the IRF is to calculate the accumulated impact of each shock on the endogenous variables in the long run. Assuming that the long run value of an endogenous variable is represented by x^{LR} (*where* x represents consumption, investment and the unemployment rate, respectively), let $\theta_t = x_t - x^{LR}$ be the difference between the actual value of variable x and its long run value in each period t, $t=1,2\ldots$ once that a shock in the system has occurred. If the shock took place in period $t=j$ then it is possible to compute the *accumulated short run effect* on variable x as:

$$x = \sum_{i=j}^{n} \theta t \qquad (10)$$

Note that this measure is the area below the IRF, therefore the greater the former, the larger the accumulated effect of a given shock on the variable under study. We may complement this measure with a temporal quantification of the shock. In this case, we would analyze the number of periods required for an endogenous variable to reach again its long run value (or a neighborhood of it). The first type of measures will be referred to as *quantitative persistence*, while the latter will be referred to as *temporal persistence*. Table 4 summarizes these calculations[18].

Table 4. Measures of quantitative and temporal persistence

	Consumption shock			Investment shock		
	Cons.	*Invest.*	*Unemp.*	*Cons.*	*Invest.*	*Unemp.*
Quantitative persistence ([illegible]	8.25	0.00	-1.46	4.31	3.33	-0.31
Temporal persistence ([illegible])	13	11	4	6	6	13

Notes: Cons., Invest. and Unemp. refer to consumption, investment and unemployment, respectively. Temporal persistence is computed as the number of periods required for the system to absorb 90% of the initial impact of the shock.

Source: Author's own calculations.

These results reinforce those provided by the IRF. The shocks generating more unemployment quantitative persistence are the consumption shocks, in spite of the larger effect in temporal terms of investment shocks. As regards the response of the two other endogenous variables, the quantitative effect on consumption of self and investment shocks is similar, while the self-effect of investment shocks is greater than consumption shocks.

In any case, these results suggest that should persistent falls in consumption levels occur (as it is the case in the Spanish economy), the unemployment rate would be largely and persistently affected, which would further complicate the labor market adjustment.

An additional exercise allows analyzing the effects on the equilibrium unemployment rate of *permanent* changes in consumption. The idea is to simulate what would happen with the equilibrium unemployment rate when a permanent fall in consumption or in investment levels occurs. Which of these aggregate demand shocks would affect unemployment most in the long run? To answer this question we use the model presented in Table 3 and introduce a permanent shock in the consumption equation and in the investment equation (in different simulations), and compute the responses of the unemployment rate, which are depicted in Figure 5.

From this figure we conclude that there are very different long run effects. Thus, a permanent unit shock on consumption increases equilibrium unemployment by 1.5 points, whereas in the case of investment shocks the unemployment rise is of 0.31, i.e., a permanent consumption shocks generates a long run effect which is 5 times larger than that of investment shocks. Secondly, the dynamic adjustment towards the new long run unemployment rate is remarkably different. In the short run, the unemployment rate overreacts to the consumption shock, increasing by 2.3 points, and then it progressively approaches its new long run value. In the case of an investment shock, the dynamic response is softer, approaching smoothly the new equilibrium unemployment rate. This indicates that in the case of consumption, the short run effects are greater than the long run effect, while the contrary holds for the investment shocks. This results calls for caution in the design of public budget adjustment policies, since any measure targeted at increasing public revenues taxing (and therefore lowering) consumption are likely to exert catastrophic effects on the equilibrium unemployment rate.

[18] Given that we have considered three types of shocks, we should provide persistence measures for each. However, autoregresive shocks are persistent temporary shocks, and thus the persistence measures would only reflect an amplification of the effects of a temporary one-off shocks. For this reason we provide the results for this latter type of innovations.

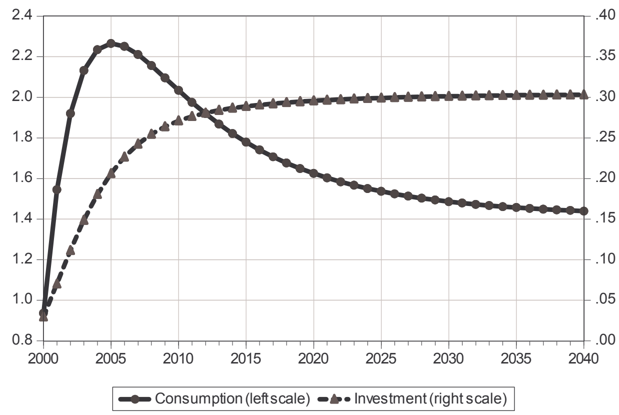

Figure 5. Impulse - Response functions to permanent shocks in consumption and investment

Conclusions

This paper provides new evidence as regards the existence of an empirical relationship between the main components of aggregate demand (consumption and investment) and the labor market equilibrium. Specifically, and taking into account that the main economic thought paradigms in macroeconomics (the New Keynesian Macroeconomics and the New Classical Macroeconomics) do not support the idea that aggregate demand shocks have prolonged labor market effects, we have shown that through a highly stylized macroeconomic model it is possible to establish a number of transmission mechanisms of aggregate demand shocks to the equilibrium level of employment, as a function of the adjustment of the capital stock and its level of utilization. In essence, this model indicates that if changes in the aggregate demand are associated to permanent shifts in the labor demand curve (due for instance to increases in the marginal product of labor or to an increase in the number of firms), there will be permanent effects on the equilibrium level of employment. These permanent effects would also be present if the labor demand function has positive slope and the change in aggregate demand affects the wage setting curve (or the labor supply curve). Combining this model with standard macroeconomics models of investment and consumption behavior allows us to establish a close relationship between consumption, investment and the labor market equilibrium (measured through the unemployment rate). The empirical evidence provided in Section 4 proves the existence of such relationship for the Spanish economy, while the dynamic simulations in Section 5 allowed quantifying the potential effects of demand shocks. Thus, we found that temporary shocks on consumption exert greater accumulated effects on unemployment than investment shocks, due to the feedback between both variables. In the presence of permanent consumption shocks, the equilibrium unemployment rate increases more than with investment shocks, as well as triggering stronger short run effects.

The implications for policy making are very relevant, especially in the current recessionary context of the Spanish economy, which is joined by a strong fiscal adjustment process. As in Malley and Moutos (2001), from our results we may deduce that any policy mix targeted at reducing the unemployment rate should include measures to incentive capital accumulation. In this sense, we agree with Kapadia (2005) in that policies fostering investment would help in reducing unemployment, and that in the case that policy focuses exclusively on labor market reforms, the effect on the employment level may be negative. In this same line, Arestis *et al.* (2007) conclude that policies should not be focused on the deregulation of the labor market and the promotion of flexibility, but should incentive sufficient capital accumulation.

Notwithstanding, from this paper we may also conclude that focusing exclusively on capital accumulation and leaving aside measures to stimulate aggregate consumption may not solve the labor market adjustment problem. Our empirical results suggest that current falls in consumption during the present recession may generate a very strong effect on the unemployment rate, through the accumulated first and second round effects (via investment). At the same time, credit restrictions which are characterizing this turmoil will not allow for increases in investment, even with historically low real interest rates. Lastly, fiscal adjustment will lead to further falls in aggregate demand and tax rises, which will curtail disposable income even further. Therefore, the outlook as regards the evolution of the unemployment rate is quite pessimistic.

The negative recent behavior of the Spanish unemployment rate is, no doubt, the result of a series of unfavorable factors (excessive dependence of low-productivity sectors, high temporary employment rate, duality in the labor market, excessive rigidities in the wage bargaining processes, etc.), but the insufficient private spending in consumption and investment, which far from being outweighed by public spending, has been reinforced by recent public budget adjustment plans, has contributed to amplify the magnitude of the problem in recent years. It seems clear that the chronic unemployment problem in Spain (and its deterioration in the last years) is not uniquely attributable to labor market institutions and regulations. Therefore, any measure focused on those aspects, affecting negatively consumption and investment will not be able to solve the unemployment problem.

Measures targeted at increasing aggregate demand components without compromising the public budget balance are, therefore, needed. In this sense, a deep reform of the tax system which lowers tax pressure on the income levels with greater propensity to consume would be an adequate stimulus to start with the multiplier effects. At the same time, deepening into the financial system reform is essential to revert credit flows towards the real sector of the economy, such that economic activity starts creating employment again.

Note that in the context of a monetary union there would be an additional adjustment mechanism, through an internal devaluation via wage cuts that if the inflation rate is to be kept stable, should be achieved through nominal wage reductions. However, our results show that this type of adjustment, being an attack to the welfare system, also would be associated to falls in the disposable income of households, with a likely deterioration of the unemployment problem. Therefore, these types of measures should be avoided by policymakers. We hope that in this context, the option of fostering labor supply reductions via external out-migration (which seems to be already in place) instead of via employment creation is not chosen by policy makers as the via to solve the labor market adjustment problem in Spain.

Acknowledgements

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APPENDIX

Table A1. Definitions and data sources

	Variable	Definition	Source
REGIONAL VARIABLES	C_{it}	Final consumption expenditure by households	BD-MORES, Dirección general de Presupuestos, Instituto de Economía Internacional, University of Valencia
	Yd_{it}	Gross household disposable income	BD-MORES
	Y_{it}	GDP at market prices	BD-MORES
	I_{it}	Gross Fixed Capital Formation	BD-MORES
	u_{it}	Regional unemployment rate	Labour Force Survey (EPA), INE
	NFW_{it}	Non financial wealth: value of the residential capital stock= residential capital stock x average price of squared meter of real estate	▪ Residential capital stock: IVIE, University of Valencia; ▪ Averag esq. M. Price: Sociedad de Tasación.
NATIONAL VARIABLES	FWt	Financial wealth: anual average of the Madrid stock index	Madrid Stock
	Bt	Social benefits per inhabitant	OCDE, Economic Outlook
	RSHIRt	Real short run interest rate: nominal short run interest rate-rate of inflation	OCDE, Economic Outlook
	DEBTt	Public debt stock as a % of GDP	OECD, Economic Outlook

Table A2. Estimation of the model. OLS

	Consumption			Investment			Unemployment	
	Coefficient	*p*-value		Coefficient	*p*-value		Coefficient	*p*-value
c_{it-1}	0.844 (0.02)	0.00	I_{it-1}	0.754 (0.02)	0.00	u_{it-1}	0.75 (0.02)	0.00
Yd_{it}	0.326 (0.04)	0.00	y_{it}	0.321 (0.05)	0.00	I_{it}	-0.033 (0.007)	0.00
Yd_{it-1}	-0.270 (0.04)	0.00	c_{it}	1.51 (0.15)	0.00	c_{it}	-0.243 (0.03)	0.00
NFW_{it}	0.042 (0.01)	0.00	$RSHIR_t$	-0.352 (0.15)	0.02	c_{it-1}	0.196 (0.03)	0.00
u_{it}	-0.141 (0.04)	0.01	$YVOL_t$	-0.856 (0.59)	0.10	b_t	0.068 (0.01)	0.00
FW_t	0.030 (0.003)	0.00						
$DEBT_t$	-0.06 (0.05)	0.00						
R^2	0.99		R^2	0.99		R^2	0.94	
SER	0.01		SER	0.07		SER	0.01	
DW	2.05		DW	2.12		DW	1.94	
NxT	374		NxT	459		NxT	459	

Notes. Standard errors in parentheses. SER is the standard error of regression, while DW refers to the Durbin-Watson statistic.

COMPARATIVE STUDIES ON COOPERATIVE STOCHASTIC DIFFERENTIAL GAME AND DYNAMIC SEQUENTIAL GAME OF ECONOMIC MATURITY

Darong DAI
Department of Economics, Nanjing University, **China**
daidarong998@163.com
Jun YIN
Department of Economics, Nanjing University, **China**

Abstract:

In the paper, we are encouraged to investigate the effect of game structure imposed on the minimum-time needed to economic maturity in a dynamic macroeconomic model. Indeed, we have established a basic framework for the comparative study of the cooperative stochastic differential game and dynamic sequential game of economic maturity. Moreover, in a simple stochastic growth model, closed-form solution of the minimum-time needed to economic maturity has been derived with the explicit condition, under which it is confirmed that cooperation between the representative household and the self-interested politician will definitely lead us to much faster economic maturity than that of sequential action, supplied, too. Finally, our model supports the comparative study of the minimum-time needed to economic maturity under different political-institution constraints.

Keywords: economic maturity, minimum-time objective, political economy, sequential equilibrium, cooperative stochastic differential game.

JEL Classification: C70, D72, O11.

1. Introduction

It is widely noted that institutional difference is one of the major differences between the developing economies and the developed economies. Usually, different institutional arrangements will form different incentive structures, induce different economic behaviors of the individuals, different fiscal policies of the government, and hence different speeds and levels of economic maturity. That is, different game structures lead to different institutional arrangements (e.g., North, 1990; Hurwicz, 1996; Williamson, 2000; Amable, 2003), hence producing different economic performances (see, North, 1994; Acemoglu *et al.,* 2005a, 2005b). The major goal of the current exploration is to construct a basic framework to comparatively study the minimum-time needed to economic maturity under different game structures, i.e., dynamic sequential game and cooperative stochastic differential game. Indeed, we have derived closed-form solution of the minimum-time needed to economic maturity in a simple model of endogenous economic growth (e.g., Barro, 1990; Rebelo, 1991; Turnovsky, 2000; Aghion, 2004; Wälde, 2011; Dai, 2012, 2013), where competitive assumption is employed for the firm, endogenous savings rate is determined by the representative household and the goal of the self-interested politician is to choose a tax policy such that the utility from tax revenue, which can be viewed as the rent, is maximized. Leong and Huang (2010) confirm that uncertainty will produce more realistic solution than that of the deterministic case (see, Kaitala and Pohjola, 1990). We also consider a stochastic environment as in Merton (1975), i.e., the source of uncertainty is the population size. In addition to that, the explicit condition, under which cooperation between the representative household and the self-interested politician will lead to much faster economic maturity than that of sequential action, has been supplied for the first time. And in fact, the explicit condition is determined by the relevant model parameters, such as discount factor, technology parameter, depreciation factor, variance term, and the natural rate of population growth.

The current investigation focuses on the issue of economic maturity for any underdeveloped economy. We argue that the state of economic maturity can be interpreted as a Golden Age (e.g., Phelps, 1961) or a turnpike (see, McKenzie, 1963a; Dai, 2012) of the economy and the formal definition of the minimum-time needed to economic maturity has been stated using mathematical language. Indeed, we pay more attention to economic development rather than purely economic growth (see, Solow, 2003; Aghion, 2004; Dai, 2013). Moreover, it's easy to notice that our paper is a natural extension of the seminal and interesting paper of Kurz (1965), where optimal paths of capital accumulation under the minimum-time objective are thoroughly investigated. It is, nevertheless, worthwhile emphasizing that the minimum-time needed to economic maturity is endogenously determined in our model. And this would be regarded as an advantage of the optimal stopping theory used here.

Some seminal papers (see, Judd, 1985; Chamley, 1986; Phelan and Stacchetti, 2001, and among others) study dynamic optimal Ramsey taxation under the crucial assumption that taxes are set by benevolent governments. Nevertheless, in practice and also in line with the public choice theory (e.g., Buchanan and Tullock, 1962; Barro, 1973; Ferejohn, 1986), the politician's preferences may diverge from those of his constituents and that he may pursue his self-interest. Indeed, some existing literatures study the dynamic taxation under the assumption that taxes are decided by a self-interested politician. For example, Acemoglu *et al.* (2008, 2010, 2011) consider the case where the self-interested politicians have the power to set taxes and meanwhile the citizens can discipline politicians using elections or other means.

Moreover, Acemoglu *et al.* (2008, 2010, 2011) analyze the political economy distortions by supplying that the politician has the power to allocate some of the tax revenue to himself as rents or government consumption, and also a formal politician utility, which is usually different from that of the individual or citizen, is supplied. Yared (2010) characterizes optimal tax policies in the presence of rent-seeking politicians whose utilities increase in rents, which are defined as excessive public spending with no social value, and also highlights how the incentives of rent-seeking politicians affect optimal policy prescriptions. As you can see below, we also suppose a self-interested politician in our model. And we further consider three types of self-interested politician, i.e., strongly self-interested politician, semi-strongly self- interested politician, and weakly self-interested politician, in order to sufficiently reflect different political institutions in reality. That is, we have provided a general framework for the study of the economic effect of the minimum-time needed to economic maturity with respect to different political institutions. Noting that North (1994) has emphasized that political and economic institutions are the underlying determinants of economic performance, hence our model provides us with a useful framework in which we can explicitly explore in which way and to what extent political institutions affect economic performance in a specific growth model.

Starting with time inconsistency being introduced by the seminal paper of Kydland and Prescott (1977), latter papers, such as Chari and Kehoe (1990, 1993), argue that fiscal-policy problems should be better studied as a dynamic game between the government and the households. For instance, in a repeated-game framework, Chari and Kehoe (1990) focus on sustainable plans characterized by symmetric perfect Bayesian equilibria. Similar to the sustainable equilibrium defined and analyzed by Chari and Kehoe (1990), Phelan and Stacchetti (2001) provide a formal definition of a sequential equilibrium for the dynamic policy game between the government and the households, and also develop a strategic dynamic programming method. Acemoglu *et al.* (2008, 2010, 2011) study dynamic taxation policy in the context of a dynamic game between a self-interested government and citizens, and characterize the best sub-game perfect equilibrium of this game from the viewpoint of the citizens. Yared (2010) considers an infinitely repeated game between citizens and rent-seeking politicians with double-sided lack of commitment in which reputation mechanism sustains efficient equilibrium policies.

Also, Farhi and Werning (2008) study efficient nonlinear taxation in a dynamic game with political economy constraints and without commitment, it is revealed that reputational mechanism induces trigger-strategy equilibrium, where a deviation would be followed by the worst possible continuation equilibrium.

In our study, it is however illustrated that under certain conditions the unique sub-game perfect equilibrium may result in dynamic inefficiency when compared to the cooperative equilibrium, and also sub-game consistency, which is a much stronger concept than that of time consistency (see, Fischer, 1980; Klein *et al.*, 2008) in some sense, has been demonstrated to be met for the current model by heavily employing the technique developed by Yeung and Petrosyan (2006). In other words, we employ backward induction principle to ensure time consistency in the dynamic sequential game while using sub-game consistency to ensure time consistency in the cooperative stochastic differential game. To sum up, the present model has supplied a useful framework for the comparative study of the dynamic sequential game emphasized by Phelan and Stacchetti (2001), Acemoglu *et al.* (2008, 2010, 2011), Dai *et al.* (2013) and the cooperative differential game studied by Kaitala and Pohjola (1990), Yeung and Petrosyan (2006) and Leong and Huang (2010) in a stochastic growth model under political-economy constraint. And hence our study would be regarded as a natural extension of existing literatures.

The current paper proceeds as follows. Section 2 thoroughly introduces some basic definitions and the computation algorithm of the sequential-equilibrium minimum-time needed to economic maturity. Sections 3 and 4 derive the sequential-equilibrium and cooperative-equilibrium minimum-time needed to economic maturity, respectively, in a very general framework. In section 5, we discuss some specific examples with closed-form solutions derived by applying the general model established in sections 3 and 4. And we finally close this study with some concluding remarks.

2. Computation algorithm of sequential equilibrium

2.1 Minimum-time needed to economic maturity

Suppose that we are given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, the optimization problem facing the economic agent is expressed as follows:

$$\max_{c(t)\geq 0} \mathbb{E}_s \left[\int_s^{\tau^*} e^{-\rho(t-s)} u\big(c(t)\big) dt \ \mid U^{\tau^*} \chi_{\{\tau^* < \infty\}} \right] \tag{1}$$

subject to the corresponding law of motion of capital accumulation with c denoting per capita consumption and with U^{τ^*} given by:

$$U^{\tau^*} \chi_{\{\tau^* < \infty\}} := \sup_{\tau \in \mathcal{T}} \mathbb{E}_s \left[e^{-\rho(\tau - s)} u\big(c(\tau)\big) \chi_{\{\tau < \infty\}} \right] \tag{2}$$

subject to the law of motion of capital accumulation and $\mathcal{T} := \{\mathcal{F} - \text{stopping times}\}$.

Hence, we give,

Definition 1 (Minimum-Time Needed to Economic Maturity). The optimal stooping time τ^* determined by (2) defines a minimum-time needed to economic maturity in the sense of Radner Preference.

About the definition of Radner Preference, one may refer to the classical paper of Radner (1961). And one can easily notice that the specification in (2) efficiently captures the Ratchet effect emphasized by traditional consumption theory and hence we would also call it the “peak preference” with the purpose of reflecting the psychological effect in consumption. As is well-known, when discussing efficient capital accumulation, efficiency is usually defined with reference to the final state (see, Radner, 1961; Kurz, 1965; Dai, 2012, 2013) or the terminal stock (see, McKenzie, 1963b, 1976). In this paper, the terminal stock, equivalent to efficient capital accumulation in some sense, is endogenously determined as well as the minimum-time needed to economic maturity, which is an optimal stopping time that maximizes the final-state objective function of the economic agent, i.e., choosing a minimum time so as to maximize the discounted utility function, which, to some extent, resembles Kurz’s (1965) specification, that is, minimizing the time needed to reach the state of economic maturity.

2.2 Types of self-interested politician

Now, we introduce three types of self-interested politician according to the above specification. Firstly, we give the respective preference of the representative household and the self-interested politician as follows,

$$\max_{c^H(t)\geq 0} \mathbb{E}_s\left[\int_s^{\tau} e^{-\rho(t-s)} u^H\left(c^H(t)\right)dt + e^{-\rho(\tau-s)} u^H\left(c^H(\tau)\right)\chi_{\{\tau<\infty\}}\right] \tag{3}$$

and,

$$\max_{c^P(t)\geq 0} \mathbb{E}_s\left[\int_s^{\tau} e^{-\rho(t-s)} u^P\left(c^P(t)\right)dt + e^{-\rho(\tau-s)} u^P\left(c^P(\tau)\right)\chi_{\{\tau<\infty\}}\right] \tag{4}$$

for any $\tau \subset \mathcal{T}$.

Then, we give the following definitions:

Definition 2 (Self-Interested Politician). We call the politician the self-interested politician when he consumes the tax revenue as rent. That is, the politician is not benevolent in the usual sense.

Definition 3 (Strongly Self-Interested Politician). We call the politician the strongly self-interested politician when he is self-interested and also the minimum-time needed to economic maturity τ^* is determined by:

$$U^{P,\tau^*}\chi_{\{\tau^*<\infty\}} := \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\left[e^{-\rho(\tau-s)} u^P\left(c^P(\tau)\right)\chi_{\{\tau<\infty\}}\right]$$

subject to the law of motion of capital accumulation.

Remark 2.1 In this case, the preference of the representative household is given by (3) while the preference of the politician given by:

$$\mathbb{E}_s\left[\int_s^{\tau} e^{-\rho(t-s)} u^P\left(c^P(t)\right)dt + U^{P,\tau}\chi_{\{\tau<\infty\}}\right]$$

Definition 4 (Semi-Strongly Self-Interested Politician). We call the politician the semi-strongly self-interested politician when he is self-interested and also the minimum-time needed to economic maturity $\tau^* := \tau^{P*} \wedge \tau^{H*}$ is determined by:

$$U^{P,\tau^{P*}}\chi_{\{\tau^{P*}<\infty\}} := \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\left[e^{-\rho(\tau-s)} u^P\left(c^P(\tau)\right)\chi_{\{\tau<\infty\}}\right]$$

and

$$U^{H,\tau^{H*}}\chi_{\{\tau^{H*}<\infty\}} := \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\left[e^{-\rho(\tau-s)} u^H\left(c^H(\tau)\right)\chi_{\{\tau<\infty\}}\right]$$

subject to the corresponding law of motion of capital accumulation, respectively.

Remark 2.2 In this case, the preferences of the politician and the representative household are respectively given as follows,

$$\mathbb{E}_s\left[\int_s^{\tau} e^{-\rho(t-s)} u^P\left(c^P(t)\right)dt + U^{P,\tau}\chi_{\{\tau<\infty\}}\right]$$

and,

$$\mathbb{E}_s\left[\int_s^{\tau} e^{-\rho(t-s)} u^H\left(c^H(t)\right)dt + U^{H,\tau}\chi_{\{\tau<\infty\}}\right]$$

with $\tau \in \mathcal{T}$.

Definition 5 (Weakly Self-Interested Politician). We call the politician the weakly self-interested politician when he is self-interested and also the minimum-time needed to economic maturity τ^* is determined by:

$$U^{H,\tau^*}\chi_{\{\tau^*<\infty\}} := \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\left[e^{-\rho(\tau-s)}u^H\left(c^H(\tau)\right)\chi_{\{\tau<\infty\}}\right]$$

subject to the law of motion of capital accumulation.

Remark 2.3 In this case, the preferences of the politician and the representative household are respectively given as follows,

$$\mathbb{E}_s\left[\int_s^\tau e^{-\rho(t-s)}u^P\left(c^P(t)\right)dt + e^{-\rho(\tau-s)}u^P\left(c^P(\tau)\right)\chi_{\{\tau<\infty\}}\right]$$

and

$$\mathbb{E}_s\left[\int_s^\tau e^{-\rho(t-s)}u^H\left(c^H(t)\right)dt + U^{H,\tau}\chi_{\{\tau<\infty\}}\right]$$

with $\tau \in \mathcal{T}$.

2.3 Computation Algorithm

We introduce the following computation algorithm by employing the well-known backward induction principle:

Case 1. There is a strongly self-interested politician in the economy. The economic agents will act sequentially and the order of action reads as follows:

- The politician determines the minimum-time needed to economic maturity based upon any given taxation policy and any given consumption strategy of the representative household.
- Based on i), the politician chooses the taxation policy to maximize his welfare given any possible consumption strategy of the representative household.
- Based upon i) and ii), the representative household determines his optimal consumption.

And hence the corresponding computation algorithm is given by:

Computation Algorithm I

Step 1. The representative household chooses his\her optimal consumption strategy given the taxation policy of the politician and the time horizon of the program.

Step 2. The self-interested politician chooses the taxation policy to maximize his welfare\utility given the optimal consumption strategy of the representative household derived in Step 1 and any possible time horizon of the program.

Step 3. Based upon the results derived in Steps 1 and 2, the minimum-time needed to economic maturity is established by the strongly self-interested politician.

Case 2. There is a semi-strongly self-interested politician in the economy. Now, the order of action reads as follows:

- The politician determines the minimum-time needed to economic maturity based upon any given taxation policy of himself and any given consumption strategy of the representative household.
- The politician chooses the taxation policy to maximize his welfare based on i) and given any possible consumption strategy of the representative household.
- The representative household determines the minimum-time needed to economic maturity based upon the taxation policy derived in ii).

- The representative household chooses his optimal consumption strategy based upon the taxation policy given by ii).

It follows from the well-known backward-induction rational principle that the computation algorithm is given as follows:

Computation Algorithm II

- *Step 1*. The representative household chooses his\her optimal consumption strategy given the taxation policy of the politician and the time horizon of the program.
- *Step 2*. Based upon Step 1, the minimum-time needed to economic maturity τ^{H*} is determined given any taxation policy of the politician.
- *Step 3*. Based upon the result derived in Step 1, the taxation policy is determined by the semi-strongly self-interested politician.
- *Step 4*. The minimum-time needed to economic maturity τ^{P*} is determined by the politician based upon Steps 1 and 3.
- *Step 5*. Given the minimum-time needed to economic maturity τ^{H*} and τ^{P*} derived in Steps 2 and 4, respectively, we define the unique minimum-time needed to economic maturity as $\tau^* := \tau^{P*} \wedge \tau^{H*}$.

Case 3. There is a weakly self-interested politician in the economy. Noting that the economic agents will act sequentially, then the order of action reads as follows:

- The minimum-time needed to economic maturity is derived by the representative household for any given taxation policy of the politician and any given consumption strategy of the representative household.
- Based on i), the taxation policy is determined by the politician to maximize his welfare for any given consumption strategy of the representative household.
- Based upon i) and ii), the optimal consumption strategy is determined by the representative household.

So, by applying the backward induction principle, we get the following computation algorithm:

Computation Algorithm III.

Step 1. The representative household chooses his\her optimal consumption strategy based upon any given taxation policy of the politician and any given time horizon of the program.

Step 2. Provided the result derived in Step 1, the taxation policy is chosen by the self-interested politician to maximize his welfare\utility for any possible time horizon of the economy.

Step 3. The minimum-time needed to economic maturity is established by the representative household based upon the results given in Steps 1 and Step 2.

Therefore, we have stated all the computation algorithms for the sequential-equilibrium minimum-time needed to economic maturity for the above three cases corresponding to three types of self-interested politician.

3. Sequential-equilibrium economic maturity

3.1 Basic environment

We consider the following neoclassical production function:

$$Y(t) = F\big(K(t), L(t)\big) \tag{5}$$

which is a strictly concave function, and it also exhibits constant returns to scale effect with K denoting the aggregate capital stock and L representing the labor force or population size. Thus, the following law of motion of capital accumulation is derived,

$$\dot{K}(t) := \frac{dK}{dt} = \left(1 - \tau_K(t)\right)\left(F_K\left(K(t), L(t)\right) - \delta\right)K(t) + \left(1 - \tau_W(t)\right)F_L\left(K(t), L(t)\right)L(t) - \left(1 + \tau_c(t)\right)C(t) \tag{6}$$

where δ, a given constant, denotes the depreciation factor, $F_K\left(K(t), L(t)\right) := \frac{\partial F}{\partial K}\left(K(t), L(t)\right)$, $F_L\left(K(t), L(t)\right) := \frac{\partial F}{\partial L}\left(K(t), L(t)\right)$, $C(t)$ stands for aggregate consumption, and $\tau_K(t)$, $\tau_W(t)$ and $\tau_C(t)$ represent capital-income tax rate, labor-income tax rate, and consumption tax rate, respectively, at period t.

Now, suppose that $(B(t), s \leq t \leq T)$ stands for a standard Brownian motion defined on the following filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{s \leq t \leq T}, \mathbb{P})$ with $\mathbb{F} := \{\mathcal{F}_t\}_{s \leq t \leq T}$ the $\mathbb{P}$ −augmented filtration generated by $(B(t), s \leq t \leq T)$ with $\mathcal{F} := \mathcal{F}_T$ for $\forall T > 0$, that is, the underlying stochastic basis satisfies the well-known usual conditions. Then, based upon the given probability space and also in line with Merton (1975), we define,

$$dL(t) = nL(t)dt + \sigma L(t)dB(t) \tag{7}$$

subject to $B(s) = 0$ and $\sigma \in \mathbb{R}_0 := \mathbb{R} - \{0\}$, a constant. Thus, combining (6) with (7) and applying Itô's Rule leads us to,

$$dk(t) = \left\{\left[\left(1 - \tau_K(t)\right)\left(F_K\left(K(t), L(t)\right) - \delta\right) + \sigma^2 - n\right]k(t) + \left(1 - \tau_W(t)\right)F_L\left(K(t), L(t)\right) - \left(1 + \tau_C(t)\right)c(t)\right\}dt - \sigma k(t)dB(t) \tag{8}$$

with initial value $k(s) := k_0 > 0$ and $k(t) := \frac{K(t)}{L(t)}$, $c(t) := \frac{C(t)}{L(t)}$ denoting the capital-labor ratio and per capita consumption, respectively, at time t.

So, based upon (8), we give the following differential operator for the new process $\zeta(t) := (t - s, k(t))$,

$$\mathcal{A}\phi(\zeta_0) := \frac{\partial \phi}{\partial s}(\zeta_0) + \left\{\left[\left(1 - \tau_K(0)\right)\left(F_K\left(K(0), L(0)\right) - \delta\right) + \sigma^2 - n\right]k_0 + \left(1 - \tau_W(0)\right)F_L\left(K(0), L(0)\right) - \left(1 + \tau_C(0)\right)c(0)\right\}\frac{\partial \phi}{\partial k_0}(\zeta_0) + \frac{1}{2}\sigma^2 k_0^2 \frac{\partial^2 \phi}{\partial k_0^2}(\zeta_0) \tag{9}$$

for $\zeta_0 := (0, k_0)$ and $\forall \phi \in C_0^2(\mathbb{R} \times \mathbb{R}_+)$.

Noting that both continuity and differentiability are neighborhood properties, we hence fix a domain D in $\mathbb{R} \times \mathbb{R}_+$ and the probability law of $\zeta(t)$ starting at $\zeta_0 := (0, k_0)$ for $t = s$ is (with slight abuse of notation) also denoted by $\mathbb{P}^{\zeta_0}$.

And define: $\tau_D := \inf\{t > s;\ \zeta(t) \notin D\}$

Definition 6 (Regular Boundary). Let ∂D denote the boundary of the domain D, a point $\zeta_0 \in \partial D$ is called regular for D (w.r.t. $\zeta(t)$) if $\mathbb{P}^{\zeta_0}(\tau_D = s) = 1$.

This definition implies that a.a. paths of $\zeta(t)$ starting from ζ_0 leave D immediately.

Assumption 1. Without loss of generality, D is chosen such that τ_D is sufficiently large a.s. $\mathbb{P}^{\zeta_0}$.

3.2 Sequential-equilibrium minimum-time needed to economic maturity

Case 1. Suppose that there is a strongly self-interested politician in the economy.

Applying Computation Algorithm I, we then obtain,

Problem 1. It is assumed that the economy consists of $L(t)$ identical individuals, each of whom possesses perfect foresight in period t. Thus, the optimization problem facing the representative household is expressed as follows,

$$\max_{c(t)\geq 0} J^c(\zeta_0) := \mathbb{E}_s\left[\int_s^{\tau^*} e^{-\rho(t-s)} u^H\big(c(t)\big)dt + e^{-\rho(\tau^*-s)} u^H\big(c(\tau^*)\big)\chi_{\{\tau^*<\infty\}}\right]$$

subject to (8) with $\mathbb{E}_s$ denoting the expectation operator depending on $\mathcal{F}_s$ and $u^H(\cdot)$ the strictly increasing, strictly concave instantaneous utility function of per capita consumption with the well-known Inada conditions satisfied.

So, we get,

Theorem 1 (Necessity). Define,

$$\psi^H(\zeta_0) = \sup\{J^c(\zeta_0); c = c(\zeta) Markov\ control\}$$

Suppose that $\psi^H \in C^2(D) \cap C(\overline{D})$ satisfies,

$$\mathbb{E}_s\left[\int_s^{\tau^*} |\mathcal{A}\psi^H(\zeta(t))|dt + |\psi^H(\zeta(\tau^*))|\right] < \infty$$

for all bounded stopping times $\tau^* \leq \tau_D$. Moreover, suppose that an optimal Markov control c^* exists and that ∂D is regular in the sense of Definition 6 for $\zeta(t)$. Then,

$$\sup_{c\geq 0}\{e^{\rho s} u^H(c) + \mathcal{A}\psi^H(\zeta_0)\} = 0$$

for all $\zeta_0 \in D$, and,

$$\psi^H(\zeta_0) = e^{\rho s} u^H(c)$$

for all $\zeta_0 \in \partial D$. In other words, the optimal consumption c^* meets,

$$e^{\rho s} u^H(c^*) + \mathcal{A}J^{c^*}(\zeta_0) = 0$$

for all $\zeta_0 \in D$.

Proof. This is a direct application of the Theorem of Hamilton-Jacobi-Bellman (HJB) Equation (see, Øksendal, 2003).

Theorem 2 (Sufficiency). Let φ^H be a function in $C^2(D) \cap C(\overline{D})$ such that for all $c \geq 0$,

$$e^{\rho s} u^H(c) + \mathcal{A}\varphi^H(\zeta_0) \leq 0$$

for all $\zeta_0 \in D$ with boundary values,

$$\lim_{t \to \tau_D} \varphi^H(\zeta(t)) = e^{-\rho(\tau_D - s)} u^H\left(c(\tau_D)\right)\chi_{\{\tau_D < \infty\}}$$

a.s. $\mathbb{P}^{\zeta_0}$, and such that,

$$\{\varphi^{H-}(\zeta(\tau^*));\ \tau^* stopping\ time\ with\ \tau^* \le \tau_D\}$$

is uniformly $\mathbb{P}^{\zeta_0}$ −integrable for all Markov controls c and all $\zeta_0 \in D$. Then,

$$\varphi^H(\zeta_0) \ge J^c(\zeta_0)$$

for all Markov controls c and all $\zeta_0 \in D$. Moreover, if for each $\zeta_0 \in D$ we have found $c^*(\zeta_0)$ such that,

$$e^{\rho s} u^H\left(c^*(\zeta_0)\right) + \mathcal{A}\varphi^H(\zeta_0) = 0$$

and,

$$\left\{\varphi^{H,c^*}(\zeta(\tau^*));\ \tau^* stopping\ time\ with\ \tau^* \le \tau_D\right\}$$

is uniformly $\mathbb{P}^{\zeta_0}$ −integrable for all $\zeta_0 \in D$. Then, $c^* = c^*(\zeta)$ is a Markov control such that,

$\varphi^H(\zeta_0) = J^{c^*}(\zeta_0)$ *and hence if c^* is admissible then c^* must be an optimal control and $\varphi^H(\zeta_0) = \psi^H(\zeta_0)$, which appears in Theorem 1.*

Proof. A canonical application of the Verification Theorem of HJB Equation (see, Øksendal, 2003) shows the desired assertion.

Some papers such as Karatzas and Wang (2000), Jeanblanc *et al.* (2004), and also the textbook of Øksendal and Sulem (2005) study utility maximization with discretionary stopping. Instead of deriving the optimal stopping time and the optimal controls simultaneously, Theorems 1 and 2 establish optimal consumption for any given stopping time based upon our Computation Algorithm defined in Section 2. In other words, the dynamic sequential game structure between the representative household and the self-interested politician will naturally make the corresponding computation of the optimal controls much easier. And this would be regarded as a byproduct of the dynamic sequential game discussed here. Moreover, it would be interesting to notice that optimal controls indeed interact with each other when the economic agents are faced with various types of decisions, i.e., optimal stopping time and optimal consumption appear in Theorems 1 and 2 can be regarded as totally different control variables in some sense.

Thus, in contrast to the traditional consumption theory, Theorems 1 and 2 show us that optimal consumption will endogenously affect the underlying minimum-time needed to economic maturity on the one hand, and on the other hand, the minimum-time needed to economic maturity will in turn constraint the choice of optimal consumption behavior as a stochastic boundary condition in the corresponding optimization problem facing the representative household. And this would be interpreted as the new characteristic of Theorems 1 and 2 when compared to existing papers focusing on optimal consumption theory.

Thus, in what follows, we substitute c^* into (9) and we will use $\mathcal{A}^{c^*}\phi(\zeta_0)$ instead of $\mathcal{A}\phi(\zeta_0)$ for all $\zeta_0 \in D$. And also (8) would be rewritten as follows,

$$dk(t) = \{[(1-\tau_K(t))(F_K(K(t),L(t))-\delta)+\sigma^2-n]k(t) + (1-\tau_W(t))F_L(K(t),L(t))-(1+\tau_C(t))c^*(t)\}dt - \sigma k(t)dB(t) \quad (10)$$

Now, the optimization problem facing the self-interested politician can be expressed as follows:
Problem 2. Here, we particularly consider the taxation-revenue consumption per capita for the politician. That is, the self-interested politician faces the following optimization problem,

$$\max_{\tau_K(t),\tau_W(t),\tau_C(t)} J^{(\tau_K,\tau_W,\tau_C)}(\zeta_0) := \mathbb{E}_s\left[\int_s^{\tau^*} e^{-\rho(t-s)} u^P\left(\tau_K(t)(F_K(K(t),L(t)) - \delta)k(t), \tau_W(t)F_L(K(t),L(t)), \tau_C(t)c^*(t)\right)dt + U^{P,\tau^*}(\zeta_0)\chi_{\{\tau^*<\infty\}}\right]$$

subject to (10) with $\mathbb{E}_s$ denoting the expectation operator depending on $\mathcal{F}_s$ and $u^P(\cdot,\cdot,\cdot)$ the smooth and increasing instantaneous utility function. Indeed, the specification of u^P can efficiently reflect the type of politician in the sense of preference, i.e., risk-aversion politician, risk-neutral politician, and risk-preference politician.

So, quite similar to Theorems 1 and 2, we obtain:
Theorem 3 (Necessity). Define,

$$\psi^P(\zeta_0) = \sup\{J^{(\tau_K,\tau_W,\tau_C)}(\zeta_0); \tau_K = \tau_K(\zeta), \tau_W = \tau_W(\zeta), \tau_C = \tau_C(\zeta) Markov\ controls\}$$

Suppose that $\psi^P \in C^2(D) \cap C(\overline{D})$ *satisfies,*

$$\mathbb{E}_s\left[\int_s^{\tau^*} \left|\mathcal{A}^{c^*}\psi^P(\zeta(t))\right|dt + \left|U^{P,\tau^*}(\zeta_0)\chi_{\{\tau^*<\infty\}}\right|\right] < \infty$$

for all bounded stopping times $\tau^* \leq \tau_D$. *Moreover, suppose that an optimal Markov control* $(\tau_K^*, \tau_W^*, \tau_C^*)$ *exists and that* ∂D *is regular in the sense of Definition 6 for* $\zeta(t)$. *Then,*

$$\sup_{\tau_K,\tau_W,\tau_C}\{e^{\rho s}u^P(\tau_K(F_K(K,L)-\delta)k_0, \tau_W F_L(K,L), \tau_C c^*) + \mathcal{A}^{c^*}\psi^P(\zeta_0)\} = 0$$

for all $\zeta_0 \in D$, *and,*

$$\psi^P(\zeta_0) = U^{P,\tau^*}(\zeta_0) \quad (11)$$

for all $\zeta_0 \in \partial D$. *In other words, the optimal control* $(\tau_K^*, \tau_W^*, \tau_C^*)$ *fulfills,*

$$e^{\rho s}u^P(\tau_K^*(F_K(K,L)-\delta)k_0,\tau_W^*F_L(K,L),\tau_C^*c^*)+\mathcal{A}^{c^*}J^{(\tau_K^*,\tau_W^*,\tau_C^*)}(\zeta_0)=0$$

for all $\zeta_0 \in D$.

And also, *Theorem 4 (Sufficiency). Let* φ^P *be a function in* $C^2(D)\cap C(\overline{D})$ *such that for all* (τ_K,τ_W,τ_C),

$$e^{\rho s}u^P(\tau_K(F_K(K,L)-\delta)k_0,\tau_W F_L(K,L),\tau_C c^*)+\mathcal{A}^{c^*}\varphi^P(\zeta_0)\leq 0$$

for all $\zeta_0 \in D$ *with boundary values,*

$$\lim_{t\to\tau_D}\varphi^P(\zeta(t))=U^{P,\tau_D}(\zeta_0)\chi_{\{\tau_D<\infty\}}$$

a.s. $\mathbb{P}^{\zeta_0}$, *and such that,*

$$\{\varphi^{P-}(\zeta(\tau^*));\ \tau^*\mathit{stopping\ time\ with}\ \tau^*\leq\tau_D\}$$

is uniformly $\mathbb{P}^{\zeta_0}$ *–integrable for all Markov controls* (τ_K,τ_W,τ_C) *and all* $\zeta_0 \in D$. *Then,*

$$\varphi^P(\zeta_0)\geq J^{(\tau_K,\tau_W,\tau_C)}(\zeta_0)$$

for all Markov controls (τ_K,τ_W,τ_C) *and all* $\zeta_0 \in D$. *Moreover, if for each* $\zeta_0 \in D$ *we have found that* $(\tau_K^*(\zeta_0),\tau_W^*(\zeta_0),\tau_C^*(\zeta_0))$ *such that,*

$$e^{\rho s}u^P\left(\tau_K^*(\zeta_0)(F_K(K,L)-\delta)k_0,\tau_W^*(\zeta_0)F_L(K,L),\tau_C^*(\zeta_0)c^*(\zeta_0)\right)+\mathcal{A}^{c^*}\varphi^P(\zeta_0)=0$$

and,

$$\left\{\varphi^{P,(\tau_K^*,\tau_W^*,\tau_C^*)}(\zeta(\tau^*));\ \tau^*\mathit{stopping\ time\ with}\ \tau^*\leq\tau_D\right\}$$

is uniformly $\mathbb{P}^{\zeta_0}$ *–integrable for all* $\zeta_0 \in D$. *Then,*

$$(\tau_K^*,\tau_W^*,\tau_C^*)=\left(\tau_K^*(\zeta),\tau_W^*(\zeta),\tau_C^*(\zeta)\right)$$

is a Markov control such that,

$$\varphi^P(\zeta_0)=J^{(\tau_K^*,\tau_W^*,\tau_C^*)}(\zeta_0)$$

and hence if $(\tau_K^*,\tau_W^*,\tau_C^*)$ *is admissible then* $(\tau_K^*,\tau_W^*,\tau_C^*)$ *must be an optimal control and* $\varphi^P(\zeta_0)=\psi^P(\zeta_0)$, *which appears in Theorem 3.*

Many existing literatures (e.g., Chamley, 1986; Jones *et al.*, 1993; Phelan and Stacchetti, 2001; Kocherlakota, 2005; Acemoglu *et al.*, 2011, and among others) focusing on taxation theory build up discrete-time models with exogenously prescribed time horizon. And some seminal papers (see, Chamley, 1986; Jones *et al.*, 1993; Acemoglu *et al.*, 2011) would heavily depend on the existence of the long-run steady state of the economy while Theorems 3 and 4 holding along the whole path of capital accumulation with the tax rates exhibiting Markov properties. And also the time horizon of the planning problem is endogenously determined in our model. That is to say, Theorems 3 and 4 show us formulas characterizing the taxation rates under political-economy constraint and also for very general preference functions, technology functions and endogenous time horizon. Generally, economic intuition will lead us to investigate how tax rates would affect the equilibrium minimum-time needed to economic maturity. In other words, we are usually inclined to focus on the policy effect of the government imposed on the equilibrium minimum-time needed to economic maturity. The results presented in Theorems 3 and 4, however, show us the inverse effect, i.e., the equilibrium minimum-time needed to economic maturity, characterized via a stochastic stopping time, as a stochastic boundary condition will also affect the equilibrium (in the sense of sub-game perfect) choice of tax rates of the self-interested politician. And hence such kind of interaction would shed some new insights into taxation theory from the viewpoint of economic development.

Hence, in what follows, we use the characteristic operator $\mathcal{A}^{c^*,(\tau_K^*,\tau_W^*,\tau_C^*)}\phi(\zeta_0)$ instead of $\mathcal{A}^{c^*}\phi(\zeta_0)$ for all $\zeta_0 \in D$. Also, inserting $(\tau_K^*, \tau_W^*, \tau_C^*)$ into (10) produces,

$$
\begin{aligned}
&dk(t) \\
&= \{[(1-\tau_K^*(t))(F_K(K(t),L(t))-\delta)+\sigma^2-n]k(t) \\
&+(1-\tau_W^*(t))F_L(K(t),L(t))-(1+\tau_C^*(t))c^*(t)\}dt \\
&-\sigma k(t)dB(t)
\end{aligned}
\tag{12}
$$

Thus, we can give,

Problem 3. Let $\mathcal{T}$ denote the set of all $\mathcal{F}-$stopping times $\tau \leq \tau_D$. Consider the following problem facing the self-interested politician,

$$
\begin{aligned}
&U^{p,\tau^*}(\zeta_0)\chi_{\{\tau^*<\infty\}} \\
&\qquad := \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\Big[e^{-\rho(\tau-s)}u^p\Big(\tau_K^*(\tau)\big(F_K(K(\tau),L(\tau)) \\
&\qquad -\delta\big)k(\tau),\tau_W^*(\tau)F_L(K(\tau),L(\tau)),\tau_C^*(\tau)c^*(\tau)\Big)\chi_{\{\tau<\infty\}}\Big] \\
&\qquad := \sup_{\tau\in\mathcal{T}}\mathbb{E}_s\big[e^{\rho s}\tilde{u}^p(\zeta(\tau))\chi_{\{\tau<\infty\}}\big]
\end{aligned}
$$

subject to (12).

It follows from Problem 3 that we have extended the concept of self-interested politician widely used by Acemoglu *et al.* (2008, 2010, 2011) and Yared (2010), and among others. Since the major issue of the present exploration is to compute the minimum-time needed to economic maturity for underdeveloped economies, the strongly self-interested politician rather than the representative household will determine the optimal stopping time. That is, the corresponding minimum-time needed to economic maturity only takes into account the utility or welfare of the self-interested politician. Indeed, this specification reflects certain type of political institutional arrangement of planning economies in reality. In addition, it is easily seen that the specification in Problem 3 is totally different from that in Dai (2012, 2013), where there is a benevolent government in the underlying economy. And it is insisted that such kind of difference indeed reflects different institutional arrangements in reality. For example, in many planning economies, it is the politician's or the government's interests instead of the households' interests that will determine the long-run economic development policy, i.e., the minimum-time needed to economic maturity. Obviously, such kind of institutional arrangement will induce an incentive structure among the economic agents leading to very poor economic performance, especially in the long run.

Undoubtedly, both Dai (2012, 2013) and Problem 3 just consider special or extreme cases. Nevertheless, what's the corresponding lesson? For underdeveloped economies, in order to promote long-run and sustainable economic development, good institutions such as democratic institutions and market- economy institutions in Western world should be established first with the purpose of endogenously producing efficient incentive structure in the economy. In other words, the corresponding political and economic institutions should play a quite positive role in increasing the encompassing interests (see, Olson, 2000) between the politician and the household.

We then obtain by solving Problem 3,

Theorem 5 (Sequential-Equilibrium Minimum-Time Needed to Economic Maturity: Existence).

a) *Suppose that we can find a function* $\phi^P : \bar{D} \to \mathbb{R}$ *such that,*

$\phi^P \in C^1(D) \cap C(\bar{D})$

$\phi^P \geq e^{\rho s}\tilde{u}^P$ *on* D *and* $\lim_{t\to\tau_D} \phi^P(\zeta(t)) = e^{\rho s}\tilde{u}^P(\zeta(\tau_D))\chi_{\{\tau_D<\infty\}}$ *a.s.* $\mathbb{P}^{\zeta_0}$

Define $G := \{\zeta_0 \in D;\ \phi^P(\zeta_0) > e^{\rho s}\tilde{u}^P(\zeta_0)\}$ *and suppose* $\zeta(t)$ *spends 0 time on* ∂G *a.s.* $\mathbb{P}^{\zeta_0}$, *i.e.,*

$\mathbb{E}_{\zeta_0}\left[\int_s^{\tau_D} \chi_{\partial G}(\zeta(t))dt\right] = 0$ *for all* $\zeta_0 \in D$, *and suppose that,* ∂G *is a Lipschitz surface.*

Moreover, suppose the following conditions: $\phi^P \in C^2(D \setminus \partial G)$ *and the second order derivatives of* ϕ^P *are locally bounded near* ∂G

$\mathcal{A}^{c^*,(\tau_K^*,\tau_W^*,\tau_C^*)}\phi^P \leq 0$ *on* $D \setminus \partial G$

Then, $\phi^P(\zeta_0) \geq U^{P,\tau^*}(\zeta_0)$ *for all* $\zeta_0 \in D$.

b) *Suppose, in addition to the above conditions, that:*

$\mathcal{A}^{c^*,(\tau_K^*,\tau_W^*,\tau_C^*)}\phi^P = 0$ *on* G

$\tau_G := \inf\{t > s;\ \zeta(t) \notin G\} < \infty$ *a.s.* $\mathbb{P}^{\zeta_0}$ *for* $\zeta_0 \in D$, *and the family* $\{\phi^{P-}(\zeta(\tau));\ \tau \leq \tau_G, \tau \in \mathcal{T}\}$ *is uniformly integrable w.r.t.* $\mathbb{P}^{\zeta_0}$ *for all* $\zeta_0 \in D$. *Then,* $\phi^P(\zeta_0) = U^{P,\tau^*}(\zeta_0) = \sup_{\tau\in\mathcal{T}}\mathbb{E}_s[e^{\rho s}\tilde{u}^P(\zeta(\tau))]$ *for all* $\zeta_0 \in D$ *and,* $\tau^* = \tau_G$ *is an optimal stopping time for this problem.*

Proof. A direct application of the variational inequalities for optimal stopping (see, Øksendal, 2003) produces the required assertion.

While one may notice certain similarity of the present approach to those literatures studying endogenous lifetime or endogenous longevity in growth models (see, Chakraborty, 2004; de la Croix and Ponthiere, 2010, and among others), there exist obvious differences especially when referring to economic intuitions and economic implications behind the formal models. For example, existing studies mainly focus on OLG models and health-investment behaviors while the current exploration emphasizing issues of macroeconomic development, i.e., formal characterization of economic maturity for underdeveloped economies and the corresponding characteristics of their optimal paths of capital accumulation. Furthermore, it is easily seen that the maximum sustainable capital-labor ratio corresponding to the state of economic maturity as well as the minimum time needed to economic maturity is endogenously determined by using stochastic optimal stopping theory that is widely applied in mathematical finance (see, Øksendal and Sulem (2005) and references therein). As is well known, in Kurz's (1965) study, the targets or the maximum sustainable level of terminal path capital-labor ratios are exogenously specified, and the corresponding minimum time problem is expressed as: for any given initial capital-labor ratios, to choose strategies so that the prescribed targets can be reached as soon as possible. The major innovation of the present approach, therefore, is that we endogenously determine the terminal path, the minimum time and also take the economic-welfare considerations of the strongly self-interested politician into account in solving the minimum-time problem. Last but not least, Theorem 5 indeed provides us with a general and complete characterization of the minimum-time needed to economic maturity when compared to the corresponding result in Dai (2012, 2013). And most importantly, this kind of generalization will sufficiently capture the economic effects of preferences and technologies on the minimum-time needed to economic maturity, which hence implies that Theorem 5 would be of independent interest.

Corollary 1. I*n principle, the sequential-equilibrium minimum-time needed to economic maturity* τ^* *can be further computed by the following equality,*

$$\psi^P(\zeta_0) = \phi^P(\zeta_0)$$

Proof. Combining (11) with Theorem 5 produces the desired result.

It is particularly worth emphasizing that Corollary 1 as well as the corollaries in Section 4 is one major innovation of the model because these corollaries provide simple conditions under which the corresponding minimum-time needed to economic maturity can be explicitly computed as is shown in Section 5. In addition to that, one may easily notice that the equilibrium minimum-time needed to economic maturity in Corollary 1 clearly reflects the reasonable combination of the optimal stopping theory and the stochastic dynamic programming method.

Case 2. Suppose that there is a semi-strongly self-interested politician in the economy.

Case 3. Suppose that there is a weakly self-interested politician in the economy.

Noting that the discussions corresponding to Cases 2 and 3 are quite similar to that of Case 1, we hence take them omitted and leave them to the interested readers.

4. Cooperative-equilibrium economic maturity

In the present section, we will introduce a new approach to economic maturity, i.e., cooperative-equilibrium economic maturity. Kaitala and Pohjola (1990), and Leong and Huang (2010) study the differential cooperative game between the firm and the government in deterministic and stochastic environments, respectively. However, we will investigate the differential cooperative game between the representative household and the self-interested politician with the time horizon endogenously determined. As a result, our following theorems are new relative to those of Kaitala and Pohjola (1990), and Leong and Huang (2010). Additionally, the following results will be much more complicated owing to the general preference and technology functions we employed here.

We will first introduce Markov feedback Nash equilibrium solution, and then cooperative equilibrium which fulfills the following requirements: individual rationality, group rationality, sub-game consistency and also Pareto-optimality. Moreover, we derive the payoff distribution procedure (PDP) of the cooperative game based upon the sub-game consistent imputation and provided that the players agree to act according to all agreed upon Pareto-optimal principle, for example, Nash bargaining solution and Shapley value. In particular, we give,

Assumption 2. Here, and throughout the current paper, it is assumed that payoffs\ utilities are transferable across players, i.e., the representative household and the self-interested politician, and over time.

Case 1. Suppose that there is a strongly self-interested politician in the economy.

Theorem 6 (Markov Feedback Nash Equilibrium Solution). We denote by $\Gamma(k_0, \hat{\tau} - s)$ *the differential game between the representative household and the self-interested politician, and hence a set of feedback strategies* $\left[\hat{c}^{(s)}(t,k); \hat{\tau}_K^{(s)}(t,k), \hat{\tau}_W^{(s)}(t,k), \hat{\tau}_C^{(s)}(t,k)\right]$ *provides a Nash equilibrium solution* *to the game* $\Gamma(k_0, \hat{\tau} - s)$*, if there exist continuously differentiable functions,* $V^{(s)i}(t,k): [s, \hat{\tau}] \times \mathbb{R}_+ \to \mathbb{R}$*,* $i \in \{H, P\}$*, satisfying the following Fleming-Bellman-Isaacs partial differential equations,*

$$-V_t^{(s)H}(t,k) - \frac{1}{2}\sigma^2 k^2 V_{kk}^{(s)H}(t,k)$$
$$= \max_{c \geq 0}\Big(e^{-\rho(t-s)}u^H(c)$$
$$+ V_k^{(s)H}(t,k)\{[(1-\hat{\tau}_K)(F_K(K,L)-\delta)+\sigma^2-n]k$$
$$+ (1-\hat{\tau}_W)F_L(K,L)-(1+\hat{\tau}_C)c\}\Big)$$

and,

$$-V_t^{(s)P}(t,k) - \frac{1}{2}\sigma^2 k^2 V_{kk}^{(s)P}(t,k)$$
$$= \max_{\tau_K,\tau_W,\tau_C}\Big(e^{-\rho(t-s)}u^P(\tau_K(F_K(K,L)-\delta)k, \tau_W F_L(K,L), \tau_C\hat{c})$$
$$+ V_k^{(s)P}(t,k)\{[(1-\tau_K)(F_K(K,L)-\delta)+\sigma^2-n]k$$
$$+ (1-\tau_W)F_L(K,L)-(1+\tau_C)\hat{c}\}\Big)$$

with the following boundary conditions,

$$V^{(s)H}(\hat{\tau}, k(\hat{\tau})) = e^{-\rho(\hat{\tau}-s)}u^H(c(\hat{\tau}))$$
$$V^{(s)P}(\hat{\tau}, k(\hat{\tau})) = U^{P,\hat{\tau}}(\zeta_0) \tag{13}$$

As argued by Fischer (1980), the problem of dynamic inconsistency can arise if the policy maker's utility function differs from that of the representative household. That is, there will be no dynamic inconsistency if the politician and the representative household face exactly the same optimization problem except for the variables they control.

Noting that the Markov feedback Nash equilibrium $\left[\hat{c}^{(s)}(t,k); \hat{\tau}_K^{(s)}(t,k), \hat{\tau}_W^{(s)}(t,k), \hat{\tau}_C^{(s)}(t,k)\right]$ given in Theorem 6 is Markovian in the sense that they are functions of current time t and current state $k = k(t)$, and thus independent of past values of state. This implies that the optimal solutions do not depend on the choice of the starting time of the optimal path, and accordingly the problem of dynamic inconsistency disappears even though the self-interested politician and the representative household face totally different optimization problems in Theorem 6. Moreover, besides the Feedback-Nash equilibrium solution established in Theorem 6, many literatures such as Pohjola (1983), and Başar *et al.* (1985) also studied Feedback-Stackelberg solution (see, Simaan and Cruz, 1973) in a differential game model of capitalism (e.g., Lancaster, 1973; Hoel, 1978). It is therefore asserted that Theorem 6 can also be extended to derive the corresponding Feedback-Stackelberg solution and one, if motivated, may also investigate the difference and similarity between the two kinds of solution in the present framework.

Now, inserting the feedback strategies derived in Theorem 6 into (8) gives rise to:

$$dk(t) = \left\{\left[\left(1-\hat{\tau}_K^{(s)}(t,k(t))\right)\left(F_K(K(t),L(t))-\delta\right)+\sigma^2-n\right]k(t) + \left(1-\hat{\tau}_W^{(s)}(t,k(t))\right)F_L(K(t),L(t)) - \left(1+\hat{\tau}_C^{(s)}(t,k(t))\right)\hat{c}^{(s)}(t,k(t))\right\}dt - \sigma k(t)dB(t) \quad (14)$$

Provided the Markov feedback Nash equilibrium $\left[\hat{c}^{(s)}(t,k);\hat{\tau}_K^{(s)}(t,k),\hat{\tau}_W^{(s)}(t,k),\hat{\tau}_C^{(s)}(t,k)\right]$ given in Theorem 6, then the corresponding stopping time $\hat{\tau}$ given in Theorem 6 is a solution to the following problem,

Problem 4. Similar to Problem 3, let $\mathcal{T}$ denote the set of all $\mathcal{F}$ −stopping times $\tau \leq \tau_D$. Then the optimal stopping problem facing the strongly self-interested politician reads as follows,

$$U^{P,\hat{\tau}}(\zeta_0)\chi_{\{\hat{\tau}<\infty\}} := \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\left[e^{-\rho(\tau-s)}u^P\left(\hat{\tau}_K^{(s)}(\tau,k(\tau))(F_K(K(\tau),L(\tau)) - \delta)k(\tau),\hat{\tau}_W^{(s)}(\tau,k(\tau))F_L(K(\tau),L(\tau)),\hat{\tau}_C^{(s)}(\tau,k(\tau))\hat{c}^{(s)}(\tau,k(\tau))\right)\chi_{\{\tau<\infty\}}\right]$$

$$:= \sup_{\tau\in\mathcal{T}}\mathbb{E}_s\left[e^{\rho s}\tilde{u}^P(\zeta(\tau))\chi_{\{\tau<\infty\}}\right]$$

subject to (14).

Solving Problem 4 establishes the following theorem, which is quite similar to Theorem 5.

Theorem 7 (Markov-Equilibrium Minimum-Time Needed to Economic Maturity: Existence).

a) Suppose that we can find a function $\phi^P:\bar{D}\to\mathbb{R}$ *such that,*

$\phi^P \in C^1(D)\cap C(\bar{D})$

$\phi^P \geq e^{\rho s}\tilde{u}^P$ *on* D *and* $\lim_{t\to\tau_D}\phi^P(\zeta(t)) = e^{\rho s}\tilde{u}^P(\zeta(\tau_D))\chi_{\{\tau_D<\infty\}}$ *a.s.* $\mathbb{P}^{\zeta_0}$

Define $G := \{\zeta_0\in D;\ \phi^P(\zeta_0) > e^{\rho s}\tilde{u}^P(\zeta_0)\}$ *and suppose* $\zeta(t)$ *spends 0 time on* ∂G *a.s.* $\mathbb{P}^{\zeta_0}$*, i.e.,*

$\mathbb{E}_{\zeta_0}\left[\int_s^{\tau_D}\chi_{\partial G}(\zeta(t))dt\right] = 0$ *for all* $\zeta_0\in D$*, and suppose that,*

∂G *is a Lipschitz surface.*

Moreover, suppose the following conditions: $\phi^P \in C^2(D\setminus\partial G)$ *and the second order derivatives of* ϕ^P *are locally bounded near* ∂G $\mathcal{A}^{\hat{c},(\hat{\tau}_K,\hat{\tau}_W,\hat{\tau}_C)}\phi^P \leq 0$ *on* $D\setminus\partial G$

Then, $\phi^P(\zeta_0) \geq U^{P,\hat{\tau}}(\zeta_0)$ *for all* $\zeta_0\in D$.

b) Suppose, in addition to the above conditions, that,

$\mathcal{A}^{\hat{c},(\hat{\tau}_K,\hat{\tau}_W,\hat{\tau}_C)}\phi^P = 0$ *on* G

$\tau_G := \inf\{t > s;\ \zeta(t)\notin G\} < \infty$ *a.s.* $\mathbb{P}^{\zeta_0}$ *for* $\zeta_0\in D$*, and*

The family $\{\phi^{P-}(\zeta(\tau));\ \tau\leq\tau_G, \tau\in\mathcal{T}\}$ *is uniformly integrable w.r.t.* $\mathbb{P}^{\zeta_0}$ *for all* $\zeta_0\in D$

Then, $\phi^P(\zeta_0) = U^{P,\hat{\tau}}(\zeta_0) = \sup_{\tau\in\mathcal{T}}\mathbb{E}_s[e^{\rho s}\tilde{u}^P(\zeta(\tau))]$ *for all* $\zeta_0\in D$

and, $\hat{\tau} = \tau_G$ *is an optimal stopping time for this problem.*

Corollary 2. I*n principle, the Markov-equilibrium minimum-time needed to economic maturity* $\hat{\tau}$ *can be further computed by the following equality,*

$$V^{(s)P}\left(\hat{\tau}, k(\hat{\tau})\right) = \phi^P(\zeta_0)$$

Proof. Combining (13) with Theorem 7 produces the desired result. Generally, the set of sub-game perfect sequential equilibrium is a subset of that of Nash equilibrium. For example, in many interesting games, there exist multiple Nash equilibrium while the uniqueness of the sub-game perfect Nash equilibrium can be ensured. Therefore, the Markov-equilibrium minimum time in Theorem 7 may rightly coincide with the sequential equilibrium minimum time in Theorem 5 on the one hand, while on the other hand, the Markov equilibrium minimum time may be also a relatively new concept under certain specifications of preference and technology in the model.

Now, we focus on the following cooperative stochastic differential game. First, we introduce,

Problem 5. Based upon Assumption 2, and suppose that the representative household and the self-interested politician agree to maximize the sum of their expected payoffs, i.e.

$$\max_{c,\tau_K,\tau_W,\tau_C} \mathbb{E}_s\Bigg\{\int_s^{\tau^{**}} e^{-\rho(t-s)}\Big[u^H(c(t)) + u^P\Big(\tau_K(t)\big(F_K(K(t),L(t)) - \delta\big)k(t), \tau_W(t)F_L\big(K(t),L(t)\big), \tau_C(t)c(t)\Big)\Big]dt + U^{Cooperation,\tau^{**}}(\zeta_0)\chi_{\{\tau^{**}<\infty\}}\Bigg\}$$

subject to (8).

In particular, both $U^{Cooperation,\tau^{**}}(\zeta_0)$ and τ^{**} are determined by the following problem:

Problem 6. When there is cooperation between the representative household and the strongly self-interested politician, then the minimum-time needed to economic maturity τ^{**} is determined by solving the following problem:

$$\begin{aligned}
&U^{Cooperation,\tau^{**}}(\zeta_0)\chi_{\{\tau^{**}<\infty\}} \\
&:= \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\Big\{e^{-\rho(\tau-s)}\Big[u^H\big(c^{**(s)}(\tau,k(\tau))\big) \\
&+ u^P\Big(\tau_K^{**(s)}(\tau,k(\tau))\big(F_K(K(\tau),L(\tau)) \\
&- \delta\big)k(\tau), \tau_W^{**(s)}(\tau,k(\tau))F_L\big(K(\tau),L(\tau)\big), \tau_C^{**(s)}(\tau,k(\tau))c^{**(s)}(\tau,k(\tau))\Big)\Big]\Big\} \\
&:= \sup_{\tau\in\mathcal{T}} \mathbb{E}_s\left[e^{\rho s}\tilde{u}^{Cooperation}(\zeta(\tau))\chi_{\{\tau<\infty\}}\right]
\end{aligned}$$

subject to,

$$
\begin{aligned}
&k(t) \\
&= \Big\{\Big[\Big(1-\tau_K^{**(s)}(t,k(t))\Big)\Big(F_K\big(K(t),L(t)\big)-\delta\Big)+\sigma^2-n\Big]k(t) \\
&+\Big(1-\tau_W^{**(s)}(t,k(t))\Big)F_L\big(K(t),L(t)\big) \\
&-\Big(1+\tau_C^{**(s)}(t,k(t))\Big)c^{**(s)}(t,k(t))\Big\}dt \\
&-\sigma k(t)dB(t)
\end{aligned} \tag{15}
$$

with $\tau_K^{**(s)}(t,k(t))$, $\tau_W^{**(s)}(t,k(t))$, $\tau_C^{**(s)}(t,k(t))$ and $c^{**(s)}(t,k(t))$ determined by Problem 5.

By solving Problem 5, we derive:

Theorem 8. We denote by $\Gamma^C(k_0,\tau^{**}-s)$ the cooperative differential game between the representative household and the strongly self-interested politician, and consequently a set of Markov feedback strategies: $\Big[c^{**(s)}(t,k(t));\tau_K^{**(s)}(t,k(t)),\tau_W^{**(s)}(t,k(t)),\tau_C^{**(s)}(t,k(t))\Big]$ provides a cooperative equilibrium solution to the cooperative game $\Gamma^C(k_0,\tau^{**}-s)$, if there exist continuously differentiable functions $W^{(s)}(t,k):[s,\tau^{**}]\times\mathbb{R}_+\to\mathbb{R}$, $i\in\{H,P\}$, satisfying the following Fleming-Bellman-Isaacs partial differential equation,

$$
\begin{aligned}
-W_t^{(s)}(t,k)-\frac{1}{2}\sigma^2k^2W_{kk}^{(s)}(t,k)
&=\max_{c,\tau_K,\tau_W,\tau_C}\Big(e^{-\rho(t-s)}[u^H(c) \\
&+u^P(\tau_K(F_K(K,L)-\delta)k,\tau_W F_L(K,L),\tau_C c)] \\
&+W_k^{(s)}(t,k)\{[(1-\tau_K)(F_K(K,L)-\delta)+\sigma^2-n]k \\
&+(1-\tau_W)F_L(K,L)-(1+\tau_C)c\}\Big)
\end{aligned}
$$

with the following boundary condition,

$$
W^{(s)}\big(\tau^{**},k(\tau^{**})\big)=U^{Cooperation,\tau^{**}}(\zeta_0) \tag{16}
$$

It will be shown below that the boundary condition in (16) is of great importance in identifying the cooperative equilibrium minimum-time needed to economic maturity. As you may notice, Theorem 8 is established relying on Assumption 2, i.e., payoffs\ utilities are transferable across players and over time. Nonetheless, technically, Theorem 8 can also be extended to study the case of nontransferable utilities\payoffs (see, Yeung and Petrosyan, 2006) across players and over time. For example, here we may consider the weighted social welfare function (see, Harsanyi, 1955, and among others) regarding the representative household and the self-interested politician. And one may further interpret such kind of specification from the following viewpoints: first, the choice of the social welfare function will to some extent reflect the social institution or social structure of the underlying economy (see, for example, Akerlof (1997) and references therein), for instance, the representative household and the self-interested politician share asymmetric social status, and therefore asymmetric bargaining power in the game of resource allocation; second, here we specifically employ the methodology that utility is comparable among the economic individuals (e.g., Harsanyi, 1955; Sen, 1970; Kalai, 1977, and among others). To sum up, Theorem 8 has provided us with a useful starting point in this direction for future exploration.

In order to make sure that $\tau_K^{**(s)}(t,k(t))$, $\tau_W^{**(s)}(t,k(t))$, $\tau_C^{**(s)}(t,k(t))$ and $c^{**(s)}(t,k(t))$ derived in Theorem 8 indeed provides us with a cooperative equilibrium solution, we need to introduce the following definitions and theorems:

Definition 7 (Group Rationality). If it is confirmed in the present cooperative model that $W^{(s)}(t,k(t)) > \sum_{i\in\{H,P\}} V^{(s)i}(t,k(t))$ along the trajectory $\{k(t)\}_{t=s}^{\tau^{**}}$ that is given by (15), then we claim that the optimal solution:

$\left[c^{**(s)}(t,k(t));\tau_K^{**(s)}(t,k(t)),\tau_W^{**(s)}(t,k(t)),\tau_C^{**(s)}(t,k(t))\right]$ satisfies group rationality.

Chang and Malliaris (1987), by using the Reflection Principle, demonstrated the existence and uniqueness of the solution to the classic Solow equation under continuous time uncertainty for the class of strictly concave production functions which are continuously differentiable on the non-negative real numbers. This class contains all CES functions with elasticity of substitution less than unity. Hence, we directly give,

Assumption 3. Suppose that the solution to the SDE given in (15) exists and it can be expressed as follows,

$$\begin{aligned}
&k^{**}(t)\\
&= k_0\\
&+\int_s^t \Big\{\Big[\Big(1-\tau_K^{**(s)}(\lambda,k^{**}(\lambda))\Big)\big(F_K(K(\lambda),L(\lambda))-\delta\big)+\sigma^2-n\Big]k^{**}(\lambda)\\
&+\Big(1-\tau_W^{**(s)}(\lambda,k^{**}(\lambda))\Big)F_L(K(\lambda),L(\lambda))\\
&\quad\Big(1+\tau_C^{**(s)}(\lambda,k^{**}(\lambda))\Big)c^{**(s)}(\lambda,k^{**}(\lambda))\Big\}d\lambda\\
&-\int_s^t \sigma k^{**}(\lambda)dB(\lambda)
\end{aligned}\tag{17}$$

We let Ξ_t^{**} denote the set of reliable values of $k^{**}(t)$ at time t generated by (17). The term k_t^{**} is also used to represent an element in the set Ξ_t^{**}. Let $\eta(t_0) := [\eta_H(t_0),\eta_P(t_0)]$ denote the instantaneous payoff of the cooperative game $\Gamma^C(s,k_s^{**})$ at time $t_0\in[s,\tau^{**}]$ with $k_s^{**}\in\Xi_s^{**}$. In particular, along the cooperative trajectory $\{k^{**}(t)\}_{t=s}^{\tau^{**}}$ we put,

$$\xi^{(s)i}(t_0,k_{t_0}^{**}) := \mathbb{E}_{t_0}\left[\int_{t_0}^{\tau^{**}} e^{-\rho(\lambda-t_0)}\eta_i(\lambda)\,d\lambda\,\middle|\,k(t_0)=k_{t_0}^{**}\right]$$

$$\xi^{(s)i}(t,k_t^{**}) := \mathbb{E}_t\left[\int_t^{\tau^{**}} e^{-\rho(\lambda-t)}\eta_i(\lambda)\,d\lambda\,\middle|\,k(t)=k_t^{**}\right]$$

for $i\in\{H,P\}$, $k_{t_0}^{**}\in\Xi_{t_0}^{**}$, $k_t^{**}\in\Xi_t^{**}$ and $t\geq t_0\geq s$.

Thus, based upon an agreed-upon optimality principle such as Nash bargaining solution or Shapley value introduced below, the vectors $\xi^{(s)}(t_0, k_{t_0}^{**}) := [\xi^{(s)H}(t_0, k_{t_0}^{**}), \xi^{(s)P}(t_0, k_{t_0}^{**})]$ for $t_0 \geq s$, are valid imputations if the following conditions are satisfied:

Definition 8 (Valid Imputation). The vector $\xi^{(s)}(t_0, k_{t_0}^{**})$ is a valid imputation of the differential cooperative game $\Gamma^C(t_0, k_{t_0}^{**})$, for $t_0 \in [s, \tau^{**}]$ and $k_{t_0}^{**} \in \Xi_{t_0}^{**}$, if it satisfies.

- The vector $\xi^{(s)}(t_0, k_{t_0}^{**}) := [\xi^{(s)H}(t_0, k_{t_0}^{**}), \xi^{(s)P}(t_0, k_{t_0}^{**})]$ is a Pareto optimal imputation vector;
- Individual rationality requirement, that is to say, we obtain the result $\xi^{(s)i}(t_0, k_{t_0}^{**}) \geq V^{(s)i}(t_0, k_{t_0}^{**})$, for $i \in \{H, P\}$.

Moreover, we define,

$$\gamma^{(s)i}(t_0; t_0, k_{t_0}^{**}) := \mathbb{E}_{t_0}\left[\int_{t_0}^{\tau^{**}} e^{-\rho(\lambda - t_0)}\eta_i(\lambda)\,d\lambda \,\middle|\, k(t_0) = k_{t_0}^{**}\right] = \xi^{(s)i}(t_0, k_{t_0}^{**})$$

and,

$$\gamma^{(s)i}(t_0; t, k_t^{**}) := \mathbb{E}_t\left[\int_t^{\tau^{**}} e^{-\rho(\lambda - t_0)}\eta_i(\lambda)\,d\lambda \,\middle|\, k(t) = k_t^{**}\right]$$

for $i \in \{H, P\}$ and $t \geq t_0 \geq s$. Noting that,

$$\gamma^{(s)i}(t_0; t, k_t^{**}) := e^{-\rho(t - t_0)}\mathbb{E}_t\left[\int_t^{\tau^{**}} e^{-\rho(\lambda - t)}\eta_i(\lambda)\,d\lambda \,\middle|\, k(t) = k_t^{**}\right]$$
$$= e^{-\rho(t - t_0)}\xi^{(s)i}(t, k_t^{**}) = e^{-\rho(t - t_0)}\gamma^{(s)i}(t; t, k_t^{**}) \quad (18)$$

for $i \in \{H, P\}$ and $k_t^{**} \in \Xi_t^{**}$, we now give:

Definition 9 (Sub-Game Consistency). The condition in (18) guarantees sub-game consistency of the solution imputation throughout the game interval in the sense that the extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behaviors would remain optimal.

Indeed, Definition 9 is directly brought from Yeung and Petrosyan (2006). Furthermore, we can get the PDP as follows,

Theorem 9 (Sub-Game Consistent Solution). An instantaneous payment at time $t_0 \in [s, \tau^{**}]$ equaling,

$$\eta_i(t_0) = -\xi_{t_0}^{(s)i}(t_0, k_{t_0}^{**}) - \frac{1}{2}\sigma^2(k_{t_0}^{**})^2\xi_{k_{t_0}^{**}k_{t_0}^{**}}^{(s)i}(t_0, k_{t_0}^{**})$$
$$- \xi_{k_{t_0}^{**}}^{(s)i}(t_0, k_{t_0}^{**})\Big\{\Big[\Big(1 - \tau_K^{**(s)}(t_0, k_{t_0}^{**})\Big)\Big(F_K\big(K(t_0), L(t_0)\big)$$
$$- \delta\Big) + \sigma^2 - n\Big]k_{t_0}^{**} + \Big(1 - \tau_W^{**(s)}(t_0, k_{t_0}^{**})\Big)F_L\big(K(t_0), L(t_0)\big)$$
$$- \Big(1 + \tau_C^{**(s)}(t_0, k_{t_0}^{**})\Big)c^{**(s)}(t_0, k_{t_0}^{**})\Big\}$$

for $i \in \{H, P\}$ and $k_{t_0}^{**} \in \Xi_{t_0}^{**}$, and this yields a sub-game consistent solution or the PDP of the cooperative game $\Gamma^C(t_0, k_{t_0}^{**})$.

Proof. It is quite similar to the proof of Theorem 5.8.3 of Yeung and Petrosyan (2006), so we take it omitted.

As noted above, one may consider sub-game consistent solutions under specific optimality principles. For example, one may use:

Definition 10 (Nash Bargaining Solution\Shapley Value). In the cooperative game $\Gamma^C(s, k_0)$, at time s an imputation:

$$\xi^{(s)i}(s, k_0) = V^{(s)i}(s, k_0) + \frac{1}{2}\left[W^{(s)}(s, k_0) - \sum_{j \in \{H,P\}} V^{(s)j}(s, k_0)\right]$$

is assigned to player i, for $i \in \{H, P\}$; and at time $t_0 \in [s, \tau^{**}]$, an imputation,

$$\xi^{(s)i}(t_0, k_{t_0}^{**}) = V^{(s)i}(t_0, k_{t_0}^{**}) + \frac{1}{2}\left[W^{(s)}(t_0, k_{t_0}^{**}) - \sum_{j \in \{H,P\}} V^{(s)j}(t_0, k_{t_0}^{**})\right]$$

is assigned to player i, for $i \in \{H, P\}$ and $k_{t_0}^{**} \in \Xi_{t_0}^{**}$.

Here, it is especially worth emphasizing that Nash bargaining solution and Shapley value coincide with each other in the present two-player game (see, Yeung and Petrosyan, 2006) while they generally showing us different cooperative mechanisms when there are over two players in the game.

Theorem 10 (Sub-Game Consistency of the Nash Bargaining Solution\Shapley Value). It is confirmed that the Nash bargaining solution\Shapley value $\xi^{(s)i}(t_0, k_{t_0}^{**})$ given in Definition 10 is a sub-game consistent imputation for the present cooperative game $\Gamma^C(s, k_0)$ for $\tau^{**} \geq t_0 \geq s$.

Proof. Noting that the equilibrium feedback strategies or the stochastic controls in Theorems 6 and 8 are Markovian in the sense that they depend on current state and current time, one can readily observe by comparing the corresponding stochastic Bellman equations in Theorems 6 and 8 for different values of $t_0 \in [s, \hat{\tau}]$ and $t_0 \in [s, \tau^{**}]$, respectively, that:

$$\begin{pmatrix} \hat{c}^{(t_0)}(t, \hat{k}_t) \\ \hat{\tau}_K^{(t_0)}(t, \hat{k}_t) \\ \hat{\tau}_W^{(t_0)}(t, \hat{k}_t) \\ \hat{\tau}_C^{(t_0)}(t, \hat{k}_t) \end{pmatrix} = \begin{pmatrix} \hat{c}^{(s)}(t, \hat{k}_t) \\ \hat{\tau}_K^{(s)}(t, \hat{k}_t) \\ \hat{\tau}_W^{(s)}(t, \hat{k}_t) \\ \hat{\tau}_C^{(s)}(t, \hat{k}_t) \end{pmatrix}$$

for $\hat{\tau} \geq t \geq t_0 \geq s$ and $\hat{k}_t = \hat{k}(t)$ the corresponding optimal trajectory of capital-labor ratio determined by (14) at time t, and also:

$$\begin{pmatrix} c^{**(t_0)}(t, k_t^{**}) \\ \tau_K^{**(t_0)}(t, k_t^{**}) \\ \tau_W^{**(t_0)}(t, k_t^{**}) \\ \tau_C^{**(t_0)}(t, k_t^{**}) \end{pmatrix} = \begin{pmatrix} c^{**(s)}(t, k_t^{**}) \\ \tau_K^{**(s)}(t, k_t^{**}) \\ \tau_W^{**(s)}(t, k_t^{**}) \\ \tau_C^{**(s)}(t, k_t^{**}) \end{pmatrix}$$

for $\tau^{**} \geq t \geq t_0 \geq s$ and $k_t^{**} = k^{**}(t)$ the corresponding optimal trajectory of capital-labor ratio determined by (17). Moreover, along the optimal trajectory $\{\hat{k}_t\}_{t=s}^{\hat{\tau}}$, one can obtain:

$$V^{(s)H}\left(t_0, \hat{k}_{t_0}\right) := \mathbb{E}_s\left[\int_{t_0}^{\hat{\tau}} e^{-\rho(\lambda-s)} u^H\left(\hat{c}^{(s)}\left(\lambda, \hat{k}_\lambda\right)\right) d\lambda\right.$$

$$\left. + e^{-\rho(\hat{\tau}-s)} u^H\left(\hat{c}^{(s)}\left(\hat{\tau}, \hat{k}_{\hat{\tau}}\right)\right) \right| \hat{k}(t_0) = \hat{k}_{t_0}\Big]$$

$$= \mathbb{E}_s\left[\int_{t_0}^{\hat{\tau}} e^{-\rho(\lambda-t_0)} u^H\left(\hat{c}^{(t_0)}\left(\lambda, \hat{k}_\lambda\right)\right) d\lambda\right.$$

$$\left. + e^{-\rho(\hat{\tau}-t_0)} u^H\left(\hat{c}^{(t_0)}\left(\hat{\tau}, \hat{k}_{\hat{\tau}}\right)\right) \right| \hat{k}(t_0) = \hat{k}_{t_0}\Big] e^{-\rho(t_0-s)}$$

$$= \mathbb{E}_{t_0}\left[\int_{t_0}^{\hat{\tau}} e^{-\rho(\lambda-t_0)} u^H\left(\hat{c}^{(t_0)}\left(\lambda, \hat{k}_\lambda\right)\right) d\lambda\right.$$

$$\left. + e^{-\rho(\hat{\tau}-t_0)} u^H\left(\hat{c}^{(t_0)}\left(\hat{\tau}, \hat{k}_{\hat{\tau}}\right)\right) \right| \hat{k}(t_0) = \hat{k}_{t_0}\Big] e^{-\rho(t_0-s)}$$

$$:= e^{-\rho(t_0-s)}\, V^{(t_0)H}\left(t_0, \hat{k}_{t_0}\right)$$

where: $V^{(s)H}\left(t_0, \hat{k}_{t_0}\right)$ measures the expected present value of the representative household's payoff in the time interval $[t_0, \hat{\tau}]$ when $\hat{k}(t_0) = \hat{k}_{t_0}$ and when the game starts from time $s \le t_0$. For the self-interested politician,

$$V^{(s)P}\left(t_0, \hat{k}_{t_0}\right)$$

$$:= \mathbb{E}_s\left[\int_{t_0}^{\hat{\tau}} e^{-\rho(\lambda-s)} u^P\left(\hat{\tau}_K^{(s)}\left(\lambda, \hat{k}_\lambda\right)\left(F_K\left(K(\lambda), L(\lambda)\right)\right.\right.\right.$$

$$\left.\left.\left. - \delta\right)\hat{k}_\lambda, \hat{\tau}_W^{(s)}\left(\lambda, \hat{k}_\lambda\right) F_L\left(K(\lambda), L(\lambda)\right), \hat{\tau}_C^{(s)}\left(\lambda, \hat{k}_\lambda\right) \hat{c}^{(s)}\left(\lambda, \hat{k}_\lambda\right)\right) d\lambda \right| \hat{k}(t_0) = \hat{k}_{t_0}\Big]$$

$$= \mathbb{E}_s\left[\int_{t_0}^{\hat{\tau}} e^{-\rho(\lambda-t_0)} u^P\left(\hat{\tau}_K^{(t_0)}\left(\lambda, \hat{k}_\lambda\right)\left(F_K\left(K(\lambda), L(\lambda)\right)\right.\right.\right.$$

$$\left.\left.\left. - \delta\right)\hat{k}_\lambda, \hat{\tau}_W^{(t_0)}\left(\lambda, \hat{k}_\lambda\right) F_L\left(K(\lambda), L(\lambda)\right), \hat{\tau}_C^{(t_0)}\left(\lambda, \hat{k}_\lambda\right) \hat{c}^{(t_0)}\left(\lambda, \hat{k}_\lambda\right)\right) d\lambda \right| \hat{k}(t_0)$$

$$= \hat{k}_{t_0}\Big] e^{-\rho(t_0-s)}$$

$$= \mathbb{E}_{t_0}\left[\int_{t_0}^{\hat{\tau}} e^{-\rho(\lambda-t_0)} u^P\left(\hat{\tau}_K^{(t_0)}\left(\lambda, \hat{k}_\lambda\right)\left(F_K\left(K(\lambda), L(\lambda)\right)\right.\right.\right.$$

$$\left.\left.\left. - \delta\right)\hat{k}_\lambda, \hat{\tau}_W^{(t_0)}\left(\lambda, \hat{k}_\lambda\right) F_L\left(K(\lambda), L(\lambda)\right), \hat{\tau}_C^{(t_0)}\left(\lambda, \hat{k}_\lambda\right) \hat{c}^{(t_0)}\left(\lambda, \hat{k}_\lambda\right)\right) d\lambda \right| \hat{k}(t_0)$$

$$= \hat{k}_{t_0}\Big] e^{-\rho(t_0-s)} := e^{-\rho(t_0-s)}\, V^{(t_0)P}\left(t_0, \hat{k}_{t_0}\right)$$

where $V^{(s)P}(t_0, \hat{k}_{t_0})$ measures the expected present value of the strongly self-interested politician's payoff in the time interval $[t_0, \hat{\tau})$ when $\hat{k}(t_0) = \hat{k}_{t_0}$ and when the game starts from time $s \le t_0$. Similarly, for the cooperative game, we obtain,

$$W^{(s)}(t_0, k_{t_0}^{**}) = e^{-\rho(t_0 - s)} W^{(t_0)}(t_0, k_{t_0}^{**})$$

where $W^{(s)}(t_0, k_{t_0}^{**})$ measures the expected present value of the cooperative payoff in the time interval $[t_0, \tau^{**})$ when $k^{**}(t_0) = k_{t_0}^{**}$ and when the game starts from time $s \le t_0$.

Now, we can obtain the Nash bargaining solution\Shapley value along the cooperative optimal trajectory $\{k_{t_0}^{**}\}_{t_0=s}^{\tau^{**}}$ as follows:

$$\xi^{(s)i}(t_0, k_{t_0}^{**}) = V^{(s)i}(t_0, k_{t_0}^{**}) + \frac{1}{2}\left[W^{(s)}(t_0, k_{t_0}^{**}) - \sum_{j\in\{H,P\}} V^{(s)j}(t_0, k_{t_0}^{**})\right] =$$

$$e^{-\rho(t_0-s)}\left\{V^{(t_0)i}(t_0, k_{t_0}^{**}) + \frac{1}{2}\left[W^{(t_0)}(t_0, k_{t_0}^{**}) - \sum_{j\in\{H,P\}} V^{(t_0)j}(t_0, k_{t_0}^{**})\right]\right\} =$$

$$e^{-\rho(t_0-s)}\xi^{(t_0)i}(t_0, k_{t_0}^{**})$$

for $i \in \{H, P\}$, $s \le t_0 \le \tau^{**}$and $k_{t_0}^{**} \in \Xi_{t_0}^{**}$. And this proof is complete.

As noted by Yeung and Petrosyan (2006) that though one of the most commonly used allocation principles is the Shapley value, however, equal imputation of cooperative gains may not be totally agreeable to the players when players are asymmetric in their sizes of noncooperative payoffs. For example, in the current context, the noncooperative payoffs of the representative household and the self-interested politician may be asymmetric in reality owing to unequal social status. So, to overcome this, we also consider the following allocation principle in which the players' shares of the gain from cooperation are proportional to the relative sizes of their expected noncooperative payoffs. To be exact, the corresponding imputation scheme satisfies:

Definition 11 (Proportional Distribution). In the present cooperative game denoted $\Gamma^C(k_0, \tau^{**} - s)$, an imputation,

$$\xi^{(s)i}(s, k_0) = \frac{V^{(s)i}(s, k_0)}{\sum_{j\in\{H,P\}} V^{(s)j}(s, k_0)} W^{(s)}(s, k_0)$$

should be assigned to player i, for $i \in \{H, P\}$; and in the sub-game denoted by $\Gamma^C(k_{t_0}^{**}, \tau^{**} - t_0)$ for $t_0 \in [s, \tau^{**}]$, an imputation,

$$\xi^{(s)i}(t_0, k_{t_0}^{**}) = \frac{V^{(s)i}(t_0, k_{t_0}^{**})}{\sum_{j\in\{H,P\}} V^{(s)j}(t_0, k_{t_0}^{**})} W^{(s)}(t_0, k_{t_0}^{**})$$

is assigned to player i, for $i \in \{H, P\}$ and $k_{t_0}^{**} \in \Xi_{t_0}^{**}$.

Theorem 11 (Sub-Game Consistency of the Proportional Distribution). The proportional-distribution imputation $\xi^{(s)i}(\tau_0, k^{**}_{\tau_0})$ *given in Definition 11 provides us with a sub-game consistent imputation for the cooperative game* $\Gamma^C(k_0, \tau^{**} - s)$ *for* $i \in \{H, P\}$, $s \leq t_0 \leq \tau^{**}$ *and* $k^{**}_{t_0} \in \Xi^{**}_{t_0}$.

Proof. The proof is quite similar to that of Theorem 10, so we take it omitted.

Up to the present step, we have discussed the relevant issues, i.e., group rationality, individual rationality, Pareto-optimal principle, and sub-game consistency of the above cooperative stochastic differential game between the representative household and the strongly self-interested politician. Now, we are in the position to derive the cooperative-equilibrium minimum-time needed to economic maturity.

By solving Problem 6 and also employing similar arguments as in Theorem 5, we get,

Theorem 12 (Cooperative-Equilibrium Minimum-Time Needed to Economic Maturity: Existence).

a) *Suppose that we can find a function* $\phi^C : \overline{D} \to \mathbb{R}$ *such that:*

$$\phi^C \in C^1(D) \cap C(\overline{D})$$
$$\phi^C \geq e^{\rho s} \tilde{u}^{Cooperation} \text{ on } D$$

and

$$\lim_{t \to \tau_D} \phi^C(\zeta(t)) = e^{\rho s} \tilde{u}^{Cooperation}(\zeta(\tau_D)) \chi_{\{\tau_D < \infty\}} \text{ a.s. } \mathbb{P}^{\zeta_0}$$

Define $G := \{\zeta_0 \in D;\ \phi^C(\zeta_0) > e^{\rho s} \tilde{u}^{Cooperation}(\zeta_0)\}$ *and suppose* $\zeta(t)$ *spends 0 time on* ∂G *a.s.* $\mathbb{P}^{\zeta_0}$*, i.e.,*

$\mathbb{E}_{\zeta_0}\left[\int_s^{\tau_D} \chi_{\partial G}(\zeta(t)) dt\right] = 0$ *for all* $\zeta_0 \in D$*, and suppose that,* ∂G *is a Lipschitz surface.*

Moreover, suppose the following conditions:

$\phi^C \in C^2(D \setminus \partial G)$ *and the second order derivatives of* ϕ^C *are locally bounded near* ∂G

$$\mathcal{A}^{c^{**}, (\tau_K^{**}, \tau_W^{**}, \tau_C^{**})} \phi^C \leq 0 \text{ on } D \setminus \partial G$$

Then, $\phi^C(\zeta_0) \geq U^{Cooperation, \tau^{**}}(\zeta_0)$ *for all* $\zeta_0 \in D$.

b) *Suppose, in addition to the above conditions, that:*

$$\mathcal{A}^{c^{**}, (\tau_K^{**}, \tau_W^{**}, \tau_C^{**})} \phi^C = 0 \text{ on } G;$$

$\tau_G := \inf\{t > s;\ \zeta(t) \notin G\} < \infty$ *a.s.* $\mathbb{P}^{\zeta_0}$ *for* $\zeta_0 \in D$, *and the family* $\{\phi^{C-}(\zeta(\tau));\ \tau \leq \tau_G, \tau \in \mathcal{T}\}$ *is uniformly integrable w.r.t.* $\mathbb{P}^{\zeta_0}$ *for all* $\zeta_0 \in D$

Then, $\phi^C(\zeta_0) = U^{Cooperation, \tau^{**}}(\zeta_0) = \sup_{\tau \in \mathcal{T}} \mathbb{E}_s\left[e^{\rho s} \tilde{u}^{Cooperation}(\zeta(\tau))\right]$ *for all* $\zeta_0 \in D$

and,

$\tau^{**} = \tau_G$ *is an optimal stopping time for this problem.*

In addition to that, we have:

Corollary 3. In principle, the cooperative-equilibrium minimum-time needed to economic maturity τ^{**} *can be further computed by the following equality,*

$$W^{(s)}(\tau^{**}, k(\tau^{**})) = \phi^C(\zeta_0)$$

Proof. Combining (16) with Theorem 12 produces the desired result.

Case 2. Suppose that there is a semi-strongly self-interested politician in the economy.

Case 3. Suppose that there is a weakly self-interested politician in the economy.

Noting that the discussions corresponding to Cases 2 and 3 are quite similar to that of Case 1, we hence take them omitted and leave them to the interested readers. Furthermore, it would be interesting to comparatively study the cooperative equilibrium minimum-time needed to economic maturity corresponding to different cases, i.e., different political institutional arrangements. That is, the framework presented here makes it possible to evaluate the economic efficiency of political institutions from the perspective of economic development. As emphasized by North (1994) that economic and political institutions are the underlying determinants of economic performance and also argued by Acemoglu *et al.* (2005b) that institutions are the fundamental cause of economic growth, the paper has built up a baseline framework for us to explore the role institutions play in economic maturity, especially from the viewpoint of time dimension. And hence our results would be seen as a supplement to those of North (1994) and Acemoglu *et al.* (2005b).

Now, provided the sequential-equilibrium minimum-time needed to economic maturity τ^*, Markov-equilibrium minimum-time needed to economic maturity $\hat{\tau}$, and the cooperative-equilibrium minimum-time needed to economic maturity τ^{**}, given in Theorems 5, 7 and 12, respectively, we can then investigate the following issue: which approach will lead us to much faster economic maturity? Lancaster (1973) and Kaitala and Pohjola (1990) argued that cooperation between the government and the firm will be more beneficial compared to the dynamic inefficiency of capitalism. Moreover, Leong and Huang (2010) demonstrates that cooperation is always Pareto optimal compared to the non-cooperative Markovian Nash equilibrium although the cooperative solution is indeterminate. Apart from these papers, the present model defines the concept of dynamic inefficiency of capitalism in the sense of the minimum-time needed to economic maturity. In other words, if the cooperation between the self-interested politician and the representative household will lead to much faster speed of economic maturity, then there exists dynamic inefficiency of capitalism in the underlying economy. Furthermore, if we interpret different game structures as different institutional arrangements (e.g., North, 1990; Hurwicz, 1996; Williamson, 2000; Amable, 2003), then we provide a basic framework to analyze different speeds of economic maturity corresponding to different institutional arrangements. This indeed shows new approach and also new perspective for those studies focused on underdeveloped economies.

Finally, it is particularly worth emphasizing that the equilibrium minimum-times needed to economic maturity derived in the above theorems strictly depend on the initial value of the underlying economic system. This has to some extent reflected the well-known path-dependence effect analyzed and emphasized by North (1990). In other words, we argue that, besides in the process of institutional changes, path-dependence effect also plays a crucial role in economic development for those underdeveloped economies. What is more, as you can see in the following section, one can even proceed to the comparative static analysis of the equilibrium minimum-time needed to economic maturity with respect to the initial capital stock of the abstract economy. This of course will show us very rich and also interesting economic intuition and economic implication of the mathematical model. And it, therefore, would be regarded as an advantage of the framework established in the paper.

5. Examples: closed-form solutions

In this section, we will take the following case for example,

Case 1. Suppose that there is a strongly self-interested politician in the economy.

In order to make things easier and also derive closed-form solutions, we adopt the following production technology instead of that in (5),

$Y(t) = AK(t)$ with $A > 0$, an exogenously given constant. Also, we shall consider a simple form of (6), i.e.

$$\dot{K}(t) = \big(1-\tau_K(t)\big)(A-\delta)K(t) - \big(1-\theta(t)\big)AK(t)$$

where θ stands for savings rate of the representative household. So, (8) becomes,

$$dk(t) = \Big[\big(1-\tau_K(t)\big)(A-\delta) + \sigma^2 - n - \big(1-\theta(t)\big)A\Big]k(t)dt \\ - \sigma k(t)dB(t) \tag{19}$$

5.1 Risk-aversion politician

It is assumed that there is a risk-aversion politician in the economy. And both the self-interested politician and the representative household exhibit log preference. In particular, here we without loss of generality put $t \geq 0$ instead of $t \geq s$ which is used in the previous sections. So, the optimization problem facing the representative household reads as follows:

$$\max_{0<\theta(t)<1} \mathbb{E}\left[\int_0^{\tau^*} e^{-\rho(s+t)} \ln\Big(\big(1-\theta(t)\big)Ak(t)\Big)\,dt \\ + e^{-\rho(s+\tau^*)} \ln\Big(\big(1-\theta(\tau^*)\big)Ak(\tau^*)\Big)\chi_{\{\tau^*<\infty\}}\right] \tag{20}$$

subject to (19). Applying Computation Algorithm I shows that,

Proposition 1. Provided the optimal stopping time τ^* and the taxation policy of the strongly self-interested politician, we can get the optimal savings rate represented by $\theta^*(t) \equiv \theta^* = 1 - \frac{\rho}{A}$ by solving the problem in (20).

Proof. The proof is quite easy and hence we take it omitted.

The optimization problem facing the self-interested politician is expressed as follows:

$$\max_{0<\tau_K(t)<1} \mathbb{E}\left[\int_0^{\tau^*} e^{-\rho(s+t)} \ln\big(\tau_K(t)(A-\delta)k(t)\big)\,dt \\ + U^{P,\tau^*}\chi_{\{\tau^*<\infty\}}\right] \tag{21}$$

subject to (19) and Proposition 1.

Proposition 2. Conditional on Computation Algorithm I and Proposition 1, we get by solving the problem in (21) the sub-game perfect Nash equilibrium capital-income tax rate as $\tau_K^*(t) \equiv \tau_K^* = \frac{\rho}{A-\delta}$ and also the following boundary condition,

$$J^P\big(\tau^*, k(\tau^*)\big) := e^{-\rho(s+\tau^*)}\left\{\left[\frac{1}{\rho}\ln\rho + \frac{1}{\rho^2}(A-\delta-n+\sigma^2-\rho) - \frac{1}{\rho} - \frac{\sigma^2}{2\rho^2}\right] + \\ \frac{1}{\rho}\ln k(\tau^*)\right\} = U^{P,\tau^*}\chi_{\{\tau^*<\infty\}}$$

Proof. Based on Proposition 1, we have the following Bellman-Isaacs-Fleming equation,

$$-J_t^P(t,k(t)) - \frac{1}{2}\sigma^2 k^2(t) J_{kk}^P(t,k(t))$$
$$= \max_{0<\tau_K(t)<1} \{e^{-\rho(s+t)} \ln(\tau_K(t)(A-\delta)k(t))$$
$$+ J_k^P(t,k(t))k(t)[(1-\tau_K(t))(A-\delta) + \sigma^2 - n$$
$$-\rho]\} \tag{22}$$

Performing the maximization gives,

$$J_k^P(t,k(t))k(t)(A-\delta) = e^{-\rho(s+t)} \frac{1}{\tau_K(t)} \tag{23}$$

If we put,

$$J^P(t,k(t)) = e^{-\rho(s+t)}[C_1 + C_2 \ln(k(t))] \tag{24}$$

which combines with (23) implies that,

$$1 = (A-\delta)\tau_K(t)C_2 \tag{25}$$

Inserting (24) and (25) into (22) yields,

$$\rho[C_1 + C_2 \ln(k(t))] + \frac{1}{2}\sigma^2 C_2$$
$$= \ln(k(t)) - \ln C_2 + C_2(A - \delta - n + \sigma^2 - \rho) - 1$$

which shows that $C_2 = \frac{1}{\rho}$ and,

$$C_1 = \frac{1}{\rho}\ln\rho + \frac{1}{\rho^2}(A - \delta - n + \sigma^2 - \rho) - \frac{1}{\rho} - \frac{\sigma^2}{2\rho^2}$$

which gives the desired result and hence the proof is complete.

Now, applying Propositions 1 and 2 reveals that (19) can be rewritten as follows:

$$dk(t) = (A - \delta - n + \sigma^2 - 2\rho)k(t)dt - \sigma k(t)dB(t) \tag{19`}$$

And so the corresponding optimal stopping problem can be written as follows,

$$U^{P,\tau^*}\chi_{\{\tau^*<\infty\}} := \sup_{\tau\in\mathcal{T}} \mathbb{E}_{(s,k)}[e^{-\rho(s+\tau)}\ln(\rho k(\tau))\chi_{\{\tau<\infty\}}]$$

subject to (19'). The generator in (9) can be written as,

$$\mathcal{A}\phi(s,k) = \frac{\partial\phi}{\partial s} + (A - \delta - n + \sigma^2 - 2\rho)k\frac{\partial\phi}{\partial k} + \frac{1}{2}\sigma^2 k^2 \frac{\partial^2\phi}{\partial k^2}$$

If we try a function ϕ of the form,

$\phi(s,k) = e^{-\rho s}k^{\mu}$ for some constant $\mu \in \mathbb{R}$. We thus get,

$$\mathcal{A}\phi(s,k) = e^{-\rho s}k^{\mu}\left[-\rho + (A - \delta - n + \sigma^2 - 2\rho)\mu + \frac{1}{2}\sigma^2\mu(\mu-1)\right]$$
$$:= e^{-\rho s}k^{\mu}h(\mu)$$

Solving equation $h(\mu) = 0$ gives the unique positive root, $\mu =$

$$\frac{-[2(A-\delta-n-2\rho)+\sigma^2]+\sqrt{[2(A-\delta-n-2\rho)+\sigma^2]^2+8\sigma^2\rho}}{2\sigma^2} \tag{26}$$

With this value of μ we put,

$$\phi(s,k) = \begin{cases} e^{-\rho s}Ck^{\mu}, & (s,k) \in G \\ e^{-\rho s}\ln(\rho k), & (s,k) \notin G \end{cases} \tag{27}$$

for some constant C, to be determined. If we let $g(s,k) := e^{-\rho s}\ln(\rho k)$, we have,

$$\mathcal{A}g(s,k) = e^{-\rho s}\left[-\rho\ln(\rho k) + \left(A-\delta-n-2\rho+\frac{1}{2}\sigma^2\right)\right] > 0 \Leftrightarrow k < \frac{1}{\rho}\exp\left[\frac{1}{\rho}\left(A-\delta-n-2\rho+\frac{1}{2}\sigma^2\right)\right]$$

Therefore, we put,

$$\Sigma := \left\{(s,k);\ k < \frac{1}{\rho}\exp\left[\frac{1}{\rho}\left(A-\delta-n-2\rho+\frac{1}{2}\sigma^2\right)\right]\right\}$$

Thus, we guess that the continuation region G has the form,

$$G := \{(s,k);\ 0 < k < k^*\} \tag{28}$$

for some k^* such that $\Sigma \subseteq G$, i.e.,

$$k^* \geq \frac{1}{\rho}\exp\left[\frac{1}{\rho}\left(A-\delta-n-2\rho+\frac{1}{2}\sigma^2\right)\right]$$

Hence, by (28) we can rewrite (27) as follows,

$$\phi(s,k) = \begin{cases} e^{-\rho s}Ck^{\mu}, & 0 < k < k^* \\ e^{-\rho s}\ln(\rho k), & k \geq k^* \end{cases}$$

We without loss of generality guess that the value function ϕ is C^1 at $k = k^*$ and this will naturally lead to the following smooth-fit conditions,

$C(k^*)^{\mu} = \ln(\rho k^*)$ (continuity at $k = k^*$)
$C\mu(k^*)^{\mu-1} = (k^*)^{-1}$ (differentiability at $k = k^*$)

from which we thus derive,

$$k^* = \frac{1}{\rho}\exp\left(\frac{1}{\mu}\right) \ \&\ C = \mu^{-1}\left[\frac{1}{\rho}\exp\left(\frac{1}{\mu}\right)\right]^{-\mu} \tag{29}$$

Proposition 3. Under the above constructions and certain parameter constraints, we obtain the sequential-equilibrium minimum-time needed to economic maturity denoted by: $\tau^* = \tau_G := \inf\{t > 0; k(t) = k^*\}$.

In other words, $g^*(s,k) := e^{-\rho s}\mu^{-1}(k^*)^{-\mu}k^{\mu} = U^{p,\tau^*}$ *is a supermeanvalued majorant of* $g(s,k)$ *with* k^* *and* μ *given by (29) and (26), respectively.*

Proof. See the proof of Theorem 1 of Dai (2012).

Corollary 4. There is a closed-form solution for the sequential-equilibrium minimum-time needed to economic maturity τ^*, and indeed,

$$\tau^* = \frac{1}{\rho}\ln\left(\mu(k^*)^{\mu}k^{-\mu}\left\{\left[\frac{1}{\rho}\ln\rho + \frac{1}{\rho^2}(A-\delta-n+\sigma^2-\rho) - \frac{1}{\rho} - \frac{\sigma^2}{2\rho^2}\right] + \frac{1}{\rho}\ln k^*\right\}\right)$$

where k^* and μ are given by (29) and (26), respectively, and k denotes the initial condition.

Proof. Combining the boundary condition in Proposition 2 with Proposition 3 easily confirms the required assertion.

In what follows, we will derive the closed-form solution corresponding to cooperative economic maturity. Before doing this, we establish,

Proposition 4. There exists a Markov feedback Nash equilibrium solution denoted by:

$\{\hat{\theta}, \hat{\tau}_K\} = \left\{1-\frac{\rho}{A}, \frac{\rho}{A-\delta}\right\}$, *and the corresponding value functions are given by:*

$$V^H(t,k(t)) = V^P(t,k(t)) = e^{-\rho(s+t)}\left\{\left[\frac{1}{\rho}\ln\rho + \frac{1}{\rho^2}(A-\delta-n+\sigma^2-\rho) - \frac{1}{\rho} - \frac{\sigma^2}{2\rho^2}\right] + \frac{1}{\rho}\ln k(t)\right\}$$

Proof. This proof is quite similar to those of Propositions 1 and 2, and hence we omit it and leave it to the interested reader.

Now, if the representative household and the strongly self-interested politician can cooperate with each other, then the corresponding optimization problem amounts to,

$$\max_{\substack{0<\tau_K(t)<1\\0<\theta(t)<1}} \mathbb{E}\left\{\int_0^{\tau^{**}} e^{-\rho(s+t)}\left[\ln\big(\tau_K(t)(A-\delta)k(t)\big) + \ln\Big(\big(1-\theta(t)\big)Ak(t)\Big)\right]dt + U^{Cooperation,\tau^{**}}\chi_{\{\tau^{**}<\infty\}}\right\} \quad (30)$$

subject to (19). By solving the problem in (30), one can establish:

Proposition 5. Provided the above constructions, there is a cooperative solution denoted by $\{\theta^{**}, \tau_K^{**}\} = \left\{1-\frac{\rho}{2A}, \frac{\rho}{2(A-\delta)}\right\}$, *and the corresponding value function is,*

$$W(t,k(t)) = e^{-\rho(s+t)}\left\{\left[-\frac{2}{\rho}\ln\frac{2}{\rho} + \frac{2}{\rho^2}(A-\delta-n+\sigma^2) - \frac{2}{\rho} - \frac{\sigma^2}{\rho^2}\right] + \frac{2}{\rho}\ln k(t)\right\}$$

Proof. This proof is quite similar to the above propositions, so here we take it omitted.

Proposition 6. Provided the cooperative solution in Proposition 5, it is shown that group rationality, individual rationality and sub-game consistency are all fulfilled when we employ Nash bargaining solution\Shapley value as the imputation scheme.

Proof. Based upon Theorem 10, Propositions 4 and 5, the required assertions are easily demonstrated, and we hence leave the details to the interested reader.

Now, we are in the position to consider the following optimal stopping problem,

$$U^{Cooperation,\tau^{**}}\chi_{\{\tau^{**}<\infty\}} := \sup_{\tau\in\mathcal{T}} \mathbb{E}_{(s,k)}\left[e^{-\rho(s+\tau)}\ln\left(\frac{\rho}{2}k(\tau)\right)^2 \chi_{\{\tau<\infty\}}\right]$$

subject to: $dk(t) = (A-\delta-n+\sigma^2-\rho)k(t)dt - \sigma k(t)dB(t)$

So, the generator in (9) can be rewritten as follows,

$$\mathcal{A}\phi(s,k) = \frac{\partial\phi}{\partial s} + (A-\delta-n+\sigma^2-\rho)k\frac{\partial\phi}{\partial k} + \frac{1}{2}\sigma^2k^2\frac{\partial^2\phi}{\partial k^2}$$

If we try, $\phi(s,k) = e^{-\rho s}k^{\epsilon}$ for some constant $\epsilon\in\mathbb{R}$. We thus get,

$$\mathcal{A}\phi(s,k) = e^{-\rho s}k^{\epsilon}\left[-\rho + (A-\delta-n+\sigma^2-\rho)\epsilon + \frac{1}{2}\sigma^2\epsilon(\epsilon-1)\right]$$
$$:= e^{-\rho s}k^{\epsilon}h(\epsilon)$$

Solving equation $h(\epsilon) = 0$ produces,

$$\epsilon = \frac{-[2(A-\delta-n-\rho)+\sigma^2] + \sqrt{[2(A-\delta-n-\rho)+\sigma^2]^2 + 8\sigma^2\rho}}{2\sigma^2} \tag{31}$$

With this value of ϵ we put:

$$\phi(s,k) = \begin{cases} e^{-\rho s}Ck^{\epsilon}, & (s,k)\in G \\ e^{-\rho s}\ln\left(\frac{\rho}{2}k\right)^2, & (s,k)\notin G \end{cases} \tag{32}$$

for some constant C remains to be determined. If we let $g(s,k) := e^{-\rho s}\ln\left(\frac{\rho}{2}k\right)^2$, we obtain:

$$\mathcal{A}g(s,k) = e^{-\rho s}\left[-\rho\ln\left(\frac{\rho}{2}k\right)^2 + 2(A-\delta-n-\rho)+\sigma^2\right] > 0 \Leftrightarrow k < \frac{2}{\rho}\exp\left\{\frac{1}{2\rho}[2(A-\delta-n-\rho)+\sigma^2]\right\}$$

So, we put:

$$\Sigma := \left\{(s,k);\ k < \frac{2}{\rho}\exp\left(\frac{1}{2\rho}[2(A-\delta-n-\rho)+\sigma^2]\right)\right\}$$

Thus, we guess that the continuation region G has the form:

$$G := \{(s,k);\ 0<k<k^{**}\} \tag{33}$$

for some k^{**} such that $\Sigma\subseteq G$, i.e.,

$$k^{**} \geq \frac{2}{\rho}\exp\left\{\frac{1}{2\rho}\left[2(A-\delta-n-\rho)+\sigma^2\right]\right\}$$

Hence, by (33) we can rewrite (32) as follows:

$$\phi(s,k) = \begin{cases} e^{-\rho s}Ck^{\epsilon}, & 0 < k < k^{**} \\ e^{-\rho s}\ln\left(\frac{\rho}{2}k\right)^2, & k \geq k^{**} \end{cases}$$

And hence we have the following smooth-fit conditions:

$C(k^{**})^{\epsilon} = \ln\left(\frac{\rho}{2}k^{**}\right)^2$ (continuity at $k = k^{**}$)

$C\epsilon(k^{**})^{\epsilon-1} = 2(k^{**})^{-1}$ (differentiability at $k = k^{**}$)

from which we thus obtain:

$$k^{**} = \frac{2}{\rho}\exp\left(\frac{1}{\epsilon}\right) \,\&\, C = 2\epsilon^{-1} \tag{34}$$

Proposition 7. Under the above constructions and certain parameter constraints, then there exists a cooperative-equilibrium minimum-time needed to economic maturity denoted by:

$\tau^{**} = \tau_G := \inf\{t > 0; k(t) = k^{**}\}.$

In other words, $g^{**}(s,k) := 2\,e^{-\rho s}\epsilon^{-1}(k^{**})^{-\epsilon}k^{\epsilon} = U^{Cooperation,\tau^{**}}$ *is a super-mean-valued-majorant of* $g(s,k)$ *with* k^{**} *and* ϵ *given by (34) and (31), respectively.*

Proof. See the proof of Theorem 1 of Dai (2012). Similar to Corollary 4, we establish:

Corollary 5. There is a closed-form solution for the cooperative-equilibrium minimum-time needed to economic maturity τ^{**}*, and in fact:*

$$\tau^{**} = \frac{1}{\rho}\ln\left(\frac{1}{2}\epsilon(k^{**})^{\epsilon}k^{-\epsilon}\left\{\left[-\frac{2}{\rho}\ln\frac{2}{\rho}+\frac{2}{\rho^2}(A-\delta-n+\sigma^2)-\frac{2}{\rho}-\frac{\sigma^2}{\rho^2}\right] + \frac{2}{\rho}\ln k^{**}\right\}\right)$$

where k^{**} *and* ϵ *are given by (34) and (31), respectively, and* k *denotes the initial condition.*

Proof. Combining Proposition 5 with Proposition 7 easily confirms the required result.

Corollary 6. Cooperation between the representative household and the strongly self-interested politician will lead us to much faster economic maturity than that of sequential action when,

$$\epsilon(k^{**})^{\epsilon}k^{-\epsilon}\left\{\left[-\frac{1}{\rho}\ln\frac{2}{\rho}+\frac{1}{\rho^2}(A-\delta-n+\sigma^2)-\frac{1}{\rho}-\frac{\sigma^2}{2\rho^2}\right]+\frac{1}{\rho}\ln k^{**}\right\}$$
$$< \mu(k^{*})^{\mu}k^{-\mu}\left\{\left[\frac{1}{\rho}\ln\rho+\frac{1}{\rho^2}(A-\delta-n+\sigma^2-\rho)-\frac{1}{\rho}-\frac{\sigma^2}{2\rho^2}\right] + \frac{1}{\rho}\ln k^{*}\right\}$$

in which κ^{**}, ϵ, k^{*} and μ are given by (34), (31), (29) and (26), respectively. Otherwise, decentralized sequential action will do a better job than that of differential cooperation in the sense of the minimum-time needed to economic maturity.

Proof. It follows from Corollaries 4 and 5 that we have the required result.

5.2 Risk-neutral and risk-preference politician

One can still suppose that the representative household exhibits log preference while the criterion of the risk-neutral self-interested politician expressed as follows:

$$\mathbb{E}\left[\int_0^{\tau^*} e^{-\rho(s+t)}\left(\tau_K(t)(A-\delta)k(t)\right)dt + U^{P,\tau^*}\chi_{\{\tau^*<\infty\}}\right]$$

And also, the objective of the corresponding optimal stopping problem is given by:

$$\mathbb{E}_{(s,k)}\left[e^{-\rho(s+\tau)}\left(\tau_K^*(\tau)(A-\delta)k(\tau)\right)\chi_{\{\tau<\infty\}}\right]$$

Similarly, for the risk-preference self-interested politician, we have the following criterions for the politician:

$$\mathbb{E}\left[\int_0^{\tau^*} e^{-\rho(s+t)}\frac{\left(\tau_K(t)(A-\delta)k(t)\right)^{\varpi}}{\varpi}dt + U^{P,\tau^*}\chi_{\{\tau^*<\infty\}}\right]$$

and,

$$\mathbb{E}_{(s,k)}\left[e^{-\rho(s+\tau)}\frac{\left(\tau_K^*(\tau)(A-\delta)k(\tau)\right)^{\varpi}}{\varpi}\chi_{\{\tau<\infty\}}\right]$$

in which $\varpi > 1$, some given constant.

Noting that the following discussion is quite similar to that appears in Section 5.1, thus we plan to omit it and leave it to the interested reader. Undoubtedly, closed-form solutions can be derived, too. Moreover, one can comparatively study the minimum-time needed to economic maturity corresponding to different types of politician, and accordingly different types of political institution. For example, one specific type of political institution will induce much higher level of economic maturity while much slower speed of economic maturity when compared with other types of political institution.

Concluding remarks

Dai (2012), by employing optimal stopping theory, discussed efficient capital accumulation with reference to the final state or terminal stocks. And Dai derived closed-form solution by using AK production technology. Nevertheless, the present exploration indeed extends Dai's results from the following directions: first, we have provided very general conditions under which the minimum-time needed to economic maturity can be computed corresponding to a wide range of preferences and technologies; second, in the present study, the role of game structure or institutional arrangement has been sufficiently emphasized in determining the minimum-time needed to economic maturity; last but not least, we study the minimum-time needed to economic maturity for underdeveloped economies and especially under political-economy constraint, i.e., the self-interested politician indeed maximizes the corresponding utility from the rent. Dai mainly demonstrated the strong convergence of capital accumulation to the efficient capital stock while the current paper focusing on the explicit computation and complete characterization of the minimum-time needed to economic maturity for those underdeveloped economies and also under political- economy constraint.

What is more, Dai, in a given institutional arrangement and for given preference and technology, provides the condition under which the efficient state is achievable in the sense of uniform topology while the present exploration constructing a general framework in which one can comparatively evaluate the economic efficiency of different institutional arrangements from the viewpoint of the efficient speed (i.e., based on welfare maximization) of economic development. In other words, Dai strictly follows the neoclassical framework while the current paper is indeed in line with new institutional economics. In particular, we have to some extent modelled the underlying idea of Coase (1988) that we need a baseline framework to comparatively and sufficiently evaluate the economic efficiency of different institutional arrangements in order to make a wise choice during the corresponding institutional changes in reality.

Although optimal stopping theory has been widely used in mathematical finance, no literatures except for Dai (2012, 2013) notice that this mathematical technique would be very helpful in endogenously determining the minimum-time needed to economic maturity in macroeconomics or development economics. Indeed, the results stated and proved in Sections 3 and 4 are new to the best of our knowledge. In other words, these theorems should be of independent interest in macroeconomics although the major techniques are brought from stochastic analysis (see, Øksendal, 2003) and cooperative stochastic differential game (see, Yeung and Petrosyan, 2006). For example, our specification will naturally lead to the explicit computation of the minimum-time needed to economic maturity, as is shown in Section 5. We have provided a general framework by which one can establish the minimum-time needed to economic maturity with respect to different game structures (or institutional arrangements) between the representative household and the self- interested politician. Moreover, our mathematical results show us, for the first time, in which way and to what extent preference, technology, economic and political institutions affect the minimum-time needed to economic maturity in a stochastic growth model. And this would be regarded as one innovation of the paper when compared to Kurz (1965), Phelan and Stacchetti (2001), Acemoglu *et al.* (2008, 2010, 2011), Kaitala and Pohjola (1990), and Leong and Huang (2010).

It is plausible to argue that in an underdeveloped economy such as China (see, Song *et al.*, 2011), the government and the households are motivated to choose appropriate fiscal policies and investment strategies, respectively, such that the economy reaches its maturity state as soon as possible. Our study has formally modeled the state of economic maturity in a stochastic growth model. Moreover, we by employing the optimal stopping theory widely used in mathematical finance give a formal definition of the concept of minimum-time needed to economic maturity. And it would be regarded as an advantage of the stopping theory that the maximal and sustainable capital stock per capita as well as the minimum-time is endogenously determined. Indeed, the major goal of the paper is to investigate the effect of game structure on the minimum-time needed to economic maturity. That is, if we interpret different game structures as different institutional arrangements, then this study provides a basic framework for the comparative study of economic maturity under different institutional arrangements. In a simple model of endogenous growth, the closed-form solution of the minimum-time needed to economic maturity has been derived with the explicit condition, under which cooperation between the representative household and the self-interested politician will induce much faster economic maturity than that of decentralized sequential action, supplied, too. That is, we have shown an example where individual rationality results in dynamic inefficiency under certain institutional arrangement. Nonetheless, it would be noticed that our model can also produce the corresponding condition under which dynamic sequential game structure corresponding to capitalism in some sense will induce much faster economic maturity than that of cooperative stochastic differential game structure in a stochastic growth model.

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SOCIAL CONTRACT, PUBLIC CHOICE AND FISCAL REPERCUSSIONS IN THE ATHENIAN DEMOCRACY

Nicholas KYRIAZIS
University of Thessaly, Department of Economic Studies, **Greece**
nkyr@ergoman.gr
Emmanouil Marios L. ECONOMOU
University of Thessaly, Department of Economic Studies, **Greece**
emmoikon@uth.gr

Abstract:

In the present essay, by utilizing game theory we present a model of choice by actors to explain how change comes about. Then by using ancient and modern sources of literature, we analyze the theory of the social contract as a historical phenomenon that first appeared during the classical period of Athens (510-323 BC.). Then we utilize our findings to explain how public choice was practiced under a direct democracy regime in ancient Athens, by focusing on two historical cases: Eubulus and Lycurgus fiscal policy programs in the second half of the 4th century. We found that these policies can be explained as an implementation of a social contract, through which the Athenian citizens were taking decisions based on rational choice according to a wider economic prospective.

Keywords: model of choice, social contract, 4^{th} century BC Athens.

JEL Classification: H53, H83, G18, N43.

1. Introduction

In the present paper we argue that during the second half of the fourth century BC., a form of a social contract was first practiced in ancient Athens, a major Greek city-state, before taking its modern conceptual shape, which is based on the writings of the 17^{th} and 18^{th} Enlightenment philosophers like J. Locke, Montesquieu and J.J Rousseau. What we purport to show is that the expansionary fiscal policy programs that were introduced by Eubulus and Lycurgus in the 4^{th} century Athens, can be defined as the implementation of an actual social contract. To achieve this, we first provide a model of choice by actors to explain how change in the structure of societies comes about, an issue which is of major importance in the areas of modern social sciences.

An attempt to estimate how change in society's ideas and options takes place preoccupied many scientists and philosophers in various fields of research such as Karl Marx and his followers who examined the changing material conditions (mode of production), or Toynbee (1946) with his theory of External challenge and successful response, which leads to an outcome of two choices: survival and adaptation or collapse. The issue of estimating social change is still pivotal in research fields such as the New Institutional Economics school initiated by North (1978, 1981, 1990) to more recently, the analysis of the emergence of specific macrocultures that are favorable to the creation of democratic forms of government (Kyriazis and Economou, 2012)

Our analysis is organized as follows: Firstly, we provide a function which presents the choices that the warriors had during battle in order to maximize their probability of survival. Then, in order to show the variety of options that direct democracy was offering to the Athenian citizens, we present a choice set like those that were being discussed in each gathering of the Athenian assembly. Next, by examining the issue of the selection between peace or war strategy, we utilize game theory in order to show that the adoption of new proposals by the Athenian deme (the citizens) was actually a compromise between the different social groups and was based on rational choice. Then, based on ancient sources we provide all the evidence about the idea of a social contract being developed as a theoretical paradigm among a series of philosophers.

Finally, we apply our model of choice in order to analyze how the conceptual framework of a social contract was actually found a practical implementation through the expansionary fiscal policies that were introduced by Eubulus and Lycurgus in the 4th century BC.

2. A model of choice

As many scholars argue, among others (Cartledge, 1977; Raauflaub, *et al.* 2007; Fuller, 2008; Kyriazis, 2012) during the Archaic period and more intensively by the middle of the 7th century BC., a new battle formation, the *phalanx*, dominated Greek battlefields. The phalanx formation was based on the *hoplites*, a new type of heavy infantryman. Krentz (1985) and Hanson (2009) provide a detailed analysis about how the phalanx was being deployed during battle and what tactics the hoplites were following. The practical way of testing the pros and cons of the phalanx formation was during battle, by trial and error. The adoption of any new kind of military formation was mainly based on the maximization of survivability criterion for individual participants (the hoplites) as well as collective gain, e.g. victory for the city-states soldiers.

Each hoplite's purpose was to maximize his individual survival probability out of a set of given choices. These choices linked to different battle formations. Based on the existing knowledge about how battles were taking place in antiquity, function (1) presents the three choices that the hoplites had to decide upon:

$$\max f\{(ev(PF), ev(LF), ev(MF)\} \qquad (1)$$

The expected value of survival for each battle formation being adopted is described by ev. PF describes the phalanx formation, LF a linear battle formation and MF a mixing (or melee) type of battle, like those mentioned in the Iliad during Mycenaean times. Through a series of battles during the last period of the Archaic times as well as during the Classical era, surviving warriors would find out that the most efficient tactical formation of the three in terms of maximizing their survival was the phalanx. That's why the phalanx formation was gradually adopted by the majority of the Greek city-states of the classical era when being engaged in war. But by choosing the best military formation in order to maximize the possibility of individual survival, hoplites could also achieve another important element: To choose the best strategy that guaranteed collective welfare e.g. a common aim at city-state level, which was victory.

Thus, under this point of view, the adoption of the phalanx would at the same time maximize individual and collective welfare. It is also obvious that the functioning and performance of the phalanx during battle made necessary the effective cooperation and coordination among its members. In order to achieve this, phalanx members needed to have excellent physical strength, to keep the right pace while walking and to achieve perfect synchronization during maneuvering in battlefield. In order to succeed in all these elements, continuous training and military exercises were required (Cartledge, 1977).

The introduction of the phalanx and in some cases, such as Athens, of a fleet of triremes, is linked with the emergence of a particular set of values in the military field, such as cohesion, discipline, trust, courage, self-consciousness, equality, coordination and cooperation. This set of values that seem to have been emerged from the phalanx formation is also underpinned by (Roisman, 2005) and Kyriazis and Economou (2012). These values were then transferred into the political field and were transformed into the values of *isonomia* (equality in front of the law), *isegoria* (equality of the right to speak), *homomoia* (unanimity, consensus), shaping thus a particular democratic macroculture[19].

[19] In this section we introduce the concept of macroculture, for the first time as far as we know, (taken over and adapted from organization theory) into institutional economics in order to analyse structural change. A "macroculture" encompasses the common values, norms and beliefs shared among the members of a society or a state. The adaptation of the term in economics and politics has also a dynamic time characteristic that of long term periods. As it will be shown in our case study, Classical Athens, the elements of macroculture take shape over time periods of decades to centuries. Through

The analysis now focuses to a model of choice under direct democracy. Under a direct democracy regime every citizen has the right to vote in favor or against on any proposal brought by any citizen in front of the supreme body of governance of the city-state, the Assembly. We postulate that citizens are rational in the sense of maximizing their individual welfare. In this case welfare could include not just economic values (measured for example in monetary terms) but “intangibles”, values such as religion, freedom, a particular way of life etc. When each citizen votes on particular proposals, he chooses the proposal that he expects it will maximize his individual welfare. Thus, the following function is maximized:

$$\max (S_1, S_2, \ldots, S_n) \qquad (2)$$

where S_1, S_2,.......S_n, are the various strategies in his choice “set”, that he expects to maximize his welfare. Since in a direct democracy every vote counts as one, the strategy that is finally selected, depends on the possibility that it has finally received the majority of votes, e.g.

$$\sum_n S_1 > \sum_n S_2 > \cdots > \sum_n S_n \qquad (3)$$

where $\sum_n$ is the total number of votes received for each strategy.

The introduction, selection and adoption of new strategies through the procedure described above, means that new laws, institutions and policies are adopted through time, sometimes changing or abolishing old ones. This again shapes political development and its rate of change, as illustrated in the following decision tree diagram:

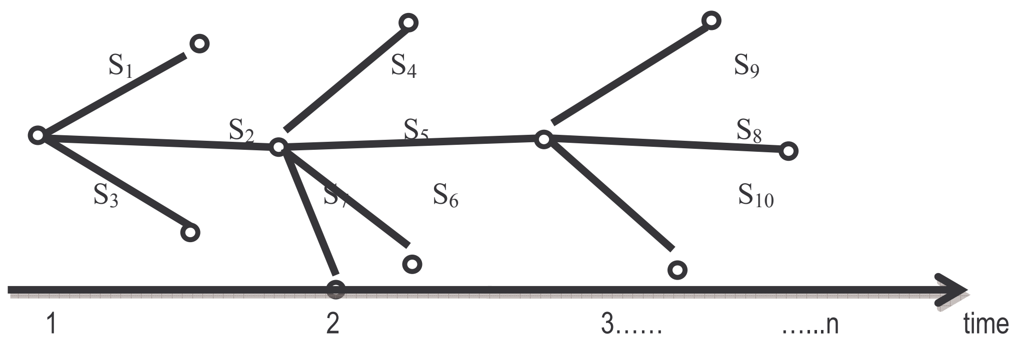

Figure 1. Decision tree probabilities and the adoption of new strategies

Figure 1 shows that if strategy S_2 is chosen at time period 1 (to the exclusion of strategies S_1 and S_3), then at time period 2 the strategies S_4, S_5, S_6 and S_7 are given as options, and if S_5 is chosen, then at time period 3 strategies S_8 to S_{10} are available, etc. Thus, the rate of change of the political framework depends on two factors: i) How many strategies are introduced into the choice set of the Assembly to be decided upon at each time period, and ii) How often new strategies are being adopted. Thus, in theory these may be two extremes: In the first case, it is possible that many new strategies can be introduced. New ones can be adopted with high frequency and older ones may be discarded. This could lead to a political system that is very adaptable, but also too fickle and variable, with a high degree of uncertainty, and low predictability.

these values, norms and beliefs, a macroculture guides actions and creates typical behaviour among independent entities, so that it coordinates their activities so that complex tasks may be completed (Abrahamson and Fombrun, 1992, 1994; Jones, Hesterly, and Borgatti, 1997). The relation between military and political values and the emergence of macrocultures has been analysed by a series of authors (Kyriazis and Paparrigopoulos, 2011; Kyriazis, 2012; Kyriazis and Economou, 2012).

In the second case, the system seems to be characterized by great stability and predictability, like the strategies S_1 and S_3 with very few new strategies being introduced, and even fewer being adopted, no breaks, low adaptability in new ideas, being unable to successfully face new challenges, a system that tends to be "ossified". Such a political system resembles to a deterministic model with perfect predictability in future steps.

The two words written above, uncertainty and predictability are keywords of the sciences that are involved into modeling of systems, including physics, philosophy, statistics, economics, sociology and economic history. Uncertainty is the lack of certainty, a state of having limited knowledge, where it is impossible to exactly describe the existing state a future outcome, or more than one possible outcome. Predictability is the degree to which a correct prediction or forecast of a system's state can be made either quantitatively or qualitatively. The equilibrium state is always aimed and investigated in all models of all sciences mentioned above.

A system may or may not evolve to an equilibrium state, but there exists no general rule to predict the time evolution of systems far from equilibrium. The most known example of such systems is the chaotic ones. Their predictability usually deteriorates with time. To quantify predictability, the rate of divergence of system trajectories in phase space can be measured (Kolmogorov-Sinai entropy, Lyapunov exponents). The main characteristic of a chaotic system is the sensitivity to initial conditions. A political system that has many strategies to choose from resembles to a mathematical model which is close to a chaotic state. In such a state the predictability of the next step or the choice of the new strategy is rather impossible. Limitations on predictability could be caused by factors such as a lack in information or excessive complexity. Two close strategies, in a chaotic system, may lead to two totally different results, two totally different political decisions (Figure 2). The one can be peace, but the other can be warfare.

On Figure 2 the famous logistic map bifurcation diagram is presented. Parameter r is varied in the interval [2.4, 4.0]. In the beginning (for $r < 3$) there is only one equilibrium state (i.e. only one strategy). As r is getting bigger (towards the right), there exist 2, 4, 8, 16, … equilibrium states (strategies). For $r = 4$, there is an infinity of possible states (strategies), thus the political system has high degree of uncertainty and low predictability.

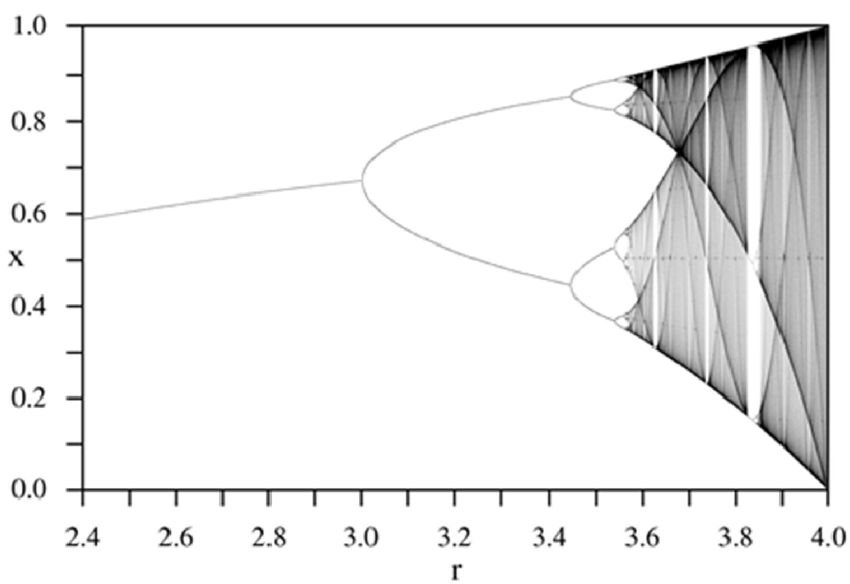

Figure 2. The famous logistic map bifurcation diagram

On the other hand, a political system that has few strategies as alternative solutions resembles to a deterministic model with perfect predictability in future steps, i.e. a non-chaotic system. This, is a central issue of all democracies old and new, to find an optimal rate of change, not too sudden and fast, not too slow and inadaptable. A system of check and balances, in modern terminology, would thus be considered as successful, if it comes close to the ideal benchmark of an optimal rate of change. A way to find an optimal rate of change, not too sudden and fast, not too slow and inadaptable is a central issue of all democracies, old and new because of its difficulty to be specified in actual terms, thus we mention it as a theoretical benchmark, inspired by neoclassical growth theory. A system of checks and balances, in modern terminology, would thus be considered as successful, if it comes close to the ideal benchmark of an optimal rate of change.

In economics an optimal rate of growth is proposed as equilibrium, (steady-state) where the interest rate equals the growth rate (Meade, 1961, ch. 4; Solow, 1970, pp. 71-97). On the other hand, the problem of fine-tuning change and stability is very real in modern democracies, as for example in the USA, where the existing system of checks and balances and the diffusion of decision making and power among too many bodies (e.g. President, two legislatives, Supreme Court, Federal Reserve- FED, Federal System of sharing power, direct and representative lawmaking), makes change in many cases extremely slow and difficult.

Going back to Ancient Greece, the two extremes are Sparta and Athens. Sparta, had in modern terminology, a political system of very strong checks and balances, with political power and decision making diffused among the two kings, the five *ephors*, (a leading group that provided balance between the two kings), *the gerousia* (the 30 elders -including the two kings) and the assembly which consisted by the Spartan deme (excluding women) called *Apella*. The Spartan system of governance guaranteed political stability for about three centuries. However, it was ill adapted to facilitate necessary change and external challenges, with the result of not being able to face the crisis of the 4th century, after which Sparta became a backwater and second rate power. Because the Spartan system did not permit adaptability and change, the two reforming kings of the 3rd century, Agis IV (without success) and Clemonenes III (successfully) had to overthrow it forcefully in order to implement reforms. Cartledge (1987) offers us an in depth analysis of the actual working of Sparta's political system.

By contrast, Athens during the fifth century was characterized by fast political change and institutional innovation. A series of capable Athenian leaders such as Themistocles, Ephialtes and Pericles, introduced, i) election by lot, ii) extension of the right to be elected and to vote to all citizens, iii) changes in the judicial system- "popular" courts by jurors elected by lot, iv) redistribution of wealth through the introduction of *liturgies* (among which the *trierarchy* was the most costly), a series of public services being financed by the richest Athenian citizens through their personal wealth, v) introduction of pay for *eklesiastika* (public offices) and for attending the theatrical plays-contests, the *theorika*[20] etc. (Kyriazis, 2009).

All these institutional innovations made the system very adaptable and changeable, but also very fickle and unpredictable. Especially after the death of Pericles and his moderating influence, the system became perhaps too volatile, more specifically concerning foreign policy and geopolitics, leading thus to wrong decisions (such as the Sicilian campaign and the recall of Alcibiades) which finally led Athenian democracy to a strategic defeat in the Peloponnesian war (431-404 BC.). After the reinstatement of democracy in 403 BC, the Athenian Assembly seemed to understand this shortcoming and introduced a moderating check in the system in the form of *graphe paranomon*, which led to a security of every new proposal before its adoption or rejection.

[20] Theorika payments, introduced probably in 350 BC by Eubulus, were a compensatory tax imposed to the well-off citizens in order to finance the four days theatrical festivals that the poor Athenians could attend if they wish so. See Kyriazis (2009). Theorika also used for various other purposes such as offering public service in the dockyards of Piraeus or in the public arsenal etc. During wartimes the financial surplus by the theorika payments could be used for military purposes, after an approval of the assembly (Pomeroy, et al. 1999).

The rate of change can be written as:

$$\frac{\Delta r}{\Delta t} = r_t = f\left(S_{1_t}, S_{2_t}, \ldots, S_{n_t}, a_t\right) \quad (4)$$

where $a_t = \frac{\Delta a}{\Delta t}$ is the rate of political change during period t, S_{it} are the new strategies introduced into the choice set during the same period, and r_t is the rate of adoption of new strategies.

Table 1 presents a game theory matrix that estimates the payoffs of three Athenian citizens: Two poor *thetes* who serve as rowers in the navy and a rich one, a *trierarch* a rich one, say a commander of a trireme warship. There are two options in the game: war (w) and peace (p) strategy. Each of the three players chooses the one that maximizes his welfare in the game calculated for simplicity as material payoffs. Let us assume (which will be explained in the section on the Social Contract) that for the two poor citizens (who have the same preferences) the war strategy maximizes their payoffs, while for the rich one, the peace strategy does so. Since every citizen has one vote, the war strategy which is selected by the two poor ones is finally chosen.

The outcome of the game is given by Table 1 below.

Table 1. Game matrix for a choice without compensatory payments

	Strategies	
Players	Peace	War
1 (poor)	2	2
2 (poor)	2	4
3 (rich)	8	-2
"Value" of the outcome	12	6

Since the payoff for each of the two poor players under the war strategy is 4, which under the peace strategy is 2, they choose the war strategy, even though this brings about a pure loss for the rich. The "value" of the game is 6 in the case of the war strategy (adding up the payoffs of the three players) which is lower than the "value" of the game under the peace strategy, which is 12. What we purport to show in this very simple game is that inferior outcomes (strategies) may be chosen under democratic voting, if no compensatory payments -"logrolling" or balancing out of interests Buchanan and Tullock (2004) can be offered as an alternative option.

The situation for the two poor citizens in the first game, which provides the outcome without compensatory payments were:

payoff (war) > payoff (peace) (5a)

and for the rich:

payoff (war) < payoff (peace) (5b)

Let us know introduce the possibility of compensatory payments by the rich to the two poor players. In the second game which is described below, the rich citizen offers compensation to the two poor voters, if they vote for peace instead of war. In the new game matrix, the peace strategy can be adopted, if the following conditions are met:

For the poor:

payoff (peace, with compensation) ≥ payoff (war) (6a) and *for the rich:*

payoff (peace, subtracting compensation) > payoff (war) (6b)

In game matrix 2 compensatory accounts are given within the parenthesis in each row of the peace strategy: Each poor voter receives (+2) from the rich one, so that he is as well off from a payoff situation point under the peace strategy as he was under the war strategy (condition 6a). The rich voter offers a total of 4 as compensation to the two poor voters to vote for peace, out of his total payoffs of 8. As table 2 presents, the "value" of the game is now again 12.

Table 2. Game matrix for a choice with compensation payments

	Strategies	
Players	Peace	War
1 (poor)	2 (+2) =4	4
2 (poor)	2 (+2) =4	4
3 (rich)	8 (-4) =4	2
"Value" of the outcome	12	6

But the important point of the second game is that once compensatory payments are introduced, the possibility of achieving Pareto improving situation is given. In the outcome of the second game, the rich player has improved his situation (from 2 to 4) so that he is Pareto efficient, while the two poor ones are no worse. Of course, through bargaining, the two poor voters could convince the rich one to give them a somewhat higher compensation, (say 2.5 to each). In this case, they would also be better off. Condition (6a) is thus a minimal condition.

It is also clear, that the second game is not a zero outcome game (where the gains of one equal the losses of the other(s) players) but a positive sum game. Everyone gains, since the total "value" of the game is higher than in the previous game matrix 1. As we will show, Eubulus and Lycurgus fiscal policy programs can be analyzed as practical social contracts with compensatory payments by the rich to the poor in order to bring about a change of strategy, from war to peace.

3. The emergence of a theory of a social contract in ancient Athens

The main idea of a "social contract" is that the society's individual members are reaching an agreement to bestow some of their freedoms to a political authority (eg. a magistrate or an elected government) in exchange for protection of their "natural and civic" individual rights. What we purport to show here, is that the idea of individual "human" and "civic" rights and a social contract was explicitly proposed in ancient Athens during the 5th century BC., before being reemerged by the 17th and 18th century Enlightenment philosophers.

The nucleus of an idea of social contract, although not explicitly stated, is inherent already in Solon's reforms of 594 BC (Forrest, 1994; Hansen, 1999). By implementing *seisachtheia* Solon abolished the ability of a lender to claim the conversion of a free citizen into slave due to its failure to repay its debts. Solon's reforms released the peasants from serfdom, restored their farms and redeemed those who had been sold into s1avery (Thompson, 1978). He believed that his reforms would balance out the various and conflicting interests of different property classes (mainly rich land owing aristocrats, "middle class", farmers and artisans and in many cases, landless poor), thus bringing about *homonoia* (same-mindness) and avoid stasis (revolt) by creating a community of interests, a version of a social contract, as far as possible. His opinion of his reform was "all people will win", which is not far from a modern interpretation of his reforms as a Pareto improving situation, but also a social contract to guarantee the states' stability.

The first assertive formulation of a theory of social contract can be found in the mid-5th century BC writings of the orator Antiphon. In On Concord, which is one of his works found in a fragment on a papyrus during the 19th century, the idea of natural rights and a social contract is clearly specified (Tsatsos, 1972, pp. 548-565). Antiphon's work contains a declaration on a natural law, as against the conventional man-made law. He, a member of the sophist movement, posed a strong criticism on the ways of implementing justice regarding them as ineffective (Moulton, 1972). By making a distinction between *physis* (what is natural and unchangeable) and *nomos* (the man-made law), Antiphon argued that people create laws which are the result of a human consent or agreement between societies and thus they may be affected by human's self-interest motives. Thus, Antiphon explained why human laws are "artificial" while the laws of nature are compulsory (Gagarin, 1997). This deduction made him also believe that human laws may be violated by people in case they will not avoid punishment.

Protagoras, another pre-Socratic philosopher (circa 490-420 BC.) also expressed some ideas compatible with the theory of a social contract. In his essay On Truth, he also uses the antithesis of law versus nature to claim, in accordance to Antiphon, that human laws are superficially imposed on citizens while those arising from physis, (nature) are unavoidable (Farenga, 2006). In his Great Speech, (Plato, Protagoras 320c-324c) he clearly specifies his views on a social contract: He believed that the main motivations of each person to enter a political community and become one of its citizens by "obeying" on certain regulations, was self-preservation and the need for survival of himself and the other members of his community (Nill, 1985). In other words, Protagoras believed that obeying the laws can be seen mainly as a compromise among people and less as a good itself in ethical terms (Mulgan, 1979; Kerferd, 1981, p. 147).

Similarly, in Plato's Republic Glaucon, Plato's older brother, and like him, amongst the inner circle of Socrates' young affluent students, believed that men found it beneficial to become members of an organized community: "in order not to suffer injuries and injustice" (Nill, 1985, p. 26). Furthermore, Xenophon's Memorabilia provides similar views to those of Plato's Republic as far as a version of a social contract is concerned. In Memorabilia (IV.4), in a dialogue between Socrates and the sophist Hippias, the later asks for Socrates to interpret the meaning of justice and Socrates responds by connecting justice with obedience to the laws as well as to *homonoia* (concord/same-mindness) among the citizens (Marchant, and Todd, 1997).

In addition, Plato, in his Kriton clearly described a picture of a society where every young Athenian, who was at the age of citizenship (only males more than 18 years old) had the right to "choose" to accept and conform to the laws and the customs of his city-state, (and thus signing and accepting a social contract of values, ethics and rules of behavior), or otherwise, to reject them. If he finally decided to reject them, he could keep his belongings, but he should abandon the city and search for an alternative settlement in an Athenian colony or in another Greek city-state of his preference. De Romilly (1992) believes that the terms of acceptance or not of a contract are clearly specified here.

Another Greek philosopher that proposed ideas that more or less could be interpreted as a social contract, is Epicurus. He believed that the purpose of a man was to live a happy and tranquil life, characterized by *ataraxia* (peace and freedom), *aponia* (absence of pain) and self-sufficiency. Epicurus believed that individuals must abstain from mutual wrong doing because this will help their own security and tranquility (Mulgan, 1979). This thesis is by itself a version of a social contract as it proposes a mutually accepted agreement by citizens not to act in a way that may harm each other (blaptein). Epicurus believed that in case of an acceptance of set of values that conform to his proposals, the society could increase its prosperity as a whole. Thus, it seems that Epicurus had understood the concept of achieving a Pareto better situation before the full expression of his theory in 1906 through the Manual of Political Economy, which is a central issue and within the core of the modern Welfare Economics (see Cirillo, 1979).

Finally, Popper (1966, ch. 3) believed that Lycophron, a pupil of Gorgias, a pro-Socratic philosopher had used the theory of the social contract under a liberal view if we interpret his ideas in terms of today. Popper argues that Lycophron considered the state laws as a "covenant by which men assure one another of justice". Popper believed that Lycophron looked upon the state as "an instrument for the protection of its citizens against acts of injustice", demanding that the state should be a "cooperative association for the prevention of crime". By interpreting Lycophron, Popper argued that Lycophron's aim was to find a way to "protect the weak from being bullied by the strong" as well as to define the rights of an institutionalized government to act only in the limited role of the physical protection of its citizens. It is obvious that this final thesis is undeniably a belief of today's liberal doctrines.

4. The implementation of a social contract in the 4th century BC Athens

Since Kleisthenes reforms, Athens gradually developed the most advanced system of direct democracy in antiquity under which any citizen, called "ho voulomenos" (he, who wishes to make a proposal) could introduce in front of the Assembly of citizens, (requiring a quorum of 6000 present) proposals on any subject, such as external policy, (war or peace), public choice such as, the famous naval law of Themistocles (Kyriazis and Zouboulakis, 2004; Halkos and Kyriazis, 2010) or monetary currency policy, eg. Nicophon's monetary law of 376 BC., on the parallel circulation of all good coins and the state's guarantee for their acceptance (Ober, 2008). A detailed analysis of this working of direct democracy, and the initiator ("ho voulomenos") as enriching the exiting choice set of strategies, is offered by Kyriazis and Karayannis (2011).

At the beginning of the 4th century BC., Athens attempted to reconstruct the Athenian League which it had been abolished after Athens' defeat in the Peloponnesian War. This second Delian League was successful for some years, so long as some city-states felt threatened by Spartan power and thus needed Athens' protection. However, since the sudden decline of Sparta after its army defeated in two decisive battles by the Thebans (at Leuctra in 371 and at Mantinea in 362 BC), many allies considered Athenian protection not necessary anymore and wanted to get rid of the burden of payments to the Athenian war treasury linked to this.

This reluctance of the allies of Athens to contribute to the war treasury led to the so called *Social War* (circa 357-355 BC), as Athens tried to prevent them to break away but finally, without success.[21] However, due to the war, Athenian public revenues were falling to 140 talents per year (due in part to much lower custom duties from trade, since war inhibited trade) whereas expenditure soared. Despite that the state was in a situation of an economic recession, the majority of the poor Athenian citizens still voted for the continuation of the war, because many of them had found a stable and not very dangerous employment as rowers in the fleet, which during wartimes comprised between 50 to 100 ships, giving employment from 8.500 to 15.000 rowers. In other words, employment as a rower in the triremes could mean that at least the one fourth to half of the active population of Athens could find a job in the navy, as the total population of Athens is estimated to have been approximately 30.000 people in the 4th century BC. (Hansen, 1999). The fact that employment in the Athenian navy even during wartime was relatively safe may sound strange, but during the 4th century, it was so. After the victorious battle of Naxos in 376 BC., the Athenian navy had reestablished its supremacy for the next half century, till its final defeat in the battle of Amorgos in 322 BC. During this period the Athenian navy fought a series of skirmishes but no major losses and human casualties, in comparison to those of the Peloponnesian War[22].

What is important to mention here is the fact that the intervention of Athens in a series of war campaigns during the 5th and 4th centuries BC had gradually unveiled a situation of opposing interests between low income class *thetes* on the one side, and middle-class *hoplite* Athenians who could not cultivate their farms when being absent in foreign expedition as well as rich Athenians, who were losing

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[21] In a similar situation United States found themselves before and after the end of the Cold war era (1947-1991) due the trend of the global disarmament that followed the dismantle of the Soviet Union. Their European Union NATO allies drastically cut down their defence expenditures, making the US being obliged to bear the highest proportion of defence outlays within the transatlantic alliance. See Metaxas and Economou (2012).

[22] For estimates of the cost of war see Pritchard (to be published) and Arvanitides and Kyriazis (2012).

revenues from a reduction of trade, banking, exports and being burdened by *trierarchies* and *eisphora* on the other side. To solve this detrimental situation Eubulus, the leading orator and politician of the 350's proposed a compromise between the different interest groups which can be regarded as a social contract, implemented by voting in the Assembly.

Instead of continuing the war strategy, poor citizens (of which a large proportion comprised by the thetes) could choose for peace (to the benefit of the rich and the middle classes). In this case they would receive *theorika* payments as a compensation for their loss of wages as rowers and being employed in an extensive public works program held by the state in order to beautify the city, as a part of Eubulus project of rebuilding Athens strength through internal means (Allen, 2010). Eubulus also introduced a law making it difficult to use the surplus of the public finances for military operations, which ensured that it would be available for the public works. Those works included among others, a newly made network of roads, water supply of the city, new waterfronts and shipyards. The ¾ of the warships were redeployed in new ports in Zea and Mounichia so that more space would become available in the central port of Piraeus. Eubulus also improved the legislation when it comes to the commercial law (Sakellariou, 1972, pp. 40-41).

Financing increased theorika payments became feasible: i) after the implementation of the *pentekoste*, through which 2% of the sums on the value of exports and imports were collected as custom duty by the state ii) due to an increase in trade, iii) due to more intensive exploitation of the state's property such as the Laureion silver mines. Eubulus also proposed that the *eisphora*, a tax on property paid by the rich during wartime should become permanent including the peaceful era, as an additional source of revenue for the state's budget, out of which *eklesiastika* (payment for the poor so that they would attend the Assembly), *theorika*, and the public building program could be financed (Kyriazis, 2009).

It is obvious that all these institutional settlements played the compensatory role which it has already been described by the second game matrix above. The compensatory measures under a peace situation made the poor at least as well off, as during the war period. The compromise between reach and poor was successful. Thetes were less in favor of war having in mind that extra war expenses would absorb the surplus of the theorika, intended otherwise for them as compensation. On the other hand, the rich would not anymore be overburdened with war expenses. Also, through the compensatory system of theorika the danger of a possible social unrest that it may have been caused by the dissatisfied lower income classes and may have turned into a revolt against the rich and their wealth, was gradually fading away. The fact that the theorika payments safeguarded the cohesion of the Athenian society and the survival of the political regime, made the Athenian orator and politician Demades, an important figure of that period to characterize all these compensatory system from the rich to the poor citizens, as "the glue of Democracy" (Sakellariou, 1972, pp. 40-41).

The expansionary fiscal policy program that introduced by Eubulus lasted up to 340 BC. During the 355-340 BC period state revenue increased from 130 talents to 400 talents, almost four times higher than the year 340. The grand strategy of the Athenian state which was based on reaping the rewards of peace, through the impressive increase in international trade and social reconciliation was abandoned only when the geopolitical expansionism of Macedonia under king Philip become extremely difficult to be ignored while in the meantime the belligerent passionate speeches of Demosthenes were "adding fuel to the fire" in favor of the war[23].

After the battle of Chaeronea that took place in 338 BC., were the coalition of armies from Athens, Thebes and their allies defeated by the Macedonians, Lycurgus, another Athenian statesman and orator implemented another similar compromise-social contract to that of Eubulus (who probably had died before 340 BC). The new contract-agreement which adopted by the Assembly was based on the same institutional framework as its predecessor. Lycurgus plan brought the brightest period of

[23] That the Macedonian threat was real was realized by almost every Athenian, when king Philip seized in a surprise move, a fleet of 240 Athenian merchant ships carrying grain. Athenian population was dependent on the imports of cereals, since it was not self-sufficient in grain products. See Green (1998).

peace in the history of the Athenian democracy, which lasted to 322 BC. (Lycurgus died in 323 BC, the same year as Alexander the Great). By the mid of 330's BC, the public revenue had been increased to 1200 talents per year (Ober, 2008).

Lycurgus political program was highly successful for a variety of reasons: Firstly, Lycurgus program guaranteed stable employment and revenues for the majority of the poorer Athenians in a series of public work programs. Being inspired by his mentor Eubulus, Lycurgus launched a vast public works program, second only to that of Pericles, which it may be interpreted, in modern terms, as an expansionary fiscal policy program of Keynesian inspiration. The public works program included the new sewage system for Piraeus, monuments such as the theatre of Dionysius beneath the Acropolis, and the extension of the Pnyx. (Hansen, 1999; Kyriazis, 2009). Other monuments also built including a prominent water clock, the Lyceum, the Telesterion at Eleusis[24], as well as the construction of local theaters in some demes. The agora (the "centre" of the city were the most financial transaction were taking place) was provided with new temples and law court facilities, In addition, new ship sheds for warships and an arsenal for naval stores were constructed at Piraeus. City walls were modernized and enhanced. Finally, a new Panathenaic stadium indented for sport activities was constructed (Ober, 2008, pp. 68-69).

Secondly, Lycurgus followed his predecessor Eubulus doctrine to focus on "international" trade as a means of increasing public revenues. Thus, Lycurgus passed a *commercial law*, which allowed metrics and perhaps even slaves to litigate over contracts on equal terms with citizens. Through *enkteseis* he also offered special grants to non-citizens to own real estate whereas some foreigners that were accustomed to overseas trade were granted full citizenship by special decrees of the assembly. The efficient exploitation of trade transactions was also guaranteed by the use of the navy so as to suppress piracy. For this purpose, a naval station was also established on the Adriatic Sea *(ibid*., pp. 68-69). This necessary prerequisite to safeguard trade transactions approves that trade activity, if manipulated smoothly and with insight, it could also have some positive feedbacks on the economy, in our case here, by increasing employment on the navy in order to exterminate piracy and by performing public works to create a naval base to keep the seas open and safe.

Thirdly, another way of increasing public revenues it seems to had come from an increase of the sacred revenues. Revenues from temples are estimated to have been analogous to more than 2% of the annual state income (Papazarkadas, 2011). Finally, in 354 BC. Lycurgus introduced more aggressive measures to safeguard the soundness of the highly-esteemed Athenian coins, the so called "Athenian owls". In parallel he drastically took measures to face coin forgery. In the meantime, he introduced a massive new issue of money in the market (Ober, 2008, pp. 68-69). This may seem that except from of an extensive expansionary fiscal policy, Lycurgus also introduced for some period a monetary expansionary policy too in parallel.

The result of all these policies was that the economy in its totality prospered, trade, exports and GDP grew. The Athenian 4th century economy showed modern characteristics in the sense of being probably the first economy ever in which the second and third sectors of the production (manufacture and services) contributed more to the total Gross Domestic Product and employment than the primary one (agriculture). Thus, the period 338-322 BC must be regarded as a second Golden Age for Athens. A detailed estimation of sectorial GDP and employment contributions is offered by Halkos and Kyriazis (2010).

The total of 1.200 talents revenue for the period of Lycurgus is impressive since it came from Athenian own sources, without contributions by allies. Athens did no more have an empire. This

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[24] Lyceum like the Academy and the Kynosarges were extensive athletic facilities were every citizen, without socioeconomic discriminations, including *ephiboi* (young men over 18) could receive training and exercise themselves in all kinds of sports. This institution was financed by the state and gradually expanded during the fifth century providing they were not slaves. See Fisher (1998) and Kyriazis and Economou (2012). The Telesterion of Eleusis was a sanctuary, one of the primary centers of the *Eleusinian Mysteries* devoted to the goddesses Demeter and Persephone (Wilson, 2005).

revenue was higher than the 1.000 talent revenue of Athens during the 440's BC. in absolute terms, and roughly comparable in relative terms, taking into account a possible inflation[25].

Conclusion

It has already been posed above the question of an optimal rate of change in a democratic system, being fine-tuned between stability and volatility. Modern representative democracies with their checks and balances seem to enhance stability to the detriment of often necessary change and adaptation to new conditions. Bowles and Gintis (1986, p. 186) put the ideal functioning of a "democratic dynamic": "The problem of building a democratic society is…..one of a dynamic interaction of rules and actors, with the actors rendering the rules more democratic, and the increasingly democratic rules rendering the actors more firmly committed to and skilled at democratic participation and decision making", something which we call "learning by voting" (Kyriazis and Karayannis, 2011).

The Athenian democracy had achieved a good balance between these two extremes, combining during the fourth century sufficient institutional change (both political and economic) with increased stability, avoiding thus cases of extreme change that rendered the system sometimes too volatile and unpredictable, especially during the period of the Peloponnesian War. Table 3 illustrates a few important institutional changes. It presents a series of key decisions that decided by the Athenian deme (the people), for example the Naval Decree of 482-481 BC, its initiator Themistocles and the positive feedbacks that that these decisions had on introducing new institutions and political change.

Table 3. Institutional and political change in ancient Athens

Political Decision	Year(s) of introduction)	Initiator	New Institutions and policies	Political change
Naval Law	482/481	Themistocles ("politician")	▪ Trierarchy Public-Private Partnerships (PPP)	Full political rights to all citizens
Theorika	460-450 approximately	Pericles ("politician")	▪ Payment for public service and for theatre plays	
Graphe paranomon	415-403	?	▪ Less radical democracy	Constitutional legal procedure
Nicophon's monetaty law	476	Nicophon ("businessman")	▪ Monetary law; ▪ Parallel circulation of all good coins.	
1.) Expansionary fiscal policy 2.) Trade increase policy	354	Eubulus ("politician")	▪ Increased theorika payments; ▪ Extensive public works program; ▪ Nicophon's law is improved.	Peace strategy Social contract
1.) Expansionary fiscal policy 2.) Trade increase policy 3.) Nicophon's law is improved	(338)	Lycurgus ("politician")	▪ Eisphora also in peacetime; ▪ Extensive public works program; ▪ Trade increase policy (commercial law, Enkteseis)	Peace strategy Social contract

[25] While for example a stonemason received a wage of one drachma per day for the working on the Acropolis building program, which it was equal to the daily wage of a rower during the 5th century, he would receive one and a half drachma during the second half of the 4th century. For prices, wages etc. Loomis (1988) offers a detailed analysis as Burke (1985) and Humphreys (1985) for Lycurgus project.

This article has firstly presented a model of choice by individual rational actors-citizens in a direct democracy setting, showing the possibility of Pareto improving solutions if compensatory payments were allowed. Then, the model of choice was applied to the fiscal expansionary policy programs of Eubulus and Lycurgus showing that they may be interpreted as the implementation of a social contract between different groups of citizens: the poorer *thetes* who were in favor of the war strategy and on the other side, the better-off middle class (hoplite farmers, artisans etc) and the rich (bankers, ship-owners, entrepreneurs) who were in favor of the peace strategy.

The programs of Eubulus and Lycurgus can be seen as a social contract i) in the sense of balancing out the various contradictory interests through the introduction of compensatory payments by the rich to the poor to convince them to change preferences, thus bringing about a Pareto better outcome for the state-society as a whole and ii) in the sense that the real preferences of the majority of the voters were revealed and then adopted through the voting process by the Athenian Assembly. The vote in favor of the peace strategy revealed the actual preferences of the majority of people "sealing" thus the contract and giving it legitimacy and validity.

We think that Eubulus and Lycurgus social contracts amplified citizens' trust in the Athenian political regime and democratic institutions. By participating in a political system that it was taking into account their individual preferences, the Athenian citizens had the will and the motives to defend it from any possible future collapse. Our idea that a political system survives when citizens as individuals wish as a total to defend it can also be found within the pivotal core of the findings of Weingast (1997) who examined the political foundations of democracy in the seventeenth-century England, after the Glorious Revolution of 1688.

The social contracts implemented by Eubulus and Lycurgus disclose also another diastasis when comparing ancient to modern democracies: In a direct democracy the problem of revealing the actual preferences of citizens on particular issues can be efficiently managed under certain circumstances whereas in a representative democracy fails to do so, because under it, citizens-voters have to decide upon a "bundle" of all-encompassing proposals made by each political party, without having the possibility to decide upon separate issues.

Also, apart from the problem of possibly too many checks and balances, a second problem that it may reduce the rate of change in representative democracies has to do with the time factor. Under representative democracies, voters have the opportunity to express preferences only periodically, every four or five years at the elections. This can be unproductive in case of pressing issues that they may have become acute in the meantime and may need an immediate arrangement. Thus, we finally tend to consider that the implementation of a social contract in practical terms under representative democracy is almost impossible.

This article may give a stimulus to the further research on the theoretical aspects of democracy and on the actual conditions under which social contracts may be implemented in practice in modern democracies.

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MONEY FLEXIBILITY AND OPTIMAL CONSUMPTION-LEISURE CHOICE UNDER PRICE DISPERSION

Sergey MALAKHOV[26]
Université Pierre-Mendès-France, Grenoble, **France**
serguei.malakhov@orange.fr

Abstract:

The synthesis of G.Sigler's rule of the optimal search with the classical individual labor supply model can incorporate the satisficing decision procedure in the neoclassical framework. Many psychological anomalies and puzzles like the paradox of little pre-purchase search for big-ticket items and the effect of sunk costs sensitivity get the purely economic rationale. This synthesis also enlarges the understanding of the phenomenon of money flexibility under price dispersion. The specific constraints of the search model establish the correspondence between elasticity of the marginal utility of labor income with regard to price dispersion, wage rates, and the propensity to search. The money flexibility under price dispersion discovers specific features of Veblen effect. The marginal utility of labor income becomes negative, when the smart shopping of luxuries results in price reductions, which are greater than the wage rate. The marginal utility of luxuries also becomes negative. And the total consumption-leisure utility is increasing only due to the increase in leisure time. The paper argues that the same mechanism underlies the phenomenon of money illusion due to the relatively excess money balances.

Keywords: money flexibility, Veblen effect, money illusion, price dispersion.

JEL Classification: D11, D83.

1. Introduction

Although the issue of consumer search has become one of the most dynamic themes in modern economic thought during the past decades, the problem of its integration with the classical theory is still open. There is a possibility of a deeper methodological integration of J.Stigler's original model of search (Stigler 1961) and the model of the individual labor supply. One of such approaches has been introduced earlier in the general form (Malakhov 2011). That approach enabled the synthesis of the search satisficing procedure and the optimal consumption-leisure choice (Malakhov 2012b), as well as the original explanation of anomalies of consumer behavior such as the paradox of little pre-purchase search for big-ticket items (Malakhov 2012a) and Veblen effect (Malakhov 2012c). Based on a wide review of the modern economic literature, those papers have also opened up the possibility of even deeper integration of the search model and the individual labor supply model, now in terms of money. This approach makes the turn to early fundamental methodological works of the neoclassical school. In this paper we pay attention to the Lagrangian multiplier and to the marginal utility of money in the consumption-leisure choice under price dispersion.

2. Lagrangian multiplier in the search model

The optimal consumption-leisure choice under price dispersion can be presented by the utility function *(U(Q,H)* which is constrained by the equality of marginal costs of search (marginal loss of labor income) to its marginal benefit (marginal savings on price reduction), or

$$w\frac{\partial L}{\partial S} = Q\frac{\partial P}{\partial S} \tag{1}$$

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where: w – the wage rate; *Q* – the quantity purchased; *L* – the labor time: *S* - the time of search;
$\partial L/\partial S$ – the propensity to search ($\partial L/\partial S<0$);
$\partial P/\partial S$ – the price reduction during the search ($\partial P/\partial S<0$).

We can formulate the following problem:

$$\max U(Q,H) subject to w = Q\frac{\partial P/\partial S}{\partial L/\partial S} \tag{2}$$

Here we consider the value of price reduction $\partial P/\partial S$ as a variable functionally independent from both the consumption *Q* and leisure *H*. This value simply states the fact that we have concluded the search at the certain satisficing price level, where we have spent time $S=\Delta S$ on the search and we have got the price reduction ΔP with respect to our willingness to pay. At this price level we determine how much we should buy for the period of time *T* until the next purchase. The chosen quantity *Q* determines the labor time *L* we need to restore our cash-in-the-pocket or money balances (Here, we can bypass the monetary theory by the simple appeal to the H.Leibenstein's understanding of the static analysis, who presented the static situation as one in which the order of events was of no significance (Leibenstein 1950, p.187), or by the come-back to old days when debts were recorded by shopkeepers in notebooks). And for the chosen time horizon of our consumption-leisure choice *T* (a day, couple of days, a week, a year) we finally determine the leisure time $H = T - S - L$ we will need to consume the chosen quantity *Q*.[27]

This search model of the optimal consumption-leisure choice can be presented with the help of the Cobb-Douglas utility function $U(Q,H) = Q^{-\partial L/\partial S}H^{-\partial H/\partial S}$. This intuition is supported by the fact that the marginal rate of substitution of leisure for consumption in the search model (Eq.3.1) corresponds to the *MRS* value of the utility function with constant elasticity of substitution $\sigma = 1$ (Eq.3.2), because from the equation $L+S+H=T$ we get $(\partial L/\partial S+\partial H/\partial S) = -1$:

$$MRS(HforQ) = -\frac{dQ}{dH} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H \tag{3.1}$$

$$\frac{dQ}{dH} = \frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = \frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = \frac{QT}{(H-T)}\frac{1}{T} = \frac{Q}{H}\frac{H}{(H-T)} = \frac{Q}{H}\frac{(H-T+T)}{(H-T)};$$

$$\frac{dQ}{dH} = \frac{Q}{H}(1+\frac{T}{H-T}) = \frac{Q}{H}(1+\frac{1}{\partial L/\partial S}) = \frac{Q}{H}(\frac{\partial L/\partial S+1}{\partial L/\partial S});$$

$$\partial L/\partial S = -\alpha \Rightarrow \frac{dQ}{dH} = -\frac{Q}{H}(\frac{1-\alpha}{\alpha}) \tag{3.2}$$

Here the difference between the search model and the classical model of the individual labor supply is the analytical extraction of the time of search from the time of leisure and its addition, as we are going to see in the set of Eq. (4), to the time of labor.

Usually the search cuts not only the labor time but also the leisure time ($\partial H/\partial S<0$). The following pair of the inequalities ($-1<\partial L/\partial S<0$; $\partial H/\partial S<0$) describes the "common model" of consumer behavior. And this "common model" can be presented with the help of the Archimedes' principle. The search displaces both labor time and leisure time in the time horizon of the consumption-leisure choice, like ice

[27] We can present any local store by the given value of price reduction $\partial P/\partial S$. This means that we can analyze the functions $Q=Q(\partial P/\partial S)$ and $H=H(\partial P/\partial S)$. But inverse functions do not exist, because the location and the price policy of the local store don't depend on consumer's tastes and preferences. Here the consumer is the price-reduction-taker. If we presuppose that consumers can always adjust price reductions to their consumption patters ($\partial P/\partial S=\partial P/\partial S(Q)$); $\partial P/\partial S=\partial P/\partial S(H)$), consumption and leisure become perfect complements.

displaces both whiskey and soda from the glass (Eq.4.2). The Archimedes' principle enables the understanding of the relationship between the labor and the search, or the function *L= L(S)*:

$$L(S) = T - H(S) - S \Rightarrow \partial L / \partial S = -\partial H / \partial S - 1 (4.1)$$

$$dH(S) = dS \frac{\partial H}{\partial S} = -dS \frac{H}{T} (4.2)$$

$$\frac{\partial L}{\partial S} = -\frac{\partial H}{\partial S} - 1 = \frac{H}{T} - 1 = \frac{H - T}{T} = -\frac{L + S}{T} (4.3)$$

$$\frac{\partial L}{\partial S} = -\frac{L + S}{T} \Rightarrow \partial^2 L / \partial S^2 = -\frac{\partial L / \partial S + 1}{T} < 0 (4.4)$$

$$\frac{\partial L}{\partial S} = \frac{H - T}{T} \Rightarrow \partial^2 L / \partial S \partial H = 1 / T (4.5)$$

We can see that in the search model the value of the time horizon equals not to the chronological period, but to the period of the product lifecycle, i.e., to the moment of the next purchase of the same item (Eq.4.5). The greater time horizon increases not only labor time required for purchase, but also the leisure time required for consumption. So, the increase in the value of time horizon decreases the absolute value of *propensity to search* $|\partial L/\partial S|$.

The set of Eq. (4) simplifies the presentation of the values of marginal utilities and it explains the step-by-step derivation of the *MRS (H for Q)* in the Eq. (3):

$$MU_Q = \lambda \frac{\partial P / \partial S}{\partial L / \partial S} = \lambda \frac{w}{Q} \quad (5.1)$$

$$MU_H = -\lambda Q \frac{\partial P / \partial S}{(\partial L / \partial S)^2} \partial^2 L / \partial S \partial H = -\lambda \frac{w}{\partial L / \partial S} \partial^2 L / \partial S \partial H = \lambda \frac{w}{L + S} \frac{T}{T} = \lambda \frac{w}{L + S} \quad (5.2)$$

Now we can optimize the utility *U** with respect to the wage rate *w*. We also meet here the specific forms of elasticity of the search model, which we will need in the analysis of money flexibility:

$$\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda \left[\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} + \frac{w}{L + S}\frac{\partial H}{\partial w} \right] (6.1)$$

$$\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} = \frac{\partial P / \partial S}{\partial L / \partial S}\frac{1}{\partial P / \partial S}\left(\frac{\partial L}{\partial S} + w\frac{\partial(\partial L / \partial S)}{\partial w} \right) = 1 + \frac{w}{\partial L / \partial S}\frac{\partial(\partial L / \partial S)}{\partial w} = 1 + e_{\partial L / \partial S, w}$$

$$\frac{w}{L + S}\frac{\partial H}{\partial w} = -\frac{w}{L + S}\frac{\partial(L + S)}{\partial w} = -\frac{w}{(L + S)/T}\frac{\partial(L + S)/T}{\partial w} = -\frac{w}{\partial L / \partial S}\frac{\partial(\partial L / \partial S)}{\partial w} = -e_{\partial L / \partial S, w}$$

$$\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda \left[\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} + \frac{w}{L + S}\frac{\partial H}{\partial w} \right] = \lambda \left[1 + e_{\partial L / \partial S, w} - e_{\partial L / \partial S, w} \right] = \lambda (6.2)$$

Then, we can re-arrange the Eq.(1) with the help of the Eq.(4.3):

$$-T\partial P / \partial S = w(L + S) = P_0 \quad (7)$$

The Eq. (7) gives us the value of *the price equivalent of the potential labor income w(L+S),* which the consumer would have earned if he spends all time of search on working.

The value of the price equivalent of the potential labor income simplifies the presentation of the value of the marginal utility of consumption. Now we can write for consumption and leisure:

$$MU_Q = \lambda \frac{\partial P/\partial S}{\partial L/\partial S} = \lambda T \frac{\partial P/\partial S}{L+S} = \lambda \frac{P_0}{L+S} = \frac{\partial U^*}{\partial w}\frac{P_0}{T-H} \quad (8.1)$$

$$MU_H = \lambda \frac{w}{L+S} = \frac{\partial U^*}{\partial w}\frac{w}{T-H} \quad (8.2)$$

However, if we repeat this step-by-step utility optimization procedure with respect to the wage rate in the framework of the individual labor supply model, we get the same results for the classical consumption-leisure choice (Baxley and Moorhouse 1984):

$$\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda P \frac{\partial Q}{\partial w} + \lambda w \frac{\partial H}{\partial w} = \lambda \left[P\frac{\partial Q}{\partial w} + w\frac{\partial H}{\partial w} \right]$$

$$wT = PQ + wH \Rightarrow T = P\frac{\partial Q}{\partial w} + w\frac{\partial H}{\partial w} + H \Rightarrow T - H = P\frac{\partial Q}{\partial w} + w\frac{\partial H}{\partial w}$$

$$\lambda = \frac{\partial U^*}{\partial w}\frac{1}{T-H} \quad (9)$$

and finally:

$$MU_Q = \lambda P = P\frac{\partial U^*}{\partial w}\frac{1}{T-H} \quad (10.1)$$

$$MU_H = \lambda w = w\frac{\partial U^*}{\partial w}\frac{1}{T-H} \quad (10.2)$$

So, the following graphical presentation, given with unusual attributes, simply represents the specific form of the graphical resolution of the optimal consumption-leisure choice for T = *24 hours* (Figure 1):

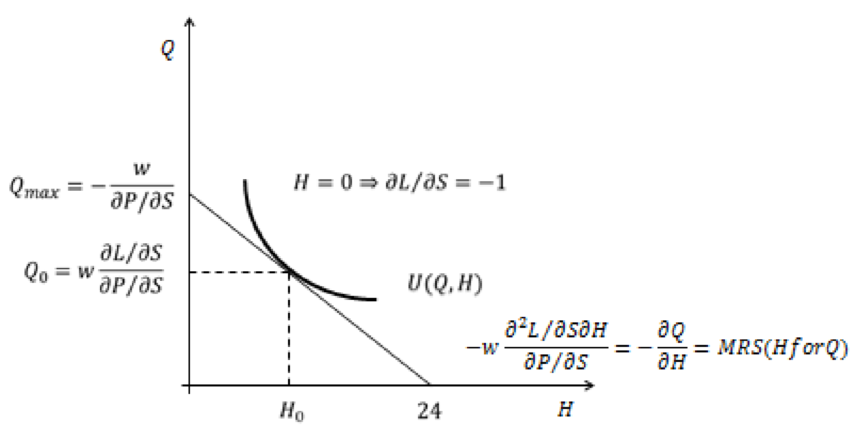

Figure 1. Common model" of search behavior

There are two important differences between two models. First, the value of the Lagrangian multiplier λ in the search model is exactly equal to the marginal utility of the wage rate, or $MU_w=\lambda$, while in the classical model we need to adjust it to the allocation of time between labor and leisure. Second, in the classical model the marginal utility of consumption is determined by the equilibrium purchase price P_e, while the marginal utility of consumption in the search model is determined by the price equivalent of the potential labor income $P_0=w(L+S)$.

If we re-arrange the Eq. (3.1) with the help of the Eq. (7), we can simplify the presentation of the marginal rate of substitution of leisure for consumption:

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=MRS(HforQ)=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H=-\frac{w}{\partial P/\partial S}\frac{1}{T}=\frac{w}{P_0} \quad (11)$$

The analysis of the satisficing decision procedure of the unit purchase (Malakhov 2012b) can clarify the concept of the price equivalent of the potential labor income (Figure2):

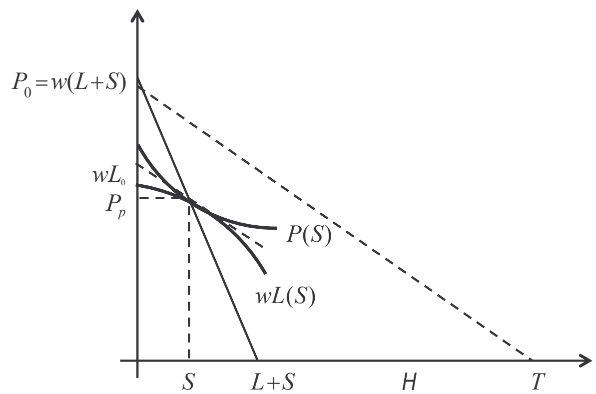

Figure 2. Satisficing decision procedure and optimal consumption-leisure choice

Here we get the shape of the *wL(S)* curve ($\partial^2 L/\partial S^2 <0$) from the Eq. (4.4). The shape of the *P(S)* curve ($\partial^2 P/\partial S^2 >0$) is based on the commonsense assumption of the diminishing efficiency of search. The both curves have the same tangent (dotted) line at the purchase price P_P. The purchase price P_P equals to the labor income *wL,* where the wage rate *w* gives us the slope of the intersection with the time horizon axis of the consumption-leisure choice. So, the value of the labor income, reserved for the purchase before we start to search, $wL_0=wL+dwL(S)$, can represent our willingness to pay. And we can say that the value of price reduction $\partial P/\partial S$ simply represents the rate of decline in the willingness to pay during the search. If we link the points $[P_0,T]$, we get another dotted line with the same $\partial P/\partial S$ slope in accordance with the Eq.(7).

This is more difficult to determine the price equivalent of the potential labor income. It looks like a high-order willingness to pay with regard to the reservation level. P. Diamond used that approach when he described the behavior of shoppers with high willingness to pay (Diamond 1987). On the other hand, the Eq.(11) looks corresponding to another P. Diamond's consideration, that positive search costs makes the equilibrium price equal to the monopoly price (Diamond 1971). However, the idea of the monopoly price seems not to be an appropriate explanation of the price equivalent of the potential labor income, because if a monopoly sets the price at the level of the potential labor income, consumers will increase their labor supply by the time of search, and this increase in labor supply will decrease the wage rate. The price equivalent of the potential labor income at lower wage rate will become unattainable.

We should keep in mind that this value is to some extent virtual, and it can be determined only *ex post*, when we conclude the search. This *ex ante – ex post* relationship between the reserved labor income and the potential labor income initiates the assumption that we have here the WTP-WTA relationship and the potential labor income represents the price that compensates not only monetary loss, which is equal to the purchase price $P_P=wL$, but also the loss of the time of search, calculated *ex post* on the base of the wage rate, or *wS*. However, this consideration needs the answer to the question whether, when search costs are positive, *the willingness to accept* can represent the equilibrium price.

Finally, it is easy to demonstrate that all above-presented equations are valid in the case when search costs are equal to zero, or *S=0*. Here, all properties of the search model become equivalent to the corresponding values of the classical individual labor supply model. The utility function takes the usual $U(Q,H)=Q^{L/T}H^{H/T}$ form, the *MRS (H for Q)* becomes equal to the *Q/L* ratio, the value of the marginal utility of consumption MU_Q and the value of the marginal utility of leisure MU_H become equal to the classical λP_P and the λw values, where the Lagrangian multiplier, according to Baxley and Moorhouse (1984), is equal to the $(\partial U^*/\partial w)/(T-H)$ ratio. Indeed, when search costs are equal to zero, the purchase price is equal to the reserved labor income as well as to the potential labor income. The willingness to pay becomes equal to the willingness to accept. In addition, the classical individual labor supply model becomes a particular case of the search model, where the propensity to search $\partial L/\partial S$ is equal to the $(- L/T)$ ratio.

3. Money flexibility and Veblen effect

The Eq. (6.2), where $MU_w=\lambda$, tells us that the marginal utility of labor income per unit of time is equal to the marginal utility of an extra unit of money. However, the search process also produces the non-labor income through the reduction in prices of purchases. And in order to compare the efficiency of shopping in different stores we need to define the marginal utility of price reduction.

For better understanding and illustrative purposes, we will use the absolute value of price reduction $|\partial P/\partial S|$, keeping in mind that due to $\partial^2 P/\partial S^2 >0$, the increase in the absolute value $|\partial P/\partial S|$ means the increase in price. (The replacement of the value $\partial P/\partial S$ to its absolute value $|\partial P/\partial S|$ as well as the replacement of the value $\partial L/\partial S$ to the absolute value $|\partial L/\partial S|$ does not matter).

$$\frac{\partial U^*}{\partial|\partial P/\partial S|}=\frac{\partial U}{\partial Q}\frac{\partial Q}{\partial|\partial P/\partial S|}+\frac{\partial U}{\partial H}\frac{\partial H}{\partial|\partial P/\partial S|}=\lambda\left[\frac{w}{Q}\frac{\partial Q}{\partial|\partial P/\partial S|}+\frac{w}{L+S}\frac{\partial H}{\partial|\partial P/\partial S|}\right]$$
$$=\lambda\frac{w}{\partial P/\partial S}\left[\frac{|\partial P/\partial S|}{Q}\frac{\partial Q}{\partial|\partial P/\partial S|}+\frac{|\partial P/\partial S|}{L+S}\frac{\partial(T-(L+S))}{\partial|\partial P/\partial S|}\right]$$
$$=\lambda\frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|}-\frac{|\partial P/\partial S|}{(L+S)/T}\frac{\partial(L+S)/T)}{\partial|\partial P/\partial S|}\right]$$
$$=\lambda\frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|}-e_{|\partial L/\partial S|,|\partial P/\partial S|}\right]\quad(12)$$

The key equation (Eq.1) of the search model can convert this result. If we present it in the elasticity form with respect to the absolute value of price reduction $|\partial P/\partial S|$, we get for the constant wage rate ($\partial w/\partial|\partial P/\partial S|=0$):

$$Q\left|\frac{\partial P}{\partial S}\right|=w\left|\frac{\partial L}{\partial S}\right|\Rightarrow e_{Q,|\partial P/\partial S|}+1=e_{|\partial L/\partial S|,|\partial P/\partial S|}$$
$$e_{Q,|\partial P/\partial S|}-e_{|\partial L/\partial S|,|\partial P/\partial S|}=-1(13)$$

This elasticity form (Eq.13) gives us the value of the marginal utility of price reduction:

$$\frac{\partial U^*}{\partial|\partial P/\partial S|}=\lambda\frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|}-e_{|\partial L/\partial S|,|\partial P/\partial S|}\right]=-\lambda\frac{w}{|\partial P/\partial S|}(14)$$

The negative value of the marginal utility of price reduction $MU_{|\partial P/\partial S|}$ needs some methodological comments. The search produces the non-labor income and the greater absolute value of price reduction increases money balances. In this sense, the marginal utility of price reduction is positive. But the assumption of the decreasing efficiency of search, or $\partial^2 P/\partial S^2>0$, establishes the direct correspondence between the value of the purchase price and the value of price reduction. The greater absolute value of the price reduction corresponds to the greater purchase price. (The case, when consumers meet big discounts and find low purchase prices, is examined with regard to the reduction in the time horizon of consumption-leisure choice or shelf lives. (Malakhov 2012b). However, the increase in price decreases the utility and, in order to avoid the confusion, we will use the notion of *the marginal disutility of price reduction* for the value $MU_{|\partial P/\partial S|}<0$.

Its unusual form raises the question whether we have the same value of marginal utility of an extra unit of money when we compare labor income with non-labor income. However, it is easy to show that *pecunia non olet* and the value of the Lagrangian multiplier, or the utility of an extra unit of money, is the same for labor income as well as for non-labor income.

The Eq. (14) is very useful. It uncovers the anatomy of the search process. First of all, we can see that different levels of wage rate result in different values of the marginal disutility of price reduction. Moreover, their relationship is reciprocal. The greater wage rate results in the greater marginal disutility of price reduction. This consideration is very important. When the value of price reduction represents the rate of reduction of the purchase price with respect to the willingness to pay, the increase in the absolute level of price reduction $|\partial P/\partial S|$ first of all means the increase in the willingness to pay.

The increase in the willingness to pay happens due to the increase in the opportunity costs of search, i.e., in the wage rate. Trying to save time, individuals begin to buy in nearby stores at high prices[28]. However, they can keep their willingness to pay constant and they can continue to buy in a distant supermarket. If the value $|\partial P/\partial S|$ is constant, we expect that the marginal disutility of price reduction $MU_{|\partial P/\partial S|}$ is also constant.

In order to verify this assumption, let's analyze the total change in the marginal disutility of price reduction with regard to the change in the wage rate:

$$\frac{\partial MU_{|\partial P/\partial S|}}{\partial w} = -\left[\frac{\partial \lambda}{\partial w}\frac{w}{|\partial P/\partial S|} + \lambda\frac{\partial\left(\frac{w}{|\partial P/\partial S|}\right)}{\partial w}\right] = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \quad (15)$$

We can see, that the marginal disutility $MU_{|\partial P/\partial S|}$ for consumers who continues to buy in the same place at the same price after the increase in their wage rates ($e_{|\partial P/\partial S|,w}=0$) is constant only if the marginal utility of the wage rate is unit elastic, or $e_{\lambda,w} = -1$. If it is not, consumers will face either the increase or the decrease in the marginal disutility of price reduction, may be in the form of *the frustration from shopping* with respect to their income levels.

Here we can address to the original R. Frisch's analysis of the phenomenon of money flexibility (Frisch 1959). It can give us some ideas about feelings of consumers with different income levels when they meet each other in the same supermarket. Things look like consumers with lower wage rate elasticity of the marginal utility of income $e_{\lambda,w}$, i.e., "the better-off part of the population", get greater disutility in supermarkets than "the median part of the population".

If we continue to follow the methodology of R. Frisch, we can simply re-write the Eq.(15), keeping in mind that the second cross derivatives of utility are equal, or $\partial MU_{|\partial P/\partial S|}/\partial w = \partial MU_w/\partial|\partial P/\partial S|$ and $MU_w=\lambda$:

$$\frac{\partial MU_{|\partial P/\partial S|}}{\partial w} = \frac{\partial \lambda}{\partial|\partial P/\partial S|} = -\left[\frac{\partial \lambda}{\partial w}\frac{w}{|\partial P/\partial S|} + \lambda\frac{\partial\left(\frac{w}{|\partial P/\partial S|}\right)}{\partial w}\right] = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \quad (16)$$

[28] Here we simply follow the Salop-Stiglitz's assumption, that only high-search-cost individuals make purchases in high-price stores (Stiglitz 1979, p. 141).

Then, we make one more step and we get the general relationship of the marginal utilities of wage rate and of consumption expenditures under the search process:

$$\frac{\partial \lambda}{\partial |\partial P/\partial S|} = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \Rightarrow \frac{\partial \lambda}{\partial |\partial P/\partial S|}\frac{|\partial P/\partial S|}{\lambda} = -\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right]$$

$$e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w} = 0 \quad (17)$$

Really, the value $e_{\lambda,|\partial P/\partial S|}$ seems to be an appropriate representation of the concept of *the marginal utility of money expenditures* carefully derived by M. Blaug from the analysis of the Marshallian legacy (Blaug 1997, pp.322-323) and, in the same scrupulous manner, compared with the marginal utility of money by A. Abouchar (Abouchar 1982). However, even when the second cross derivatives are equal, they have different economic sense. From here we can leave the concept of the marginal disutility of price reduction and begin to use the concept of the price reduction elasticity of the marginal utility of money expenditures $e_{\lambda,|\partial P/\partial S|}$.

We can see that for the unit elastic marginal utility of income ($e_{\lambda,w} = -1$) the choice of the high-price store after the increase in the wage rate makes the price reduction elasticity of the marginal utility of money expenditures $e_{\lambda,|\partial P/\partial S|}$ as expected *positive*. The purchase in the high-price store increases the marginal utility of money holdings with respect to the corresponding wage rate elasticity of price reduction, or:

$$e_{\lambda,|\partial P/\partial S|} = e_{|\partial P/\partial S|,w} \quad (18)$$

However, here we come to the real combinatorics' variety because every value of the Eq. (17) represents a pair of other variables. It is very difficult to describe all possible changes in the Eq. (17). But it is worth looking at some of them.

First of all, we should take into consideration the proportional change in the marginal utility of an item to be bought and the change in the willingness to pay. This proportional change happens, for example, when we make the trade-off between the quality and the price. Again, the shift in the $|\partial P/\partial S|$ value means here the choice of high-price store with quality items. However, the assumption of the proportional change in the marginal utility of an item and in the willingness to pay for it results in the constant value of the marginal utility of consumption expenditures. And its price reduction elasticity $e_{\lambda,|\partial P/\partial S|}$ becomes equal to zero.

The Eq. (17) takes the following form:

$$e_{\lambda,w} = e_{|\partial P/\partial S|,w} - 1 \quad (19)$$

It means, that the inappropriate ambitious choice of a high-price store after the increase in the wage rate ($\partial|\partial P/\partial S|/\partial w > 1$) can make the wage rate elasticity of price reduction greater than one, or $e_{|\partial P/\partial S|.w} > 1$. In addition, the wage rate elasticity of the marginal utility of income becomes positive, or $e_{\lambda,w} > 0$.

There are two possible explanations for the positive wage rate elasticity of the marginal utility of income $e_{\lambda,w}$. The first and most obvious explanation is that this is a symptom of the risk-seeking behavior, or $\partial\lambda/\partial w > 0$. We can find many such examples in the real estate sector.

However, not all luxury items are risky. The Chateau Lafite Rothschild 1995 from Pauillac and the Opus XA from Arturo Fuente bought to celebrate new position can hardly be regarded as investments in risky assets. There is another consideration. We can ask ourselves whether a dollar spent on those items has really increased the utility, or, more definitely, whether the marginal utility of the increase in the wage rate $MU_w = \lambda$ has been positive.

Before we reject these seemingly absurd questions, let's analyze changes in marginal utilities of both consumption and leisure under this assumption.

If we come back to our preceding results, we can see that:

$$MU_Q = \lambda \frac{\partial P / \partial S}{\partial L / \partial S} \quad (20.1)$$

$$MU_H = -\lambda \frac{w}{\partial L / \partial S} \partial^2 L / \partial S \partial H \quad (20.2)$$

Even when the price reduction is unit elastic with respect to the wage rate ($e_{|\partial P/\partial S|.w} = 1$), the ambitious choice of the high-price store ($\partial|\partial P/\partial S|/\partial w>1$), when consumers are attracted by deep discounts on big-ticket items, results in the following inequality: $w<|\partial P/\partial S|$. However, if we take the inequality $w<|\partial P/\partial S|$, we leave the "common model" of behavior and we come to the "leisure model" of behavior. The $w<|\partial P/\partial S|$ relationship changes the allocation of time. According to the key equation of the search model (Eq.1), the absolute value of the propensity to search becomes greater than one, or $|\partial L/\partial S|>1$ and $\partial L/\partial S<-1$. But now the relationship between leisure and search becomes positive, or $\partial H/\partial S>0$. The Archimedes' principle stops working. And this change means that the increase in the absolute value of the propensity to search $|\partial L/\partial S|$, i.e., its decrease in real terms, is followed now by the increase in leisure time, or $\partial^2 L/\partial S\partial H<0$.

This situation corresponds to the backward-bending effect in the classical individual labor supply model. However, there is very important distinction between the classical model and the search model. Let's come back to the Eq. (20). The value $\partial^2 L/\partial S\partial H<0$ doesn't change the marginal utility of consumption, but it dramatically changes the marginal utility of leisure. The latter becomes *negative* and the leisure becomes "bad".

That conclusion had been done in the earlier analysis of Veblen effect (Malakhov 2012c). Things looked like we wasted time at parties after the purchase of a suit. However, that consideration was *dialectically incomplete*. We told about it, when it was discussed that either the purchase of a suit made parties "reasonable", or parties made "reasonable' the purchase of a suit.

If we presuppose that the marginal utility of the wage rate, i.e., value of the Lagrangian multiplier, becomes negative, we make another representation of the marginal utilities of both consumption and leisure in the Eq.(20). Now the consumption becomes "bad" whereas the marginal utility of leisure again becomes positive.

Of course, the classical utility optimization problem doesn't take place in the "leisure model" of behavior. However, consumers can continue to increase the total utility. But now they can increase the total utility only with the increase in leisure time. The party organized in order to celebrate the new position increases the total utility, where negative marginal utilities of the Chateau Lafite Rothschild 1995 from Pauillac and of the Opus XA from Arturo Fuente become invisible.

Moreover, the increase in leisure time, required to restore the positive utility of such a "luxury" consumption-leisure choice, automatically increases the absolute value of the price reduction $|\partial P/\partial S|$, i.e., the purchase price, and Veblen effect takes place (Figure 3):[29]

It is very easy to follow all these considerations with the help of the value of the *MRS (H for Q),* derived earlier (Eq.3.1) and repeated here for illustrative purposes:

$$\frac{\partial U / \partial H}{\partial U / \partial Q} = MRS(HforQ) = -\frac{w}{\partial P / \partial S} \partial^2 L / \partial S \partial H \quad (21)$$

[29] Here we should keep in mind that usually there is a certain physical or psychological minimum level of leisure time where $\partial L/\partial S=-1$.

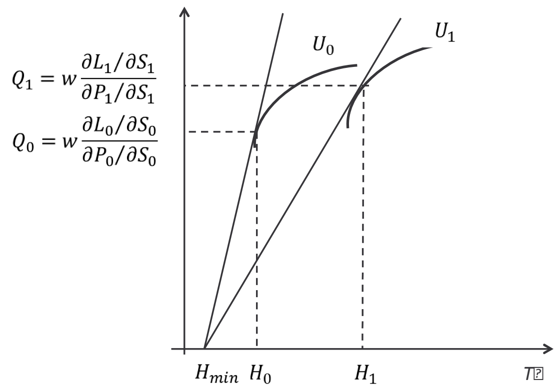

Figure 3. Leisure model" of search behavior and Veblen effect

Coming back to the key equation of the model (Eq.1) we can see that the "leisure model" of behavior (*∂L/∂S<-1; ∂H/∂S>0; ∂²L/∂S∂H<0*) can be produced not only by luxury items with corresponding high absolute values of price reductions |*∂P/∂S*|. We can also see that the value of the lower wage rate can produce the same effect even on almost perfect markets with low absolute values of price reductions, but with very important quantities to be purchased, or *w*<|*Q∂P/∂S*|. This consideration brings the Veblen effect with the Giffen case.

In addition, the "leisure model" of behavior can enlarge our understanding of many economic phenomena. For example, the need to compensate the negative utility of a luxury item by the increase in leisure time, i.e., by the increase in time and/or in intensity of consumption, when a boat is "depreciated" by the number of trips or miles, illustrates the phenomenon of sunk costs sensitivity. But the most intriguing application of the "leisure model" of behavior is the explanation of the phenomenon of money illusion.

4. Money flexibility and money illusion

If we come back to the key equation of the search model and to its elasticity form with respect to the wage rate, we can see that the proportional increase in wages *w* and in price reductions |*∂P/∂S* | doesn't exclude the possibility of the simultaneous increase in both consumption *Q* and propensity to search |*∂L/∂S*|:

$$Q\left|\frac{\partial P}{\partial S}\right| = w\left|\frac{\partial L}{\partial S}\right| \Rightarrow e_{Q,w} + e_{|\partial P/\partial S,w|} = 1 + e_{|\partial L/\partial S|,|w}$$

$$e_{Q,w} + 1 = 1 + e_{|\partial L/\partial S|,|w} (22)$$

If we re-arrange both the Eq.(13) and the Eq.(17), we have:

$$e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} = e_{|\partial P/\partial S|,w} - 1 = e_{|\partial L/\partial S|,w} - e_{Q,w} = 0 \quad (23)$$

If we take the "common model" of behavior *(-1<∂L/∂S<0*), presented graphically by the Cobb-Douglas utility function (Figure 1), where both consumption and leisure are normal goods, we can see that the Eq.(23) takes place only when *∂Q/∂w=0* and *∂*|*∂L/∂S*|*/∂w=0*. Indeed, the derivative *∂*|*∂L/∂S*|*/∂w* is negative in the "common model of behavior. The wage growth reduces *the propensity to search* and the search itself, because the latter becomes less attractive. However, the decrease in the time of search is greater than the increase in the labor time, because for the "common model" of behavior we have $dL(S)+dS+dH(S)=0$ where $\partial L/\partial S<0$ and $\partial H/\partial S<0$. And with the help of the Eq. (4.4) we can confirm the negative of the derivative $\partial|\partial L/\partial S|/\partial w$.

This consideration tells us that the phenomenon of money illusion doesn't take place in the "common model" of behavior. The proportional increase in wages w and in price reductions ($e_{|\partial P/\partial S|.w}=1$) changes neither the consumption nor the time allocation ($\partial Q/\partial w=0;\ \partial|\partial L/\partial S|/\partial w=0$), because the negative value $\partial|\partial L/\partial S|/\partial w<0$ makes its elasticity with respect to the wage rate $e_{|\partial L/\partial S|.w}$ also negative. And the Eq. (23) takes place only when $\partial Q/\partial w=0$ and $\partial|\partial L/\partial S|/\partial w=0$, of course, if consumption and leisure are not inferior.

But this is not true for the "leisure model" of behavior. There, the values $\partial|\partial L/\partial S|/\partial w$ and $e_{|\partial L/\partial S|.w}$ become positive, because now both the time of search and the time of leisure are increased by the important decrease in the labor time. So, the increase in the search time is less than the decrease in the labor time. But this means that the Eq. (23) takes place in the "leisure model" of behavior for any proportional increase in consumption Q and in propensity to search $|\partial L/\partial S|$. However, the increase in propensity to search in the "leisure model" means the *increase in leisure time*. So, the unit price reduction elasticity ($e_{|\partial P/\partial S|.w}=1$) actually can be produced by an ambitious choice of an item ($\partial|\partial P/\partial S|/\partial w>1$) in the framework of the "leisure model" of behavior ($w<|\partial P/\partial S|;\ \partial L/\partial S<-1;\ \partial H/\partial S>0;\ \partial^2 L/\partial S\partial H<0$). And the phenomenon of money illusion takes place.

It can take place either when a suit makes parties "reasonable", i.e., when parties are "bad", and the price of the suit becomes the price for the "disposal of bad leisure", or when parties make "reasonable" the purchase of the suit, i.e., when the suit is "bad". In the case of the "bad" suit the "luxury model" of behavior changes not only the sigh of the Lagrangian multiplier λ. It also changes signs of both $e_{\lambda,P}$ and $e_{\lambda,w}$ values in the Eq.(23), however, without prejudice to the Eq.(23) itself.

It seems that the "leisure model" of behavior is the result of large money balances, when the marginal usefulness of money becomes negative (Friedman 1969, pp.17-18, Walsh 2003, p.45). However, the Eq. (23) pays attention not to the absolute value of wealth but to its relative value. Poor people can also suffer from money illusion, because they can also follow the "leisure model" of behavior as well as the Veblen effect (Moav and Neeman 2012). When "public is unwilling to hold securities at interest rates lower than those corresponding to a floor level" (Fellner 1956, p.949), people couldn't wait the restoration of the aggregative equilibrium with a fall of the general price level. On the contrary people could start to buy luxury items.

Conclusion

When market environment progressively relaxes budget constraints, the development of Stigler's rule of the equality of marginal costs of search and its marginal benefit becomes more and more important in the understanding of consumer behavior. The general approach to the elasticity of the marginal utility of money (Eq.17) can be developed in two ways. First, we can use the key equation of the search model (Eq.1) in order to show the relationship between the marginal utility of money and income elasticity of both consumption and leisure for the "common model" of behavior in the following form:

$$e_{\lambda,|\partial P/\partial S|}+e_{\lambda,w}+1-e_{|\partial P/\partial S|,w}=0$$

$$1-e_{|\partial P/\partial S|,w}=e_{Q,w}-e_{|\partial L/\partial S|,w}=e_{Q,w}+\frac{H}{L+S}e_{H,w}$$

$$e_{\lambda,|\partial P/\partial S|}+e_{\lambda,w}+e_{Q,w}+\frac{H}{L+S}e_{H,w}=0 \quad (24)$$

This equation can be useful in field studies of the elasticity of the marginal utility of income in public economics as well as for studies of optimal taxation and for measurements of inequality.

Second, if we presuppose that the price reduction itself is unit elastic or $e_{|\partial P/\partial S|,P} = 1$, we can re-write the Eq. (17) in the following form:

$$e_{\lambda,P} + e_{\lambda,w} = e_{P,w} - 1 \quad (25)$$

This equation can give us the better understanding of many macroeconomic phenomena, money illusion, for example.

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ARE LARGE INNOVATIVE FIRMS MORE EFFICIENT?

Rosario SÁNCHEZ-PÉREZ
University of Valencia, **Spain**
Rosario.sanchez@uv.es
M. Ángeles DÍAZ-MAYANS
University of Valencia, **Spain**
angeles.diaz@uv.es

Abstract:

Size is one of the factors that condition the managerial organization of the firms and their efficiency and productivity. Moreover size has been found a highly significant variable in explaining differences in firm's innovative activities and the returns of R&D expenditures, and it is a well-established connection between productivity and innovative activities.

This paper analyses the relationship between innovative activities and size and their effect over firms' technical efficiency and then over their productivity. The analysis takes, also, into account other variables that could affect the relationship between productivity and innovative activities: industrial sector, market structure, or firms' financial conditions. We use a micro panel data set of Spanish manufacturing firms, during the period 2004–2009, to simultaneously estimate a stochastic frontier production function and the inefficiency determinants. The data source is published in the Spanish Industrial Survey on Business Strategies (Encuesta sobre Estrategias Empresariales, ESEE), collected by the Fundación SEPI. Our results show that innovative firms are more efficient than non-innovative firms; and that small and medium-sized firms' tent to be more efficient than large firms are.

Keywords: size, firms, technical efficiency, productivity, innovative activities, R&D expenditures.

JEL Classification: D24, L25, L60, O25.

1. Introduction

Our objective is to analyze if differences in efficiency could be explained by differences in innovative activities and if size have a significant impact on the returns of R&D expenditures. One of the characteristics of the Spanish economy is the high percentage of small and medium-sized firms. So, it is important to understand if size has a significant effect on the effectiveness of the R&D expenditure and then, on the effectiveness of the undertaken product or process innovation. Our analysis could help to design political economic measures to encourage small firms' innovation and then contribute to improve their competitiveness.

We use a micro panel data set to simultaneously estimate a stochastic frontier production function and the inefficiency determinants using an unbalanced panel of manufacturing firms. We analyze, firstly, if innovative firms are more technical efficient than non-innovative firms and finally if large firms obtain more returns from their investment on R&D. We obtain that R&D intensity is a significant determinant of efficiency for large firms but not for small companies. Moreover, capital intensity is more important for small firms than innovative activities. Then, it seems that small firms find more difficult to obtain benefits from their R&D expenses than large firms.

2. Size, innovation and technical efficiency

There is an extensive literature that analyses the effect of innovation on productivity. Also, the effect of size on innovation activities has been largely analyzed. Size has been found one of the factors that explain firms' differences in innovation activities and in the returns on R&D expenditures. Most studies found that large firms are more innovative than the small and medium sized firms. Large firms could benefit from scale economies, more qualified work force, and better access to external financial funds and better capacity to exploit an innovation and expand the new production. Some empirical papers showed that, to a threshold point, there is a linear relationship between R&D expenditures and size. Large firms innovate more and obtain higher returns from their investment. Other studies consider that new small firms are more innovative, as a way to quickly raise their size and survive. The Winter's hypothesis (Winter 1984), that innovation activities respond to different technological regimes and differences in the economic environment, has obtained empirical support as in Acs & Audretsch (1990).

We follow the frontier approach, first developed by Farrell (1957) and widely used in empirical works. This approach measures the technical inefficiency of a production unit as the ratio of a firm's production over its optimal level. The optimal behavior, the technically efficient result of the production process, is represented by a production function, a frontier, which shows the maximum level of output a firm could achieve, given the technology and a given level of inputs. The first step of this approach is to estimate the practice frontier obtained from the sample information, using their best observations. If a firm produces this optimal level of output, it is technically efficient and it will be on the frontier. If a firm produces less than is technically feasible, given both, the technology and a level of inputs, it is inefficient and we can measure the degree of technical inefficiency as the distance from each individual observation and a corresponding point on the frontier. Using frontier techniques, several studies have analyzed which are the sources of technical inefficiency. Caves and Barton (1990) examine technical inefficiency of the manufacturing industry in United States, while Green and Mayes (1991) analyze technical inefficiency for United Kingdom firms. Caves *et al.* (1992) compare inefficiency and its determinants between developed countries. Other studies focus on particular determinants of inefficiency, such as the Hay and Liu study (1997), which focuses on the relevance of a competitive environment on efficiency; Patibandla (1998), who shows the relevance of capital market imperfections on the structure of an industry; and Dilling-Hansen et al. (2003), who analyzed whether relative efficiency is due to R&D investment. Díaz and Sánchez (2008) obtain that small and medium-sized firms tend to be more efficient than the large firms are.

3. Stochastic frontier and the inefficiency model

We use the SFA to estimate a production frontier with inefficiency effects. Specifically, we use a panel data version of the Aigner *et al*. (1977) approach, following Kumbhakar and Lovell (2000), and Wang (2002) specification, in which technical inefficiency is estimated from the stochastic frontier and simultaneously explained by a set of variables representative of the firms' characteristics. This approach avoids the inconsistency problems of the two-stage approach used in previous empirical works when analyzing the inefficiency determinants[30].

The model can be expressed as:

$$Y_{it} = f(X_{it};\beta)\exp(v_{it} - u_i) \tag{3.1}$$

where i indicates firms and t represents the period, X is the set of inputs; β is the set of parameters, v_{it} is a two-sided term representing the random error, assumed to be *iid* $N(0,\sigma_v^2)$; u_i is a non-negative random variable representing the inefficiency, which is assumed to be distributed independently and obtained by truncation at zero of $N(0,\sigma_u^2)$.

We introduce some explanatory variables to explain inefficiency assuming that,

$$\sigma^2_{(u)_i} = \sigma^2_{(u)} \exp(\delta' Z) \tag{3.2}$$

where Z is a (Mx1) vector of variables that may have effects over firm efficiency, δ is a (1xM) vector of parameters to be estimated. We also control for heteroscedasticity, allowing the noise term to reflect differences between firms related to size.

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30 In a two-stage procedure, first of all a stochastic frontier production function is estimated and the inefficiency scores are obtained under the assumption of independently and identically distributed inefficiency effects. But in the second step, inefficiency effects are assumed to be a function of some firm-specific variables, which contradicts the assumption of identically distributed inefficiency effects.

$$\sigma^2_{(v)it} = \sigma^2_{(v)} \exp(\gamma' w) \qquad (3.3)$$

Given that technical efficiency is the ratio of observed production over the maximum technical output obtainable for a firm (when there is no inefficiency), the efficiency index (TE) of firm *i* in year *t* could be written as[31]:

$$TE = \frac{f(X_{it};\beta)\exp(v_{it} - u_i)}{f(X_{it};\beta)\exp(v_{it})} = \exp(-u_i) \qquad (3.4)$$

The efficiency scores obtained from expression (3.4) take value one when the firm is efficient, and less than one otherwise.

4. Data and variables

The Data source is published in the Spanish Industrial Survey on Business Strategies *(Encuesta sobre Estrategias Empresariales, ESEE).* The data is collected by the *Fundacion Empresa Pública* (FEP) and sponsored by the Spanish Ministry of Industry. This is supplied as a panel of firms' representative of twenty manufacturing sectors. A characteristic of the data set is that firms participating in the survey were chosen according to a selective sampling scheme. The sample of firms includes almost all Spanish manufacturing firms with more than two hundred employees. Firms employing between ten and two hundred employees were chosen according to a stratified random sample representative of the population of small firms. Given the procedure used to select firms participating in the survey, both samples of small and large firms can be considered representative of the Spanish firms' population. Each year a number of additional firms were selected according to a random sampling procedure among the whole population of firms. This selection is conducted using the same proportion as in the original sample (see Fariñas and Jaumandreu, 2004) for technical details of the sample)

From the original sample, a number of firms have been eliminated, most of them due to a lack of relevant data. Others were eliminated because they reported a value-added annual growth rate per worker in excess of 500% (in absolute value), and some were rejected because they have fewer than ten workers and, in both cases, they would distort the analysis. Also, we do not include firms after a merger or division process in our sample data. Our sample includes 2,247 firms from the ESEE Survey and refers to an unbalanced panel where we have eliminated those firms for which we do not have two consecutive years of data. Our period of analysis runs from 2004 to 2009. Summary statistics of the data are presented in Table 1.

Table 1. Descriptive statistics

	Min.	*Max*	*Mean*	*Standard Deviation*
*VA**	*110.29*	*10689161.42*	*162610.05*	*553841.99*
*K**	*10.94*	*33091212.35*	*357083.77*	*1609312.16*
L	*10.00*	*14400.00*	*236.90*	*724.36*
INP	*0.00*	*1.00*	*0.20*	*0.40*
INPR	*0.00*	*1.00*	*0.32*	*0.47*
Investment over capital	*0.00*	*4.58*	*0.07*	*0.14*
External funds over VA	*0.00*	*209.39*	*2.31*	*5.82*
Proportion of temporary	*0.00*	*0.97*	*0.13*	*0.17*
Innovation investment over capital	*0.00*	*3.64*	*0.02*	*0.10*
R&D expenditures	*0.00*	*4152551.57*	*11367.04*	*122314.54*

(*) Euros

[31] Individual efficiency scores u_i, which are unobservable, can be predicted by the mean or the mode of the conditional distribution of u_i given the value of (v_i-u_i) using the technique suggested by Jondrow et al (1982).

We estimate a stochastic translog production function adding a term of inefficiency, whose variance is the function of a set of inefficiency determinants[32].

$$\ln Y_{it} = \beta_0 + \sum_{j=1}^{J} \beta_j \ln X_{ijt} + \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{K} \beta_{jk} \ln X_{ijt} \ln X_{ikt} + \sum_{m=1}^{M} \varphi_m S_{im} + \theta_1 INP + \theta_2 INPR + v_{it} - u_i$$
$$\sigma^2_{(u)_i} = \sigma^2_{(u)} \exp(\delta' Z) \quad (4.1)$$

The variables used for estimation of the production frontier are the value-added, such as the output variable, and the number of employees in the firm, capital stock and trend, as input variables (X_{it}), the industrial sector dummies (S_i) and two dummies that indicate if firms have undertaken process (INPR) or product innovation (INP). Here we present a more precise definition of the variables used for estimation and the definition of the inefficiency determinants considered:

Variables of Stochastic Frontier estimations:

- VA: The value added in real terms. This is a dependent variable;
- CAPITAL STOCK (K): Inventory value of fixed assets excluding grounds and buildings;
- L: Total employment by firm;
- T: This is the time trend;
- INP: dummy that takes value 1 if there is product innovation and 0 otherwise;
- INPR: dummy that takes value 1 if there is process innovation and 0 otherwise.

Sector classification: There are seven dummy variables that take value one when the firm belongs to the corresponding sector of activity; otherwise this value is zero.

- SEC1: Meat and manufacturing of meat; food industry and tobacco drinks; textiles, clothing and shoes; leather, shoes and derivatives;
- SEC2: Wood and derivatives, paper and derivatives;
- SEC3: Chemical products; cork and plastic; non-metallic mineral products;
- SEC4: Basic metal products; manufactured metal products; industrial equipment;
- SEC5: Office machinery and others; electrical materials;
- SEC6: Cars and engines; other material transport;
- SEC7: Other manufactured products.

Determinants of efficiency:

- PROPORTION OF TEMPORARY: This is the proportion of temporary workers on total employment;
- INVESTMENT OVER CAPITAL: This is the ratio between investment expenditure in capital goods over capital;
- INNOVATION INVESTMENT OVER CAPITAL: This is the ratio between costs of purchase of capital goods for product improvement over capital;
- R&D INTENSITY: This is the ratio between R&D expenditures over Value added;
- EXTERNAL FUNDS OVER VA: This is the ratio between external total funds over added value;
- SIZE: There are six dummy variables that take value one when the firm belongs to the corresponding interval of workers, zero otherwise:
  - SIZE 1: Firms with no more than twenty workers;
  - SIZE 2: from 21 up to 50;
  - SIZE 3: from 51 up to 100;
  - SIZE 4: from 101 up to 200;
  - SIZE 5: from 201 up to 500;
  - SIZE 6: Firms with a number of workers higher than 500.

[32] We imposed the usual symmetry conditions to the translog function.

4. Empirical results

From the frontier approach, we obtain a measure of a firm's technical inefficiency compared with the best observations of the sample. The value of the estimates allows us to explain the differences in the inefficiency effects among firms. As technological and market conditions can vary over sectors, we have included sector dummy variables in the production function in order to be able to control them.

The maximum-likelihood estimates of the production frontier parameters, given the specification for the inefficiency effects, defined in equation (4.1), are presented in Table 2. We use the translog specification for the production function and we obtain the expected signs of the inputs estimates. We also get that both dummies representing firms' innovative activities have a positive and statistically significant coefficient.

Respect to the inefficiency determinants, our results show that inefficiency tends to be larger for firms with a high ratio of external financial funds over total assets. As higher is the leverage more difficult is for firms to be close to the frontier. The ratio of temporary over total employment shows, also, a negative impact over efficiency. Díaz and Sánchez (2004) obtained that a higher number of temporary workers in manufacturing firms affects negatively their technical efficiency because firms do not invest in training in this type of workers.

We find a negative and significant relationship between size and technical efficiency. There are at least two explanations for expecting a negative relationship between size and efficiency. First, large firms may suffer more from bureaucratic frictions, lack of motivation of workers, and difficulty in monitoring than smaller firms. Second, large firms are more able to remain in the market, even if they have economic problems due to a low technical efficiency, than small firms because of the existence of market imperfections. Due to this effect of market selection, the surviving small firms that we observe may on average show a higher level of technical efficiency than the larger firms do.

The R&D intensity, affects positively the firm's efficiency, that is, innovative firms tend to be closer to the frontier than those firms that do not perform R&D spending. We obtain the same significant effect for variables representing the degree of investment. These results allow us to conclude that the most innovative companies are closer to the efficient frontier than those that are not innovative

When we estimate two separate frontiers, for small and large companies we observe interesting differences. R&D intensity is a relevant determinant of efficiency for large firms but not for small companies. Moreover, capital intensity is more relevant for small firms than innovative activities. Then, it seems that for small firms it is more difficult to obtain benefits from their R&D expenses than for large firms.

Table 2. Stochastic Frontier Analysis

Translog Production function estimates				
Variables		**Coefficient**	**Standard- Error**	**T- Student**
Constant	β_0	5.883	0.142	41.302
T	β_1	0.146	0.018	7.971
L	β_2	1.074	0.050	21.592
K	β_3	-0.110	0.020	-5.508
K^2	β_{11}	0.042	0.002	23.894
L^2	β_{22}	0.076	0.007	11.661
T^2	β_{33}	-0.013	0.002	-6.824
KxL	β_{12}	-0.195	0.0130	-14.939
LxT	β_{13}	0.025	0.004	6.348
KxT	β_{23}	-0.019	0.003	-7.305

Translog Production function estimates				
Variables		**Coefficient**	**Standard- Error**	**T- Student**
INP	θ_1	0.025	0.015	1.681
INPR	θ_2	0.034	0.012	2.980
Wood and derivatives, paper and derivatives.	φ_1	-0.066	0.025	-2.651
Chemical products; non-metallic mineral products.	φ_2	-0.192	0.0045	-4.301
Basic metal products; industrial equipment.	φ_3	0.044	0.029	1.541
Office machinery and others; electric materials.	φ_4	0.095	0.025	3.754
Cars and engines; other material transport.	φ_5	0.177	0.034	5.167
Others manufactured products.	φ_6	0.053	0.041	1.301
Inefficiency model				
Variables		**Coefficient**	**Standard- Error**	**T- Student**
Investment over capital	δ_1	-1.643	0.228	-7.197
R&D intensity	δ_2	-0.186	0.039	-4.818
External funds over VA	δ_3	0.021	0.023	9.058
Proportion of temporary	δ_4	0.376	0.165	2.282
Innovation investment over capital	δ_5	-2.682	0.658	-4.078
Size1: Up to 20 workers	δ_7	-0.917	0.142	-6.473
Size2: From 21 to 50	δ_8	-0.924	0.148	-6.227
Size3: From 51 to 100	δ_9	-1.008	0.135	-7.488
Size4: From 101 to 200	δ_{10}	-0.930	0.133	-7.015
Size5: From 201 to 500	δ_{11}	-0.800	0.140	-5.719
Heteroscedasticity				
L	γ_1	0.000	0.000	0.324
INP	γ_2	-0.033	0.227	-1.209
INPR	γ_3	0.009	0.020	0.462

To sum up, the impact of the investment in R&D over efficiency and consequently over production has been positive and statistically significant. Our results indicate that innovative firms produce more efficiently than non-innovative firms. This implies that all policies conducted to incentive this kind of investment will contribute to a productivity growth in the long run.

Table 3. Large firms' inefficiency model

	Coefficient	**Standard error**	**t-Student**
Investment over capital	-3.727	0.564	-6.613
R&D intensity	-0.237	0.057	-4.190
External funds over VA	0.020	0.004	4.587
Proportion of temporary	-0.904	0.191	-4.726
Innovation investment over capital	2.681	1.634	1.641

Table 4. Small firms' inefficiency model

	Coefficient	Standard Error	T-Student
Investment over capital	-1.800	0.618	-2.913
R&D intensity	-0.128	1.088	-0.117
External funds over VA	0.050	0.008	6.039
Proportion of temporary	0.589	0.512	1.149
Innovation investment over capital	-1.835	2.035	-0.902

Conclusions

We have analyzed the impact of corporate R&D activities on firms' technical efficiency and whether large companies are more successful in achieving efficiency gains from R&D activities. Then we estimated firstly a frontier using the whole sample and, secondly, two different frontiers: one for large firms and another for small and medium sized companies. From the first estimation we obtained that R&D investment are statistically significant and their negative sign indicate that they had a positive effect over efficiency. This means that innovative firms operate closer to the frontier. We also found a negative and significant link between size and technical efficiency. When estimating two separated frontiers we appreciate, inside each homogeneous group, which are the more relevant factors to reach a higher degree of efficiency. Thus we have obtained that for small firms the intensity of capital is the significant factor to acquire a higher level of technical efficiency while large firm obtained more gains in efficiency from the R&D investment.

The question is that even though small firms face additional difficulties to obtain product and process innovation they are more technically efficient. One of the reasons could be that small and large companies face different technological and environmental regimes.

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BOUNDED RATIONALITY: PSYCHOLOGY, ECONOMICS AND THE FINANCIAL CRISES

Daniele SCHILIRÒ
Department SEAM, University of Messina, **Italy**
schi.unime@katamail.com

Abstract:

This contribution focuses on the concept of bounded rationality, highlighting the role of psychology in the economic decisions. The work analyzes Simon's approach and his notion of bounded rationality as procedural rationality. Moreover, it examines some major contributions of behavioral economics concerning cognitive biases, stressing the importance of the institutional structure in the decision process. The paper also surveys the literature of behavioral finance which has become fashionable in explaining the anomalies of financial markets, pointing out also its limits.

Keywords: Bounded rationality, rational choice, behavioral economics, behavioral finance, risk aversion.

JEL Classification: C60, B52, D81, D83.

1. Introduction

The present work criticizes the classical theory of rational choice pointing out its failures, highlighting the approach that seeks to combine economics and psychology. In particular, this contribution first analyzes the concept of bounded rationality devised by Herbert Simon. In Simon's view the rationality of the individual is bounded, since the quality of information used is poor and the cognitive capacity of the individual is limited. So the individual can make decisions that appear irrational from the perspective of conventional economic wisdom. However, these decisions are typically the right ones for the individual making them. Moreover, the paper analyzes the approach of behavioral economics which has grown in importance since the seventies.

This approach marks a return to reality from the rational optimizing model as the only framework for economics, and it also underlines that the human actions are heavily influenced by frames of reference. Thus behavioral economics maintains that institutional structure that individuals have is the basic framework for all of our economic decisions. Finally, the work refers to the literature of behavioral finance which has become widespread in explaining the behavior and the anomalies of financial markets and its crises, pointing out also its limits.

2. 'Perfect' rationality and expected utility theory

Rationality in neoclassical economics is represented by perfect rationality and it is interpreted in terms of consistency not of substance. The agents are rational if they have a coherent criterion of choice. The consistency of the choices implies that the agents are represented by a system of preference. Economics describes the choice as a rational process driven by a single cognitive process that includes the principles of the theory of rational choice and it orders the decisions on the basis of their subjective expected utility. In this view the individual has a complete knowledge and is fully rational, while his economic choices, guided by his perfect rationality, are self-contained in the economic sphere without affecting other aspects, such as the emotions, or being influenced by the environment[33].

2.1. The expected utility theory

A cornerstone of the classical theory of rational choice is the expected utility theory, which deals with the analysis of choices among risky alternatives.

[33] Hogarth and Reder (1986) argue that the paradigm of rational choice provides economics with a unity that is lacking in psychology.

Von Neumann and Morgenstern (1944) proposed their analysis of choice under uncertainty, which depends on strong assumptions of a psychological nature. The rationality is represented by the maximization of the expected utility that is a criterion that facilitates choice under risk.

According to von Neumann and Morgenstern, individuals generally move in the reality following predetermined patterns of behavior, at the base of which there is the assumption that they always prefer to have a greater wealth than less. The theory studies the preferences underlying consumer behavior under risk, i.e. when the subject is asked to make a decision without knowing with certainty which *ex ante* state of the world will happen, but he knows the probability distribution, that is, it is known to him a list of possible events, each of which he associates a probability of occurrence. This theory assumes that each individual has stable and consistent preferences, and that he makes decisions based on the principle of maximization of subjective expected utility. So given a set of options and beliefs expressed in probabilistic terms, it is assumed that the individual maximizes the expected value of a utility function u(.). The individual uses probability estimates and utility values as elements of calculation to maximize his expected utility function. Thus he evaluates the relevant probabilities and utilities on the basis of his personal opinion but also using all relevant information available.

Von Neumann and Morgenstern have proposed a well-known theorem in which they make the construction of an expected utility function possible. Any individual acting to maximize the expectation of a function u() will obey to four axioms, which are: completeness, transitivity, continuity, and independence[34]. The first two axioms, completeness and transitivity, require respectively that an individual has well defined preferences, which are therefore complete, and that preference is consistent across any three options, so the consistency requirement reminds us that intransitive preferences lead to irrational behavior.

The von Neumann-Morgenstern theorem is also based on a third axiom of continuity which states that the preferences of rational agents are ordered and without points of discontinuity. This axiom implies that for each P, Q, R $\in$ ɪ, if the lottery P is preferred to Q and Q to R, then exist $\alpha, \beta \in (0,1)$ such that you can construct a linear combination of P and R for which

$$\alpha P + (1-\alpha) R \succ Q \succ \beta P + (1-\beta) R$$

The fourth axiom of independence is crucial and it assumes that a preference holds independently of the possibility of another outcome. For each P, Q, R $\in$ ɪ, if P $\succ$ Q $\succ$ R, and for each $\alpha \in (0,1)$

$$P \succsim Q \Leftrightarrow \alpha P + (1-\alpha) R \succsim \alpha Q + (1-\alpha) R$$

The expected utility theory has been generally accepted as a normative model of rational choice, defining which decisions are rational. If an individual does not maximize his expected utility he is designed to violate in his choices some precise axiomatic principles, which are rationally binding. This theory has also been applied as a descriptive model of economic behavior (Friedman and Savage, 1948; Arrow, 1971) so as to constitute an important reference model for economic theory. Thus the standard idea of rationality in economics is represented by the maximization of subjective expected utility, which is "a combination of von Neumann-Morgenstern preferences and a Bayesian belief structure" (Kahneman, 2003, p.163).

[34] The expected utility function can take three forms: is concave when describing the preferences of a risk averse individual; it is convex type when describing the preferences of an individual willing to risk; it is linear when describing the preferences of a risk-neutral individual. In the von Neumann-Morgenstern framework, we can define individual's attitudes towards risk without making any prior assumptions about his behavior.

3. Psychology and economics: a challenging relationship

During the fifties there have been important explorations along the boundaries between economics and psychology. In particular, experimental psychology, concerned with the study of actual behavior and aware of the complexity of choices, had highlighted the systematic (and unconscious) divergence of human behavior from the postulates of economic rationality. Some economists using experimental results questioned the validity of the classical model of rational choice (Simon, 1959). Thus a new line of research, called behavioral economics, started to be developed, trying to relate psychological factors to economic behavior (Rabin, 1998). One important contribution came from Herbert Simon's approach that developed the notion of bounded rationality and the problem solving. Bounded rationality, in particular, depends – according to Simon (1972) – on the limits of attentive and computational capacity. Thus, he gave start to an approach based on the heuristics that are interpreted as a trade-off between the limits of the human mind and the computing performance required by complex problems. Simon's concept of bounded rationality can be interpreted – according to Kahneman (2003) – as defining a realistic normative standard for an organism with a finite mind. Simon essentially criticized – on the basis of analysis conducted on the field – the lack of realism of the neoclassical economic theory based on the assumption of full rationality. Another major contribution came from the pioneering experimental studies of Allais (1953), which have given a boost to the cognitive economic approach. Allais was investigating on the question if preferences are consistent with each other and with the axioms of rational choices. Allais' studies demonstrated that preferences of individuals violate expected utility theory, so he proved the systematic discrepancy between the predictions of traditional decision theory and actual behavior. The results of laboratory experiments conducted by Allais have shown that individuals chose inconsistently and that they preferred solutions which did not maximize the expected utility. In this way Allais has demonstrated that the axiomatic definition of rationality did not allow describing or even predicting economic decisions[35].

Later, Ellsberg (1961) identified another paradox. He demonstrated another type of inconsistency in preferences, showing that individuals prefer to bet on a lottery with a chance of obtaining a win already known that on a lottery with ambiguous results. This aversion to uncertainty (ambiguity) of the individual is completely ignored in the expected utility model from a descriptive point of view, while is not considered acceptable from a normative point of view.

Other challenges to utility theory and to the inconsistency in preferences came from framing effects by Tversky and Kahneman (1981).

3.1. Bounded rationality

Herbert Simon[36] proposed the idea of bounded rationality as an alternative basis for the mathematical modeling of decision making. Simon has coined the term 'bounded rationality' in *Models of Man* (1957). In his view, rationality of individuals is limited by the information they have, the cognitive limitations of their minds, and the finite amount of time they have to make decisions. Bounded rationality expresses the idea of the practical impossibility (not of the logical impossibility) of exercise of perfect (or 'global') rationality (Simon, 1955). Simon argues that most people are only partly rational while are emotional/irrational in the remaining part of their actions. He maintains that, although the classical theory with its assumptions of rationality is a powerful and useful tool, it fails to include some of the central problems of conflict and dynamics which economics has become more and more concerned with (Simon, 1959, p.255). Simon identifies a variety of ways to assume limits of rationality such as risk and uncertainty, incomplete information about alternatives, complexity (1972, pp.163-164). Furthermore, he asserts that an individual who wants to behave rationally must consider not only the objective environment, but also the subjective environment (cognitive limitations); thus you need to know

[35] Maurice Allais presented in Paris, in 1952, his famous paradox to an audience composed of the best economist of his generation; among others, Kenneth Arrow, Paul Samuelson, Milton Friedman, Jacob Marschak, Oskar Morgenstern and Leonard Savage.

[36] Simon (1955, 1956, 1957, 1972, 1979, 1982, 1997).

something about the perceptual and cognitive process of this rational individual. Simon, therefore, considers the psychological theory very important to enrich the analysis for a description of the process of choice in economics. This is why he adopts the notion of procedural rationality, a concept developed within psychology (Simon, 1976, 1997), which depends on the process that generated it, so rationality is synonym of reasoning. According to Simon (1976, p.133), a search for procedural rationality is the search for computational efficiency, and a theory of procedural rationality is a theory of efficient computational procedures to find good solutions. Procedural rationality is a form of psychological rationality which constitutes the basic concept of Simon's behavioral theory (Novarese, Castellani, Di Giovinazzo, 2009; Barros, 2010, Graziano, Schilirò, 2011; Schilirò, 2012), in contrast to economic rationality, defined by Simon as 'substantive rationality'.

Another way to look at bounded rationality is that, because individuals lack the ability and resources to arrive at the optimal solution, they instead apply their rationality only after having greatly simplified the choices available. Actually, individuals face uncertainty about the future and costs in acquiring information in the present. These two factors limit the extent to which agents can make a fully rational decision. Thus – Simon claims – agents have only bounded rationality and are forced to make decisions not by 'maximization', but rather by *satisficing*, i.e. setting an aspiration level which, if achieved, they will be happy enough with, and if they don't, they try to change either their aspiration level or their decision. "The limits of human cognitive ability for discovering alternatives, calculating their outcome and making comparison may lead the decision maker to settle for some satisficing strategy" (Simon, 1982). *Satisficing* is the hypothesis that allows to the conception of diverse decision procedures and which permits rationality to operate in an open, not predetermined, space (Barros, 2010). Real-world decisions are made using fast heuristics, 'rules of thumb', that *satisfice* rather than maximize utility over the long run. Therefore agents use the heuristics to make decisions rather than a strict rigid rule of optimization. The agents do this because of the complexity of the situation, and their inability to process and compute the expected utility of every alternative action. In fact, there are limits in the attentive, mnemonic and computational capacity binding the computational load, hence the usefulness of automatic routines. Rationality is bounded by these internal constraints in the uncertain real world. Simon then relates the concept of bounded rationality to the complementary construct of procedural rationality, which is based on cognitive processes involving detailed empirical exploration and procedures ("search processes") that are translated in algorithms. This is in contrast to the notion of perfect rationality, that is based on substantive rationality, which derives choices from deductive reasoning and from a tight system of axioms; an idea of rationality that has grown up strictly within economics (Simon, 1976, 1997).

Simon does not reject completely the neoclassical theory in fact he describes a number of dimensions along which neoclassical models of perfect rationality can be made somewhat more realistic, while sticking within the vein of fairly rigorous formalization. These include: limiting what sorts of utility functions there might be, recognizing the costs of gathering and processing information, the possibility of having a "multi-valued" utility function. However, although bounded rationality offers an alternative in the form of multi-level utility, the problem with bounded rationality is that it lacks mechanisms of comparison between alternatives.

Simon's work has been followed in the research on judgment and decision making, both in economics and psychology. Two major approaches produced important insights into perception mechanisms shaping the individual's internal representation of the problem: the ''heuristics and biases" program (Kahneman, Tversky (K&T), 1972; Tversky, Kahneman (T&K), 1974)[37], which has been

[37] The literature on bias evidence is quite large. As stated by Conlisk (1996, p. 672): "the evidence suggests that the magnitude and nature of the errors are themselves systematically related to economic conditions such as delibaration cost, incentives, and experience. In this sense, investigation of bounded rationality is not a departure from economic reasoning, but a needed extension of it".

fundamental to the contemporary development of behavioral economics[38]. The other approach, derived from Simon's work, is the ''fast and frugal heuristics" program (Gigerenzer, Goldstein, 1996; Todd, Gigerenzer, 2003).

Tversky and Kahneman (K&T, 1972; T&K, 1974, 1983) published a series of experiments in which they demonstrate significant deviations from the Bayesian theory of judgment under uncertainty. In a major article "Judgment under uncertainty: heuristics and biases" published in *Science* they offered a theoretical explanation about the observed deviations from perfect rationality, noting that people rely on "heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations" (T&K, 1974, p.1124). In other words, Tversky and Kahneman were arguing that heuristic short-cuts created probability judgments which deviated from statistical principles. Moreover, these authors explored the psychology of intuitive beliefs and choices and examined their bounded rationality (Kahneman, 2002, p.449). However, Tversky and Kahneman do not abandon the assumption that individuals are intelligent and intentional in making decisions, but they assume systematic and specific biases that move away the judgment from the perfect rationality of individuals. They highlighted that "failures" of perfect rationality depend on the specific ways in which people select and process the information mentally. Kahneman and Tversky (K&T, 1979, 1984; T&K, 1981, 1986) articulated a direct challenge to the rationality assumption itself, based on experimental demonstrations in which preferences were affected predictably by the framing of decision problems, or by the procedure used to elicit preferences[39]. One major conclusion of this alternative approach is that the susceptibility of people to framing effects violates a fundamental assumption of invariance. Kahneman and Tversky (1979, 1984) also argued that any individual has a deformation of the probability, which is different between gains and losses and, moreover, the individual has aversion to losses. A loss, in fact, is more weighted by a psychological point of view than a gain. Consequently, taking into account framing effects, aspects like loss-aversion, money illusion, etc. become relevant in strategic decision-making, macroeconomic phenomena and financial decisions, so the model of choice based on perfect rationality with its underlying expected utility theory fails as an adequate descriptive model of choice under risk[40].

Yet, critics have pointed out that behavioral economics is not a unified theory, but it is instead a collection of tools and ideas. This is true. But this is also true of neoclassical economics. However, as Kahneman admitted (2003, p.166), a real problem in economic theory is that models must satisfy the constraint of tractability when the assumptions are set forth. So the models can become complex, but the number of parameters that can be added is small. In addition, theoretical innovation in behavioral economics tends to be noncumulative. Despite this, one of the goals of behavioral economics is to develop better tools, but also normative rules that drive choice in desired directions (Camerer and Loewenstein, 2003; Shiller, 2005).

The other approach, derived from Simon's work, is the ''fast and frugal heuristics" program (Gigenzer, and Goldstein, 1996; Goldstein and Gigerenzer, 2002; Todd and Gigerenzer, 2003), that is in contrast to the theoretical position of Tversky and Kahneman and the theoretical strands of behavioral economics which, showing the distortions of judgment and choice defined as cognitive biases, highlight the negative effects and the errors that these heuristics lead in the behavior and choices of individuals. Fast and frugal heuristics refer to simple, task-specific decision strategies that are part of a decision maker's repertoire of cognitive strategies for solving judgment and decision tasks. Unlike many decision-making models in the behavioral sciences, models of fast and frugal heuristics describe not only the outcome of the decision-making process but also the process itself. Studies on fast and frugal heuristics include (a) the use of analytical methods and simulation studies to explore when and why heuristics

[38] Behavioral economics aims at increasing the realism of the psychological underpinnings of economics analysis for generating theoretical insights, making better predictions and suggesting better policy (Camerer and Loewenstein, 2003).

[39] In their 'Prospect theory' Tversky and Kahneman have shown experimentally the presence of inconsistent judgments and choices by an individual facing the same problem presented in different frames ('invariance of failures'). It follows that the frame, or the context of choice, *coeteris paribus*, helps to determine a different behavior.

[40] However, according to Gennaioli and Shleifer (2010, p.1399): "Although Kahneman and Tversky's heuristics and biases program has survived substantial experimental scrutiny, models of heuristics have proved elusive".

perform well; and (b) experimental and observational studies to explore whether and when people actually use fast and frugal heuristics. Following Simon's notion of *satisficing*, Gigenzer and Goldstein (1996), for example, have proposed a family of algorithms based on a simple psychological mechanism: one-reason decision making. These fast and frugal algorithms violate fundamental tenets of classical rationality: they neither look up nor integrate all information (Gigenzer and Goldstein, 1996). The heuristics are determined by a trade-off between the limits of the human mind and the computing performance required by complex problems[41]. Each individual is a complex system operating in a complex environment interconnected by a system of relationships that change dynamically over time the choices that each individual operates are choices with limited rationality. In the same way, the companies of all kinds are complex systems composed of individuals or complexing agents their collective behavior is in turn influenced by the bounded rationality of its components, making them agents bounded rationality. These considerations (Gigerenzer, Hertwig and Pachur, 2011) give more confirmation that the understanding of the mechanisms underlying the behavior of individuals, agents are critical to the operation of the firm. The use and study of heuristics is therefore crucial to understand and operate in these uncertain, dynamic and highly interconnected contexts. At the same time the version of bounded rationality, coupled with insights from evolutionary psychology, put forward by Gigerenzer and Selten (2001) and by Todd and Gigerenzer (2003), gives a different grasp of the functional role of emotions within the human decision machinery.

4. Behavioral finance

To understand financial markets, traditional finance paradigm use models in which agents are perfectly or fully rational. Agents' rationality means that when they receive new information, agents update their beliefs correctly, in the manner described by Bayes' law. Moreover, given their beliefs, agents make choice that are normatively acceptable, in the sense that they are consistent with the notion of subjective expected utility, implying a preference-maximizing choice, so financial decisions for the rational agent are based on the hypothesis that they calculate their rational advantage and act consistently with that.

According to Simon's bounded rationality, individuals may not have the knowledge base to make ideal choices in finance-related matters. Following this view, financial decision making can be improved by providing individuals with better quality information presented in a non-complex fashion and an institutional environment conducive to good decisions.

Yet, research in psychology have supported the view that emotional reactions to situations involving uncertainty or futurity often differ sharply from cognitive assessments of those situations, and that when such differences occur, it is often the emotional reactions that determine behavior.

From the seventies onwards there has been an increasing interest towards psychological and sociological aspects in the analysis of financial behavior. Then there has been the development of a new branch of finance: behavioral finance, which in itself combines aspects of cognitive psychology and financial theories in the strict sense. Behavioral finance argues that some financial phenomena can plausibly be understood using models in which some agents are not fully rational. One of the building blocks of behavioral finance is psychology. Then behavioral finance has become the study of the influence of psychology on the behavior of financial investors and the subsequent effect on markets.

Behavioral finance is of interest because it helps explain why and how markets might be inefficient. In practice this new approach seeks to explain the so-called financial market "anomalies" by analyzing the deviation of financial agents' behavior from full rationality and optimum choices. A way to overcome the limited or bounded rationality of the agents, also in financial markets, is to make use of rules of thumb. But, as highlighted by Tversky and Kahneman (1974), «people rely on a limited number

[41] Empirical literature has indicated that humans use fast and frugal heuristics especially when under time pressure, when information search is costly or when information has to be retrieved from memory. Gigerenzer and Brighton (2009) maintain that *Homo heuristicus* has a biased mind and ignores part of the available information, yet a biased mind can handle uncertainty more efficiently and robustlythan an unbiased mind.

of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors» (Tversky and Kahneman, 1974, p. 1124)[42]. Thus, the adoption of heuristics by individuals can be necessary to solve the problems of everyday life, but in the financial sector it can lead to biases which have proved very expensive. However, behavioral finance make use of a set of tools that include susceptibility to frames and other cognitive errors, so that the institutional structure becomes relevant, but also tools that imply varying attitudes toward risk, aversion to regret and imperfect self-control.

4.1. Behavioral finance: anomalies and biases

In the reality of financial markets the fact that the price of a stock should coincide with its fundamental value seems to be more the exception than the rule. The "anomalies" in the behavior of prices yields, in contrast to the hypothesis of efficient markets, are numerous and show that the securities are by no means in line with their fundamentals.

The efficient market theory (Fama, 1970, 1991) asserts that financial markets are informationally efficient, that is markets are efficient in the sense of information if at all times the stock prices fully and correctly reflect all the available information[43]. This theory rests on the efficient market hypothesis (EMH), which is based on three theoretical assumptions: i) market agents are perfectly rational and are able to value any security rationally; ii) even if there are some investors who are not rational, their trading activities will either cancel out one another or will be arbitraged away by rational investors; iii) market agents have well defined subjective utility functions which they will maximize. Consequently, the theoretical foundation of the EMH is the subjective utility theory. Thus, according the EMH, no investment strategy can earn average returns greater that are warrented for its risk (Barberis and Thaler, 2002).

Kahneman and Tversky (1973) had already noticed that investors systematically violate Bayes' rule and other maxims of probability theory in predicting uncertain outcomes. In forecasting future uncertain events investors usually focus on recent history and pay less attention to the possibility that such short history could be determined by chance[44].

In the eighties and nineties there have been a series of contributions in behavioral finance which have proposed models departing from economic rationality in specific contexts that explain a family of anomalies. These models do not abandon completely the rationality model as the basic framework, but they focus on some particular deviation explaining the anomalies. Behavioral finance, in fact, has argued that some features of asset prices are most plausibly interpreted as deviations from fundamental value, and that these deviations are brought about by the presence of traders that are not fully rational. For instance, the theory of market efficiency has been challenged by the discovery of some anomalies that would produce excess returns. De Bondt and Thaler (1985) in their seminal paper "Does the stock market overreact?", discovered that people systematically overreacting to unexpected and dramatic news events results in substantial inefficiencies in the stock market. These authors have shown that bonds, characterized by particularly high yields (so-called winners), record in the aftermath the worst yield and vice versa. This depends on investors' overreaction to an event. Since investors count on the

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[42] Tversky and Kahneman (1973) introduced the availability heuristic as "a judgmental heuristic in which a person evaluates the frequency of classes or the probability of events by availability, i.e. by the ease with which relevant instances come to mind." The reliance on the availability heuristic leads to systematic biases.

[43] According to the efficient market hypothesis, "it was generally believed that securities markets were extremely efficient in reflecting information about individual stocks and about the stock market as a whole. The hypothesis is associated with the idea of a "random walk", which is a term loosely used in the finance literature to characterize a price series where all subsequent price changes represent random departures from previous prices. The logic of the random walk idea is that if the flow of information is unimpeded and information is immediately reflected in stock prices, then tomorrow's price change will reflect only tomorrow's news and will be independent of the price changes today. But news is by definition unpredictable, and, thus, resulting price changes must be unpredictable and random" (Malkiel, 2003, p.59).

[44] In a later paper Kahneman and Thaler (2006) reviewing a wide empirical literature in behavioral economics noticed that since often forecasts are systematically biased, then choices may systematically fail to maximize utility.

representative heuristic, they become too optimistic about recent winners and too pessimistic about recent losers. However, over the time the investors realize the error and correct their assessments causing a reversal of returns[45]. De Bondt and Thaler (1985) made use in their paper of the notion of mental accounting which is the set of cognitive operations used by individuals and households to organize, evaluate and keep track of financial activities. Odean (1998, 1999), instead, has found that investors tend to overestimate their ability and also the precision of their own private information. Odean finds that overconfident traders trade too much, lower their expected utility and increase volatility in the markets. So he has designed a stock market in which all traders are overconfident; these traders do not properly optimize their expected utility, which are therefore lower than if the traders were rational. A consequence of this behavior is that overconfident traders hold underdiversified portfolios (Odean, 1998, p. 1912). At the same time, Barberis, Shleifer and Vishny (1998), adopting a quasi-Bayesian approach[46], present a model of investor sentiment that displays under reaction of stock prices to news such as earnings announcements in the short run and overreaction of stock prices to a series of good or bad news in the long run. Another important aspect highlighted in behavioral finance is loss-aversion. Bernatzi and Thaler (1995) presented a model of a stock market in which investors are assumed to be “loss-averse”, meaning that they are distinctly more sensitive to losses than to gains. In this model agents are taking into account of interdependencies between decisions. Another assumption of the model is that even long-term investors are assumed to evaluate their portfolios frequently. Thus they define the combination of these two assumptions “myopic loss-aversion”. The aim of this contribution is to explain the equity premium puzzle by myopic loss-aversion, adopting an alternative preference structure to the standard expected utility-maximizing paradigm (Bernatzi and Thaler, 1995, p.90).

As we mentioned in the section 2.2, Kahneman and Tversky (1979) in their prospect theory, acknowledging the bounded rationality of individuals, offered a descriptive model of risky choice in which the carries of utility are not states of wealth, but gains and losses relative to a neutral reference point. The most distinctive predictions of the theory arise from a property of preferences called loss-aversion (Kahneman, 2003, p.164), which indicates the disparity the strong aversion to losses relative to a reference point and the weaker desire for gains of equivalent magnitude. In other words, the loss-aversion of an individual is related to the fact that the sensation of loss relative to status quo and other reference points looms very large relatively to gains. This sensation of loss has been identified and emphasized in a great deal of experimental work, thus loss-aversion proves to be more realistic than the standard continuous, concave, utility function over wealth (Rabin, 2002a). Thaler (1980) was the first to extend the idea of loss-aversion to riskless choice. He used loss-aversion to explain the endowment effect – the discrepancy between willingness-to-pay and willingness-to-accept for the same good. Also Kahneman, Knetsch and Thaler (1991) analyzed the topic of loss-aversion[47], exploring other implications. They carried out a significant experiment based on the “endowment effect” where these authors demonstrated that the individuals feel a great sorrow when they lose the objects they possess, more than the pleasure would cause them to acquire those same objects, if they do not already possess them. So the “endowment effect” is an anomaly that causes a *statu quo bias* (a preference for the current state that biases the individual against both buying and selling his object). The “endowment effect” is connected to the particularly pervasive phenomenon of loss-aversion, for which the disutility of a loss is greater than the utility of a win of the same size.

Loss-aversion contributes to stickiness in markets, because loss-averse agents are much less prone to exchanges than final-states agents (Kahneman, 2003). Thus in the field of behavioral finance the loss-aversion appears to manifest itself in the investor behavior as an unwillingness to sell assets or

[45] In a subsequent article De Bondt and Thaler (1987) provide additional evidence that supports the overreaction hypothesis.

[46] A “quasi-Bayesian” approach assumes that people misspecify a set of hypotheses, or enconde new evidence incorrectly, but otherwise use Bayes’ rule. See, for instance, the model of Rabin (2002) on the “law of small numbers”.

[47] The loss-aversion is a core aspect of agent’s reference-based preferences and it is related to the fact that the sensation of loss relative to status quo and other reference points looms very large relatively to gains. This sensation of loss has been identified and emphasized in a great deal of experimental work (Rabin, 2002a, p.9).

other securities, if doing so forces the investor to achieve a nominal loss. This loss-aversion can help to explain why housing market prices do not adjust downwards during periods of low demand with the relative drying up of sales since the agents are unwilling to accept losses relative to an existing reference price (Genesove and Mayer, 2001)[48]. Barberis and Huang (2008) also provided a model incorporating loss aversion and framing into asset pricing to understand the equity premium puzzle.

Furthermore, Thaler and Shefrin (1981)[49], who gave major contributions to behavioral finance, presented their behavioral life-cycle theory arguing that economists who wish to analyze the consumption-saving decision must address the bounded rationality and impatience of consumers. In their models self-control is an acknowledged problem[50]. The behavioral-life cycle theory models consumers as responding to psychological limitations by adopting rules-of-thumb, such as mental accounts (as in De Bondt and Thaler, 1985), that are used to constrain the decision making of the myopic agent. Mental accounting is a useful way to describe the rules which govern gain/loss integration (Thaler, 1999). It predicts that people will spend money coming from different sources in different ways. Mental accounting stands in opposition to the standard view in economics that money is fungible.

Several criticisms have been made to behavioral finance. One is that theoretical behavioral models are somewhat as hoc and designed to explain specific stylized facts. This may be true, but these models are based on how agents actually behave, and their behaviors have been studied on extensive experimental evidence, so these models are able to explain evidence in financial markets better than traditional models. Another important criticism and often recurring is that behavioral finance presents no unified theory unlike expected utility theory maximization using rational beliefs. But the normative property of the traditional theory cannot justify its limitations; moreover behavioral finance proved to be able in helping us understanding financial phenomena (Subrahmanyam, 2008).

Conclusion

Psychology and economics have provided wide-ranging evidence and robust empirical findings that bounded rationality is crucial, so this notion represents a reference point for understanding economic behavior and economic choices. This paper argued that we can enrich our knowledge of the complex reality of financial markets through the fertile contribution of Simon' approach and of behavioral finance.

The work also examined some major contributions of behavioral finance which have highlighted failures in the traditional theory of rational investor, but also anomalies and biases in the behavior of financial markets that can lead to financial crises. A consequence of this analysis is that economics should aspire of making assumptions about human behavior as realistic as possible on a psychological level and that individuals with bounded rationality must be provided with better quality information presented in a non-complex fashion and an institutional environment conducive to good decisions, since the latter constitutes the basic framework for all economic decisions.

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[48] The literature of behavioral finance includes the lack of symmetry between decisions to acquire and maintain resources and the strong aversion to the loss of some (emotionally) valuable resources that could be completely lost.

[49] See also Thaler and Shefrin (1988) and Thaler (2003).

[50] Thaler (1981), discussed the theory of intertemporal choice and tested the model porposed in Thaler and Shefrin (1981) in which the hypothesis was that the discount rate will vary inversely with the size of the reward for which the individual must wait. This hypothesis is derived from viewing intertemporal choice as problem in self-control. In fact, waiting for a reward requires some mental effort. If this effort does not increase proportionally with the size of the rewards (if there are some fixed psychics to waiting) then the hypothesized result will be present.(Thaler, 1981, p. 202). Thaler (1981) also showed that gains and losses of different absolute magnitudes are discounted differently.

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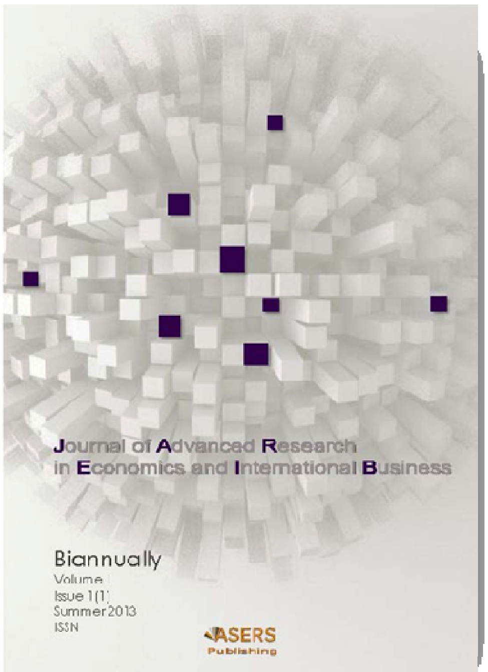

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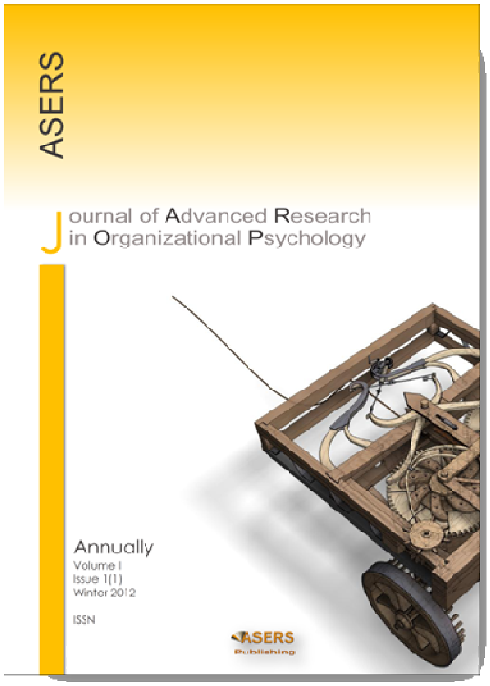

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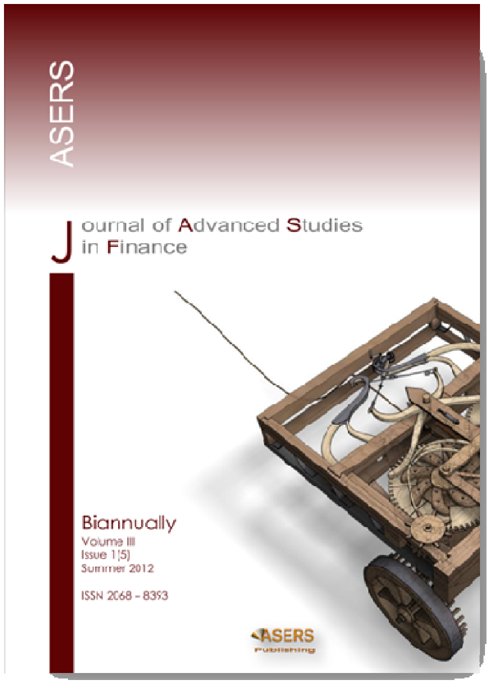

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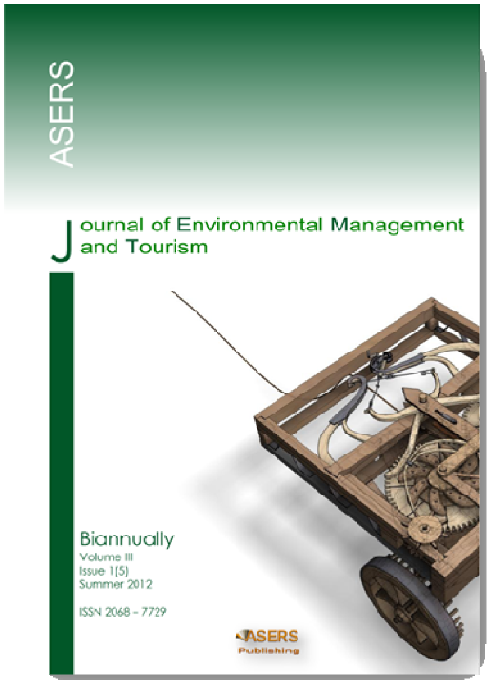

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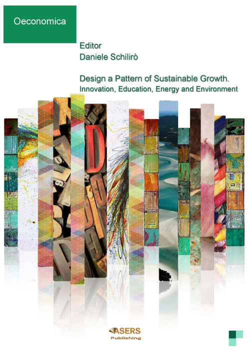

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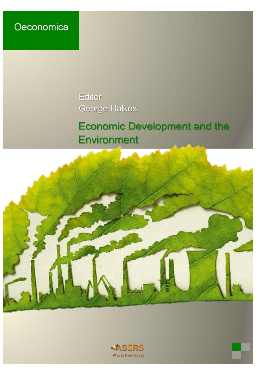

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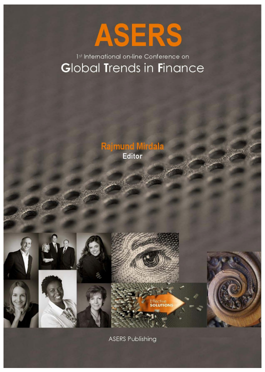

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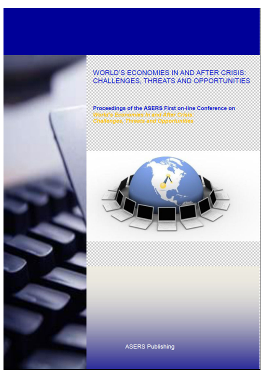

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