

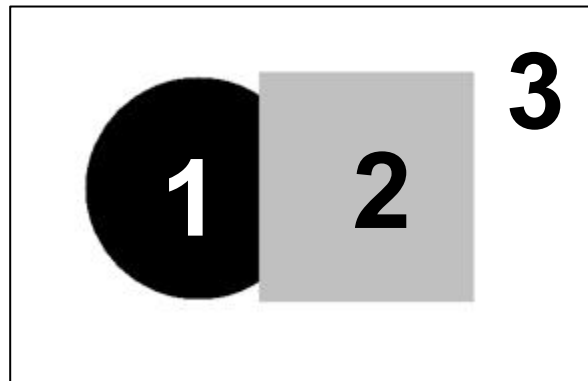
Segmentation: MRFs and Graph Cuts

Computer Vision

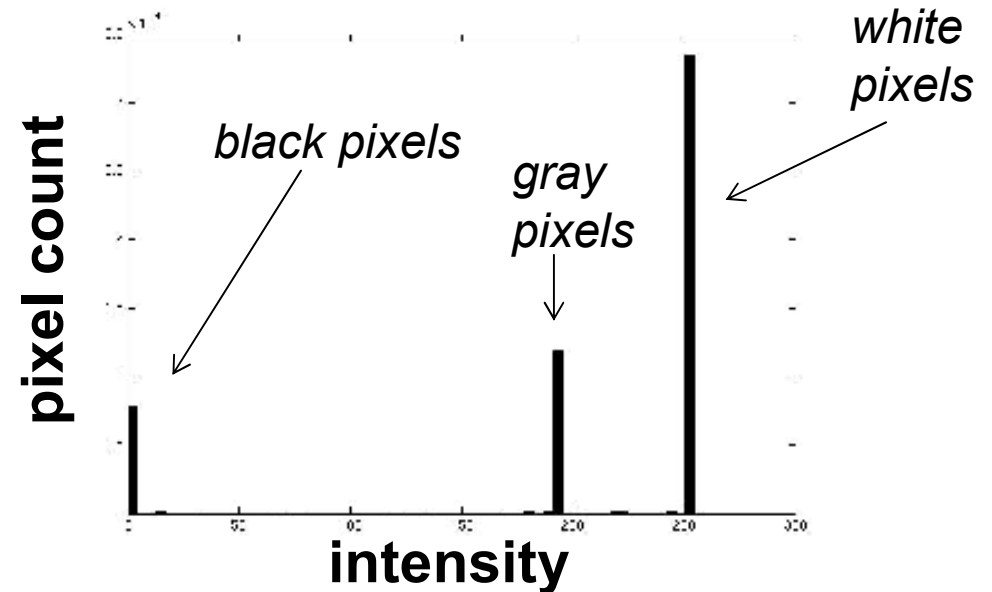
CS 143, Brown

James Hays

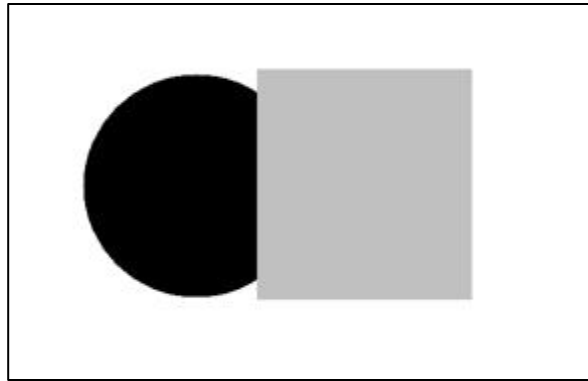
Image segmentation: toy example



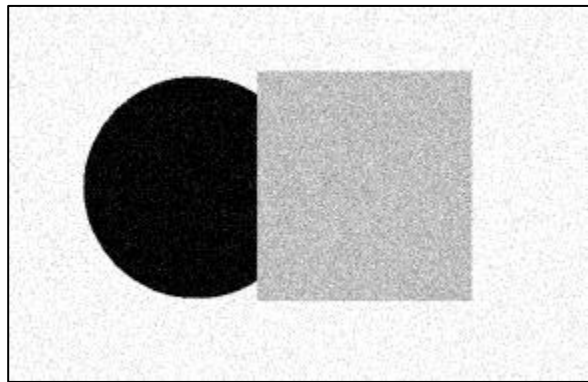
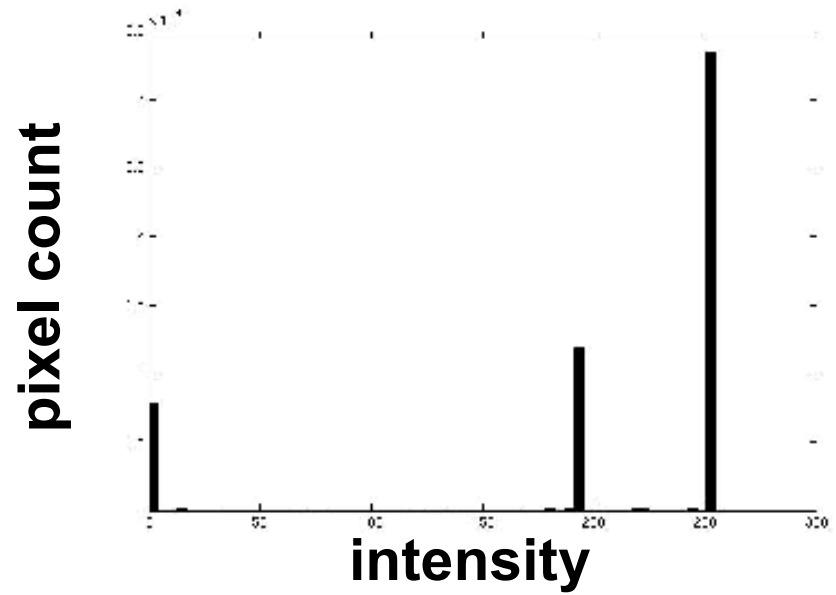
input image



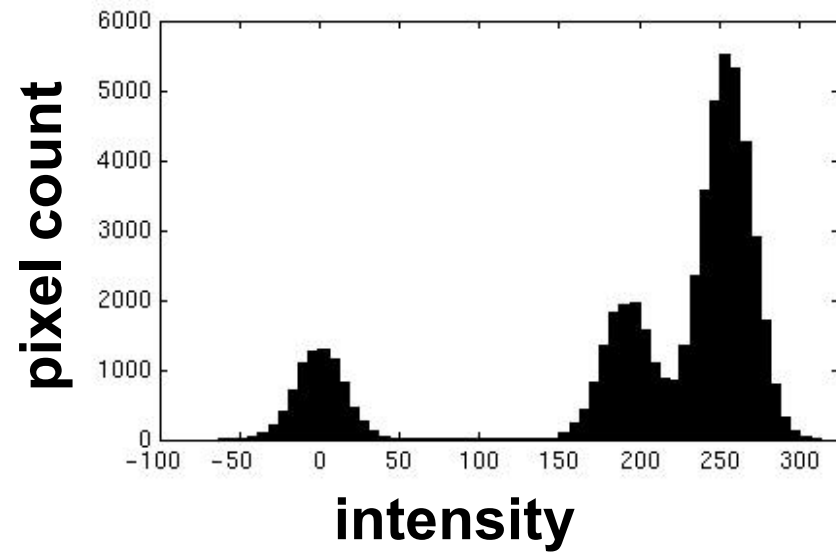
- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
 - i.e., *segment* the image based on the intensity feature.
- What if the image isn't quite so simple?

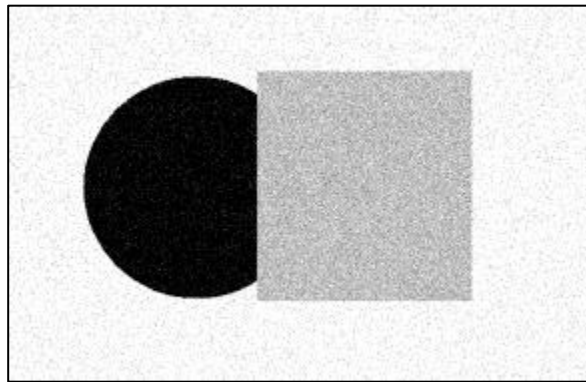


input image

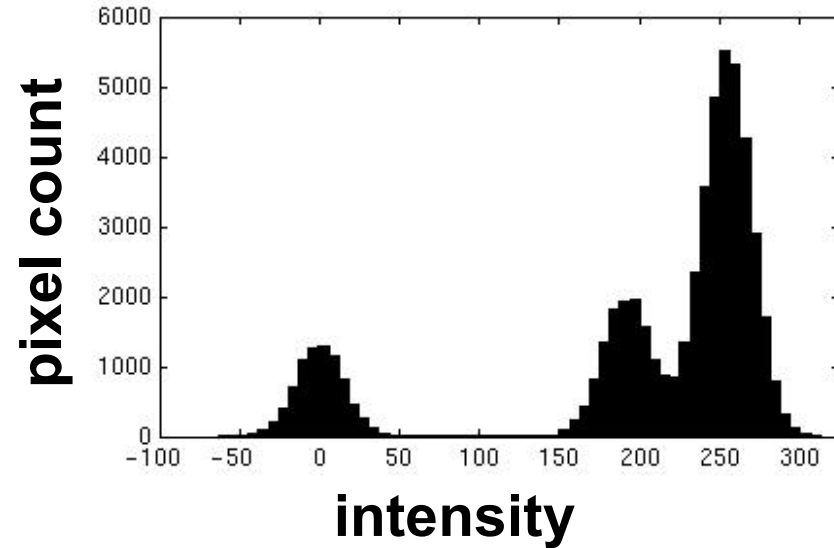


input image





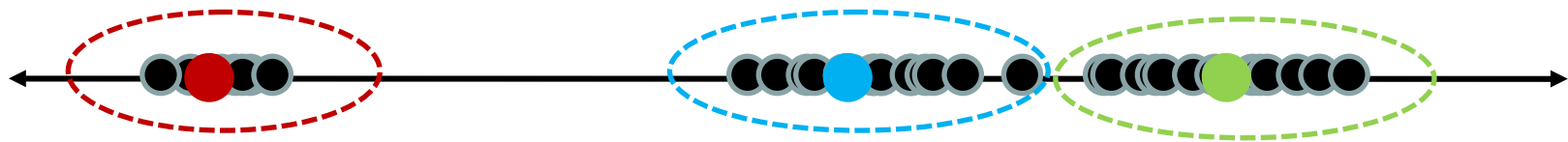
input image



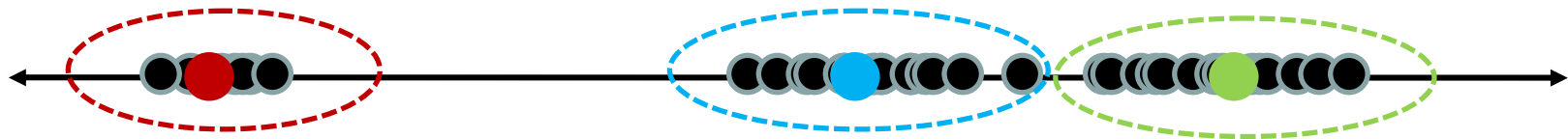
- Now how to determine the three main intensities that define our groups?
- We need to ***cluster***.

Clustering

- With this objective, it is a “chicken and egg” problem:
 - If we knew the **cluster centers**, we could allocate points to groups by assigning each to its closest center.

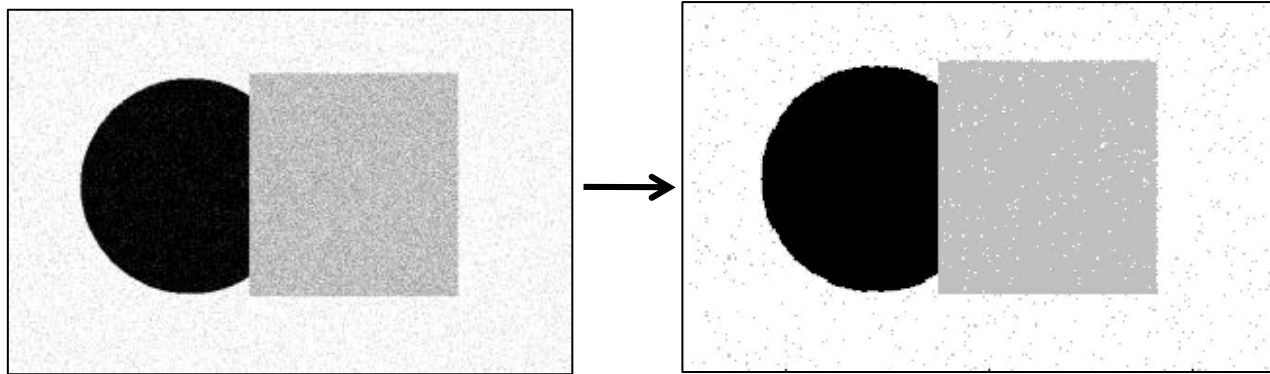


- If we knew the **group memberships**, we could get the centers by computing the mean per group.



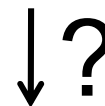
Smoothing out cluster assignments

- Assigning a cluster label per pixel may yield outliers:

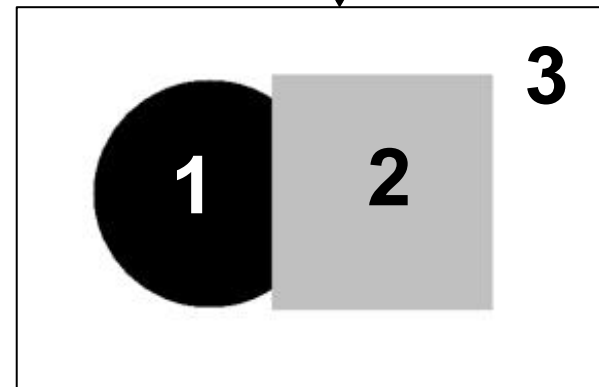


original

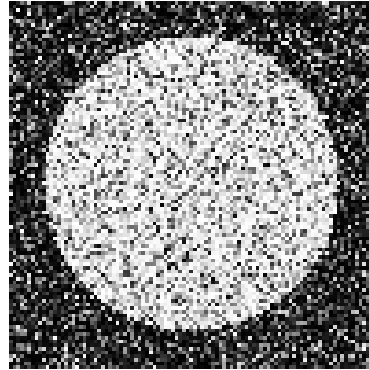
labeled by cluster center's intensity



- How to ensure they are spatially smooth?



Solution



$P(\text{foreground} \mid \text{image})$

Encode dependencies between pixels

Normalizing constant

$$P(\mathbf{y}; \theta, data) = \frac{1}{Z} \prod_{i=1..N} f_1(y_i; \theta, data) \prod_{i,j \in \text{edges}} f_2(y_i, y_j; \theta, data)$$

Labels to be predicted

Individual predictions

Pairwise predictions

Writing Likelihood as an “Energy”

$$P(\mathbf{y}; \theta, data) = \frac{1}{Z} \prod_{i=1..N} p_1(y_i; \theta, data) \prod_{i,j \in edges} p_2(y_i, y_j; \theta, data)$$

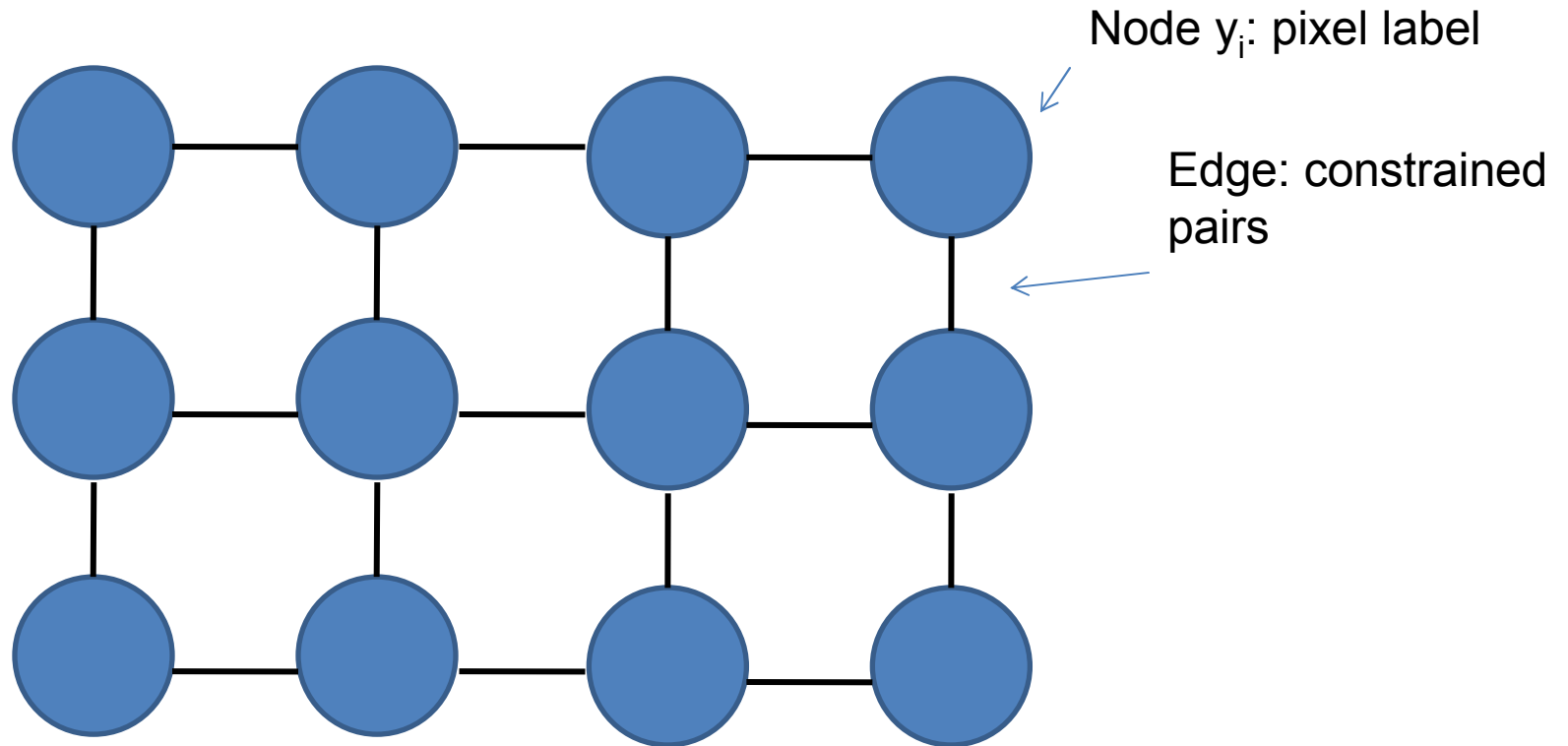


$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i,j \in edges} \psi_2(y_i, y_j; \theta, data)$$

“Cost” of assignment y_i

“Cost” of pairwise assignment y_i, y_j

Markov Random Fields



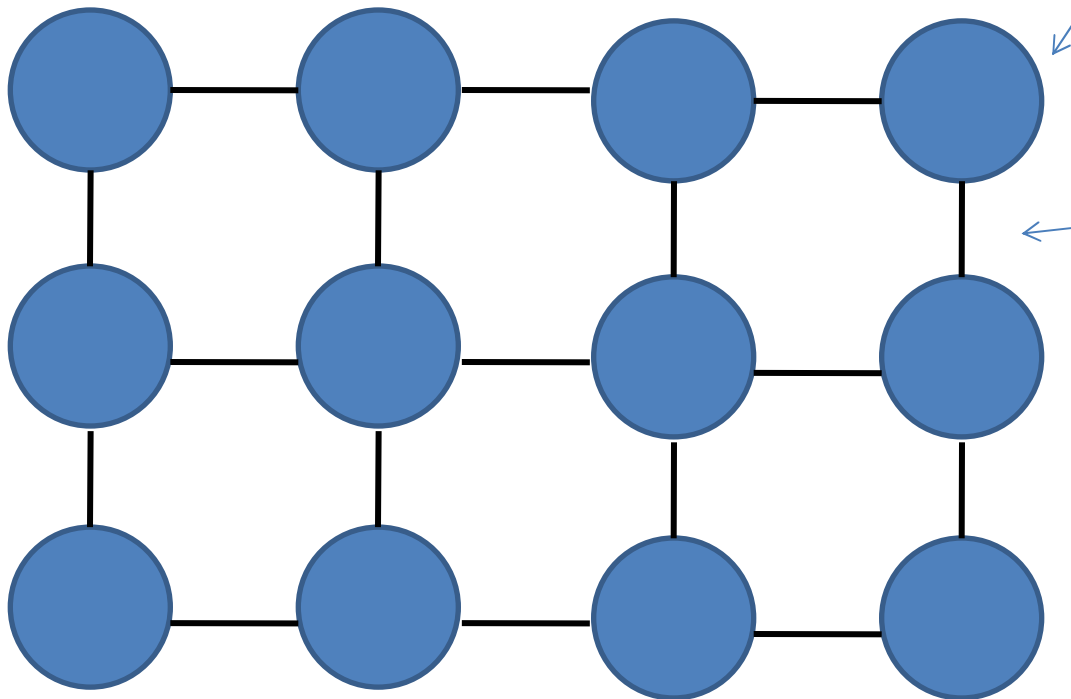
Cost to assign a label to each pixel

Cost to assign a pair of labels to connected pixels

$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i,j \in edges} \psi_2(y_i, y_j; \theta, data)$$

Markov Random Fields

- Example: “label smoothing” grid



Unary potential

0: $-\log P(y_i = 0 ; \text{data})$

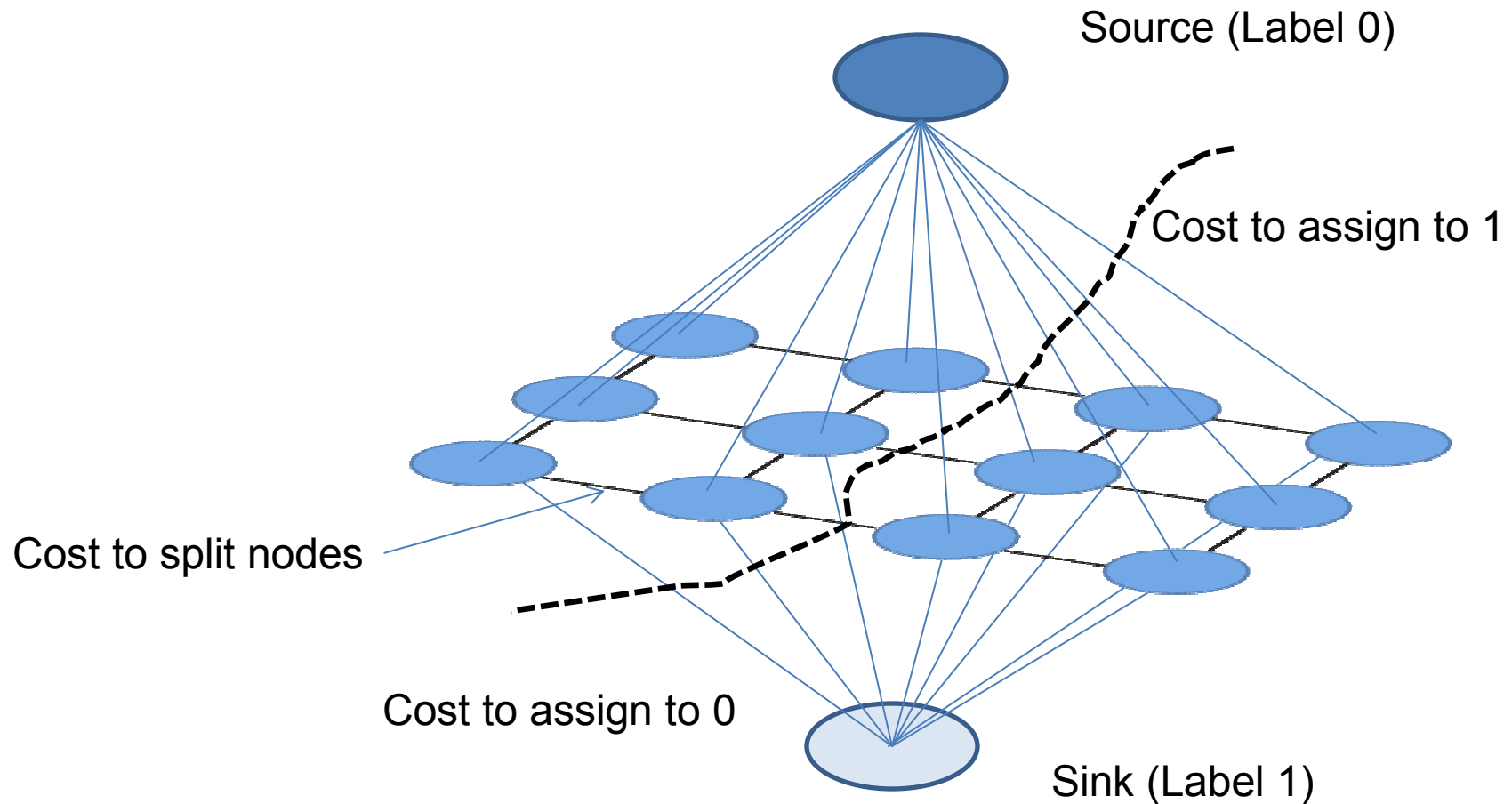
1: $-\log P(y_i = 1 ; \text{data})$

Pairwise Potential

	0	1
0	0	K
1	K	0

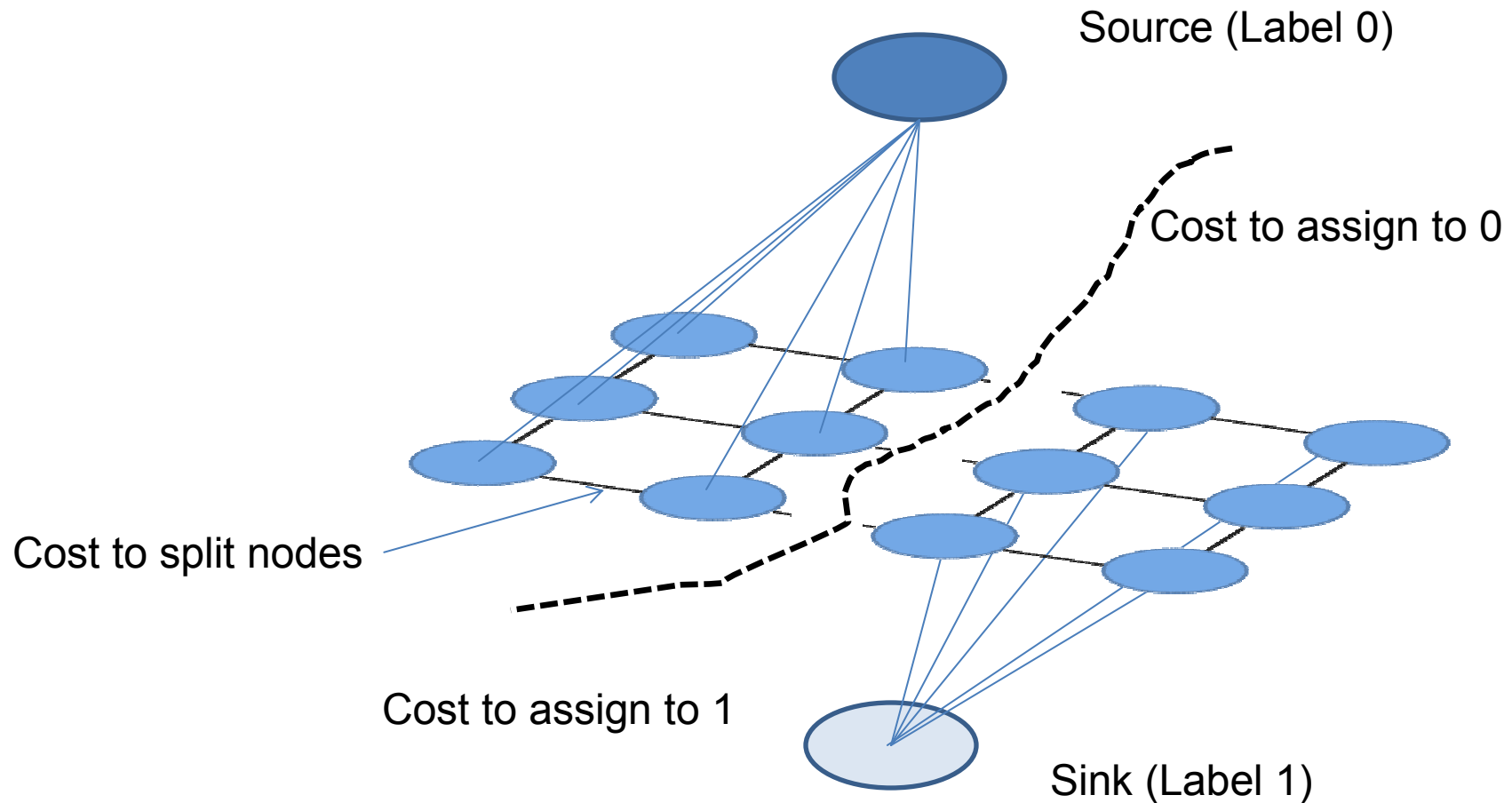
$$\text{Energy}(\mathbf{y}; \theta, \text{data}) = \sum_i \psi_1(y_i; \theta, \text{data}) + \sum_{i,j \in \text{edges}} \psi_2(y_i, y_j; \theta, \text{data})$$

Solving MRFs with graph cuts



$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i,j \in edges} \psi_2(y_i, y_j; \theta, data)$$

Solving MRFs with graph cuts



$$Energy(\mathbf{y}; \theta, data) = \sum_i \psi_1(y_i; \theta, data) + \sum_{i,j \in edges} \psi_2(y_i, y_j; \theta, data)$$