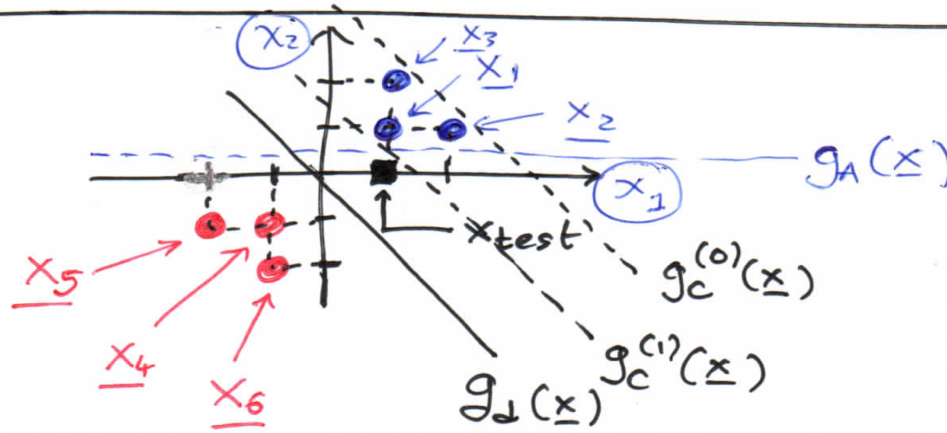
HY 345 – EXAM 1 – A /19/6/14/



1a

- Προφανώς είναι διαχωρίσιμες
- Η $x_2 = 0.5 \Leftrightarrow g_A(\underline{x}): \boxed{0x_1 + 1x_2 - \frac{1}{2}} \Leftrightarrow \underline{W} = \begin{bmatrix} 0 \\ 1 \\ -1/2 \end{bmatrix}$

επιτυγχάνει το ζητούμενο ($x_{test} \in \omega_2$)

$\left(g_A(\underline{x}) > 0, \ \underline{x} = \underline{x}_1, \underline{x}_2, \underline{x}_3 \right.$
$\left. < 0, \ \underline{x} = \underline{x}_4, \underline{x}_5, \underline{x}_6 \right)$

1b

Έστω απόσταση L_1

$d(\underline{x}_{test}, \underline{x}_1) = 1 \longrightarrow \min \Rightarrow \boxed{x_{test} \in \omega_1}$ με βάση κανόνα 1-NN

$, \underline{x}_2) = 2$

$, \underline{x}_3) = 2$

$, \underline{x}_4) = 3$

$, \underline{x}_5) = 4$

$, \underline{x}_6) = 4$

1c $\quad \underline{w}(0) = \begin{bmatrix} 1 \\ 1 \\ -\frac{15}{4} \end{bmatrix} \Leftrightarrow g_c^{(0)}(\underline{x}): x_1 + x_2 - \frac{15}{4}$

$\underline{x}_1, \underline{x}_2, \underline{x}_3 : g_c^{(0)}(\underline{x}) < 0 \longrightarrow$ λάθος ταξινόμηση

$1 + 2 - \frac{15}{4} = -3/4$

$2 + 1 - 15/4 =$

$1 + 1 - 15/4 = -7/4$

Τα $\underline{x}_4, \underline{x}_5, \underline{x}_6 : g_c^{(0)}(\underline{x}) < 0 \Longrightarrow$ OK!

Άρα: $\underline{w}(1) = \underline{w}(0) + \rho(\underline{x}_1 + \underline{x}_2 + \underline{x}_3) =$

$$= \begin{bmatrix} 1 \\ 1 \\ -\frac{15}{4} \end{bmatrix} + \frac{1}{4}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 \\ 1 \\ -\frac{15}{4} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -\frac{12}{4} \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$$

$g_c^{(1)}(\underline{x}) : \boxed{2x_1 + 2x_2 - 3}$

$\underline{x}_1, \underline{x}_2, \underline{x}_3 : g_c^{(1)}(\underline{x}) > 0$ OK

$\underline{x}_4, \underline{x}_5, \underline{x}_6 : g_c^{(1)}(\underline{x}) < 0$ OK

$\underline{x}_{test} : \boxed{g_c^{(1)}(\underline{x}_{test}) < 0 \rightsquigarrow \omega_2}$

1d Είναι προφανές ότι η σχεδιασμένη $g_d(\underline{x})$ επιτυγχάνει το ζητούμενο. (max margin $= 2\sqrt{2}$)

$$g_d(\underline{x}) : \boxed{\tfrac{1}{2}x_1 + \tfrac{1}{2}x_2 + 0} \rightsquigarrow \begin{bmatrix} +1/2 \\ +1/2 \\ 0 \end{bmatrix}$$

με SUPPORT VECS τα $[1, 1]^T$ & $[-1, -1]^T$

(δίνει: $\pm 1 = g_d(\underline{x}_1), g_d(\underline{x}_4)$)

Παρατηρούμε πως:

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} +1/2 \\ +1/2 \end{bmatrix} = \lambda_1 \underbrace{y_1}_{+1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \lambda_4 \underbrace{y_4}_{-1} \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

λ_1: πχ $1/4$ $\quad$ λ_4: πχ $1/4$

με: $\sum_{i=1}^{N} \lambda_i y_i = \frac{1}{4} - \frac{1}{4} = 0$

Επίσης:

$$\underbrace{[w_1 \ w_2]}_{1/2 \ 1/2} \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\underline{x}_1} + w_0 = 1 \iff w_0 = 0$$

Ισοδύναμα

$$\underbrace{[w_1, w_2]}_{1/2, 1/2} \begin{bmatrix} -1 \\ -1 \end{bmatrix} + w_0 = -1 \iff w_0 = 0$$

ΠΕΡΙΘΩΡΙΟ: $\frac{2}{\|w\|} = \frac{2}{\sqrt{2}\,\frac{1}{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$\underline{X}_{test} \in \omega_1$

1e. • $\boxed{\text{Μέση τιμή}}$ $\frac{1}{6}\sum_{i=1}^{6}\underline{x}_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\underline{X} = [\underline{x}_1, \underline{x}_2, \underline{x}_3, \dots, \underline{x}_6] = \begin{bmatrix} 1 & 2 & 1 & -1 & -2 & -1 \\ 1 & 1 & 2 & -1 & -1 & -2 \end{bmatrix}$$

• $\underline{X}\underline{X}^T = \begin{bmatrix} 12 & 10 \\ 10 & 12 \end{bmatrix} \Longrightarrow$

$\Longrightarrow$ ΙΔΙΟΤΙΜΕΣ: $(12-\lambda)^2 - 10^2 = 0 \Longleftrightarrow$

$\Longleftrightarrow (12-\lambda-10)(12-\lambda+10) = 0$

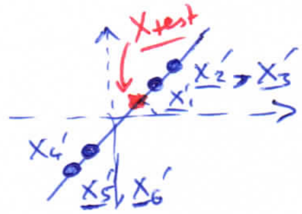
$\Longleftrightarrow \lambda_1 = 2, \lambda_2 = 22$ → ΜΕΓΑΛΥΤΕΡΗ

• ΙΔΙΟΔΙΑΝΥΣΜΑ:

$$\begin{bmatrix} 12 & 10 \\ 10 & 12 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 22\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Longleftrightarrow$$

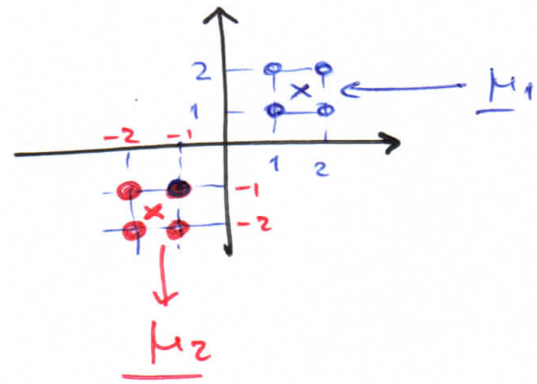
$$\Longleftrightarrow \begin{array}{l} 12x_1 + 10x_2 = 22x_1 \\ 10x_1 + 12x_2 = 22x_2 \end{array} \Longleftrightarrow \boxed{x_1 = x_2}$$

• ΠΡΟΒΟΛΕΣ



Προφανώς στην ω_1

1 f



$$\hat{\underline{\mu}}_1 = \frac{1}{4} \sum_{i=1}^{4} \underline{x}'_i = \frac{1}{4} \begin{bmatrix} 1+2+1+2 \\ 1+1+2+2 \end{bmatrix} = \frac{6}{4} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix}$$

$$\hat{\underline{\mu}}_2 = \frac{1}{4} \sum_{i=5}^{8} \underline{x}'_i = \begin{bmatrix} -3/2 \\ -3/2 \end{bmatrix}$$

$$\hat{\Sigma}_1 = \frac{1}{3} \begin{bmatrix} -0.5 & -0.5 & +0.5 & +0.5 \\ -0.5 & +0.5 & -0.5 & +0.5 \end{bmatrix} \begin{bmatrix} \; \end{bmatrix} = \frac{1}{3}\frac{1}{4} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix}$$

$$\Downarrow$$

$$|\Sigma_1| = 1/9$$

$$\hat{\underline{\Sigma}}_2 = \hat{\Sigma}_1$$

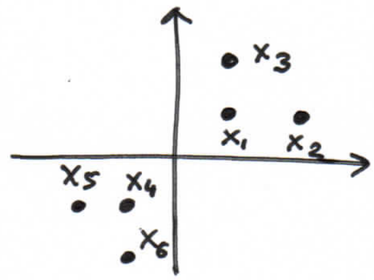
$$p(\underline{x}|\omega_1) = \frac{1}{\sqrt{2\pi}\, 9} \exp\left[-\frac{1}{2} \left(\underline{x} - \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix}\right)^T \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \left(\underline{x} - \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix}\right) \right]$$

$$p(\underline{x}|\omega_2) =$$

$$P(\omega_1) = P(\omega_2) = 1/2$$

Είναι προφανές ότι $\underline{x}_{test} \in \omega_1$ (με βάση τις συναρτήσεις διάκρισης & επιφάνεια διαχωρισμού $x_1 + x_2 = 0$)

2 (A)



1: $\{x_1\}$

2: $\{x_1, x_2\} \rightarrow$ ΜΕΣΟ $\underline{c}_1 = [1.5, 1]^T$

$\hookrightarrow d = 1.0 < 1.25$

3: $\{x_1, x_2\}, \{x_3\}$

$d = 1.5 > 1.25$

4: $\{x_1, x_2\}, \{x_3\}, \{x_4\}$

$d = 3.5 > 1.25$ $\quad d = 5 > 1.25$

5: $\{x_1, x_2\}, \{x_3\}, \{x_4, x_5\}$

$\hookrightarrow d = 1.0 < 1.25$

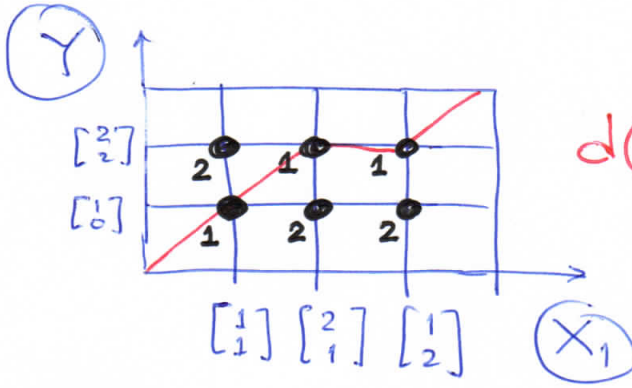
$\underline{c}_2 = [-1.5, 1]$

6: $\boxed{\{x_1, x_2\}, \{x_3\}, \{x_4, x_5\}, \{x_6\}}$

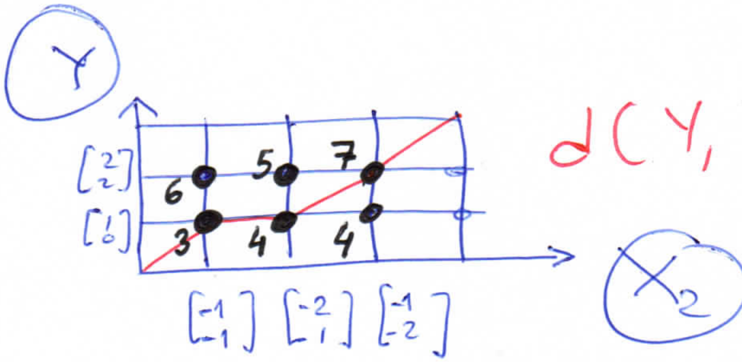
$5.5 > 1.25$ $\quad 6 > 1.25$ $\quad 1.5 > 1.25$

$\rightarrow$ 4 ΟΜΑΔΕΣ

2 B



$d(Y, X_1) = 3$



$d(Y, X_2) = 14$

Προφανώς ταξινομείται στην ω_1.