Introduction to Python for Econometrics, Statistics and Data Analysis

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Notes

These notes are not yet complete. Missing include:

1. Some of the more esoteric chapters
2. Solutions

I hope to have the addressed by the end of March 2012.

Contents

Completed

Chapter 1

Introduction

1.1 Background

These notes are designed for someone new to statistical computing wishing to develop a set of skills necessary to perform original research using Python.

Python is a popular language which is well suited to a wide range of problems. Recent progress has extended Python's range of applicability to econometrics, statistics and numerical analysis. Python – with the right set of add-ons – is comparable to MATLAB and R, among other languages. If you are wondering whether you should bother with Python (or another language), a very incomplete list of considerations includes:

You might want to consider R if:

- You want to apply statistical methods. The statistics library of R is second to none, and R is clearly at the forefront in new statistical algorithm development – meaning you are most likely to find that new(ish) procedure in R.
- Performance is of secondary importance.
- Free is important.

You might want to consider MATLAB if:

- Commercial support, and a clean channel to report issues, is important.
- Documentation and organization of modules is more important than raw routine availability.
- Performance is more important than scope of available packages. MATLAB has optimizations, such as Just-in-Time (JIT) compiling of loops, which are not available in most (possibly all) other packages.

Having read the reasons to choose another package, you may wonder why you should consider Python.

- You need a language which can act as a end-to-end solution so that everything from accessing web-based services and database servers, data management and processing and statistical computation can be accomplished in a single language.
- Performance is a concern, but not at the top of the list.
- Free is an important consideration. Python can be freely deployed, even to 100s of servers in a compute cluster.

1.2 Conventions

These notes will follow two conventions.

1. Code blocks will be used throughout.

```
"""A docstring
"""

# Comments appear in a diferent color

# Reserved keywords are highlighted
and as assert break class continue def del elif else
except exec finally for from global if import in is
lambda not or pass print raise return try while with yield

# Common functions and classes are highlighted in a
# different color. Note that these are not reserved,
# and can be used although best practice would be
# to avoid them if possible
array matrix xrange list True False None
```

2. Then a code block contains >>>, this indicates that the command is running an interactive IPython session. Output will often appear after the console command, and will *not* be preceded by a command indicator.

```
>>> x = 1.0
>>> x + 2
3.0
```

If the code block does not contain the console session indicator, the code contained in the block is designed to be in a standalone Python file.

```
from __future__ import print_function
import numpy as np

x = np.array([1,2,3,4])
y = np.sum(x)
print(x)
print(y)
```

1.3 Required Components

1.3.1 Python

Python 2.7.2 (or later, but Python 2.7.x) is required. It provides the core Python interpreter.

1.3.2 NumPy

NumPy provides a set of array and matrix data types which are essential for econometrics and data analysis.

1.3.3 SciPy

SciPy contains a large number of routines needed for analysis of data. The most important include a wide range of random number generators, linear algebra and optimizers. SciPy depends on NumPy.

1.3.4 IPython

IPython provides an interactive Python environment. It is the main environment for entering commands and getting instant results, and is a very useful tool when learning Python.

1.3.5 Distribute

Distribute provides a variety of tools which make installing other packages easy.

1.3.6 PyQt4

PyQt4 provides a set of libraries used in the Qt console mode of IPython. This component is optional, but recommended.

1.3.7 matplotlib

matplotlib provides a plotting environment for 2D plots, with limited support for 3D plotting.

1.3.8 Windows Specific

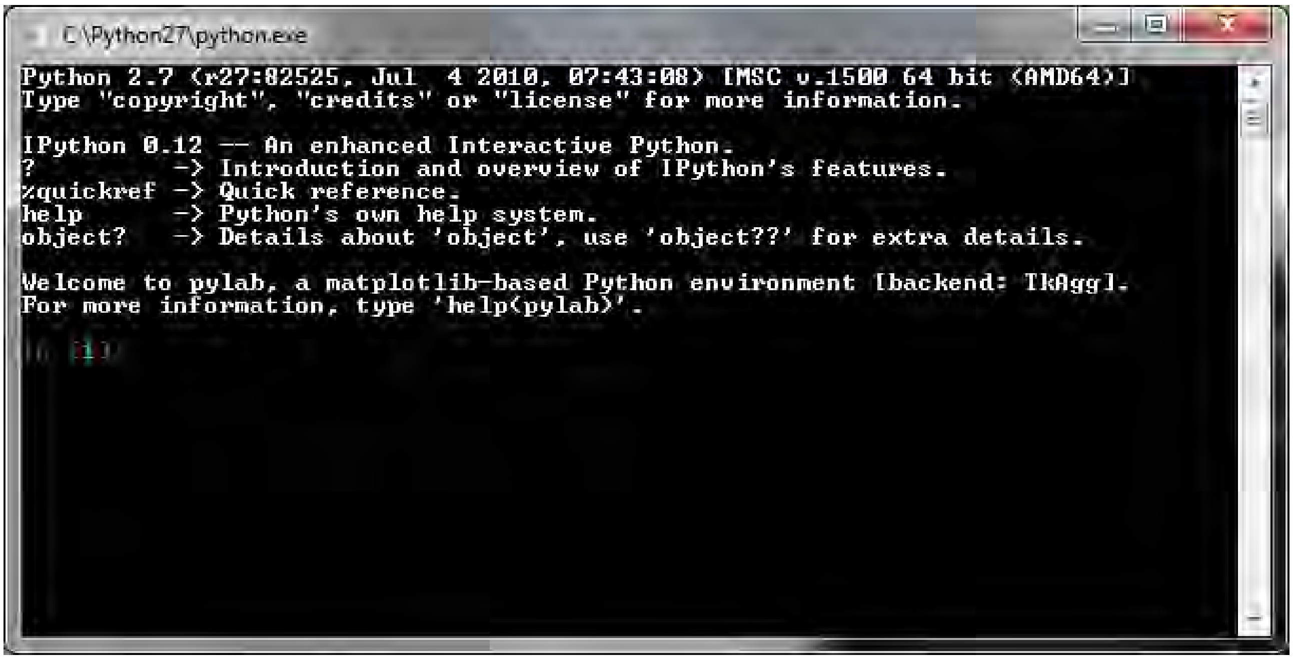
Pyreadline Pyreadline is required for windows to provide syntax highlighting in IPython.

Console2 Optional component that provides an enhanced console.

1.3.9 Package List

The list of windows packages used in writing these notes include:

Package	Version	File name
Python	2.7.2	Python 2.7
SciPy	0.10.0+	scipy-0.10.1.rc1.win-amd64-py2.7
NumPy	1.6.1	numpy-MKL-1.6.1.win-amd64-py2.7
Pyreadline*	2.0.dev	pyreadline-2.0.dev.win-amd64-py2.7
Distribute	0.6.24	*Installed using instruction below*
IPython	0.12	iPython-0.12.win-amd64-py2.7
matplotlib	1.1.0+	matplotlib-1.1.0.win-amd64-py2.7
PyQt†	4.8+	PyQt-Py2.7-x64-gpl-4.8-6-1
Pygments†	1.4.0	Pygments-1.4.tar.gz
PyZMQ	2.1.11	pyzmq-2.1.11.win-amd64-py2.7
Console2*	2.00b148	Console-2.00b148-Beta_64bit



```
Python 2.7 (r27:82525, Jul  4 2010, 07:43:08) [MSC v.1500 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 0.12 -- An enhanced Interactive Python.
?         -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help      -> Python's own help system.
object?   -> Details about 'object', use 'object??' for extra details.

Welcome to pylab, a matplotlib-based Python environment [backend: TkAgg].
For more information, type 'help(pylab)'.

In [1]:
```

Figure 1.1: The basic IPython environment running pylab inside cmd, Windows command interpreter.

† needed for IPython QtConsole, ⋆ Windows only

1.4 Setup

Setup of the required packages is straightforward. A video demonstration of the setup on Windows 7 and Fedora 16 is available on the website for these notes.

1.4.1 Windows

Begin by installing Python, NumPy, SciPy, Pyreadline, Distribute, IPython and matplotlib. These are all standard windows installers (msi or exe), and the order is not important aside from installing Python first. You should create a shortcut containing `c:\Python27\Scripts\iPython.exe --pylab`. The icon will be generic, and if you want a nice icon, select the properties of the shortcut, and then Change Icon, and navigate to `c:\Python27\DLLs` and select `pyc.ico`.

Opening the icon should produce a command window similar to that in figure

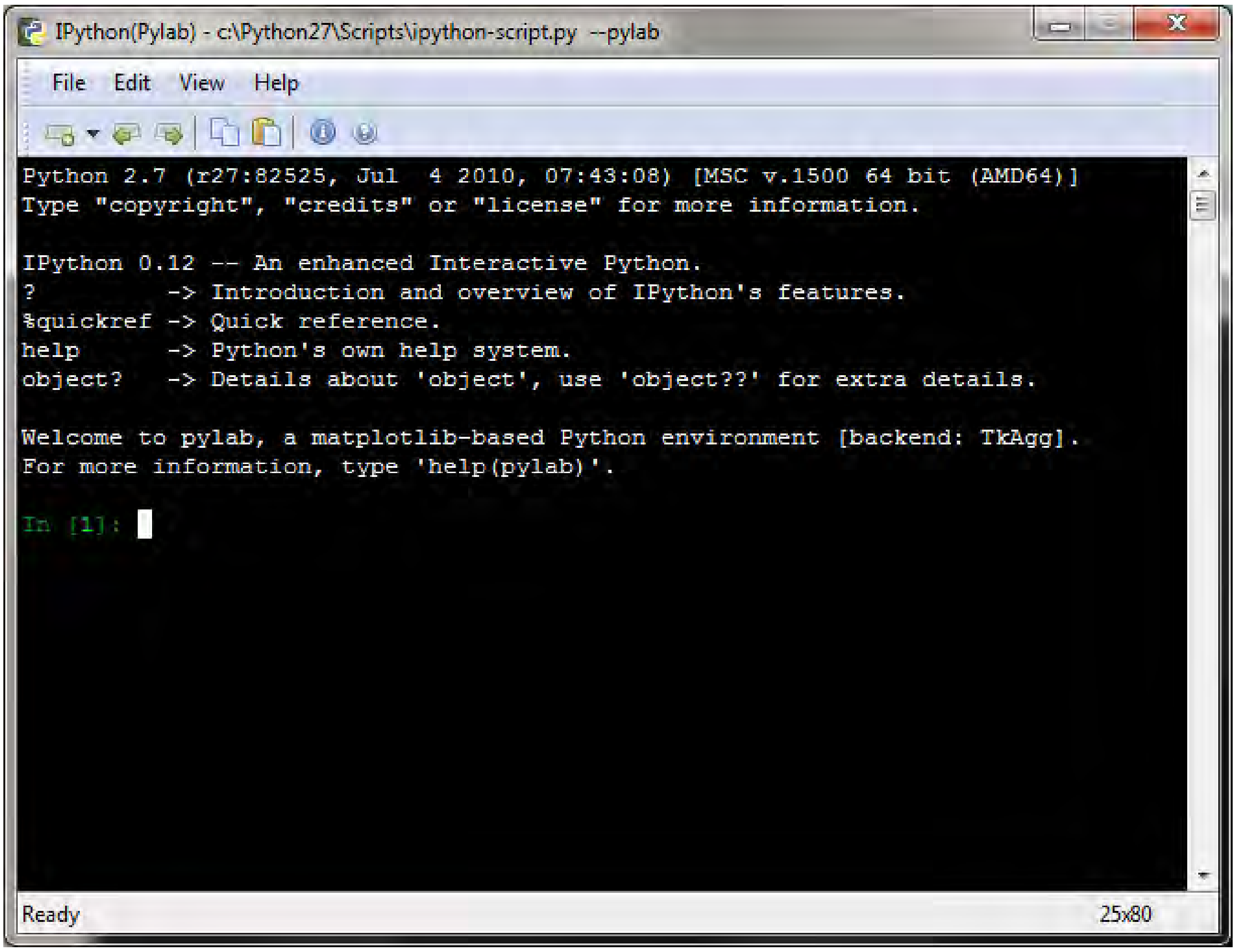
1.4.1.1 Better: Console2

The Windows command interpreter (cmd.exe) is very limited compared to other platforms. Fortunately, cmd.exe can be replaced with Console2. To use Console2, extract the contents of the zip file Console-2.00b148-Beta_64bit.zip (assumed to be `C:\Python27\Console2\`). Launch Console.exe, and select Edit > Settings > Tabs. Click on Add, and input the following:

Title IPython(Pylab)

Icon Navigate to `c:\Python27\DLLs` and select `py.ico`.

Shell `cmd.exe /k "c:\Python27\Scripts\iPython-script.py --pylab"`


IPython(Pylab) - c:\Python27\Scripts\ipython-script.py --pylab
File
Edit
View
Help
Ready
25x80

```
Python 2.7 (r27:82525, Jul  4 2010, 07:43:08) [MSC v.1500 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 0.12 -- An enhanced Interactive Python.
?         -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help      -> Python's own help system.
object?   -> Details about 'object', use 'object??' for extra details.

Welcome to pylab, a matplotlib-based Python environment [backend: TkAgg].
For more information, type 'help(pylab)'.

In [1]:
```

Figure 1.2: IPython environment running pylab inside Console2. Console2 provides a more productive environment than cmd.

Startup dir *Where data and work are stored*, e.g. `c:\Users\username\Documents\`

Finally, create a shortcut to Console with the command:

```
c:\Python27\Console2\Console.exe -t IPython(Pylab)
```

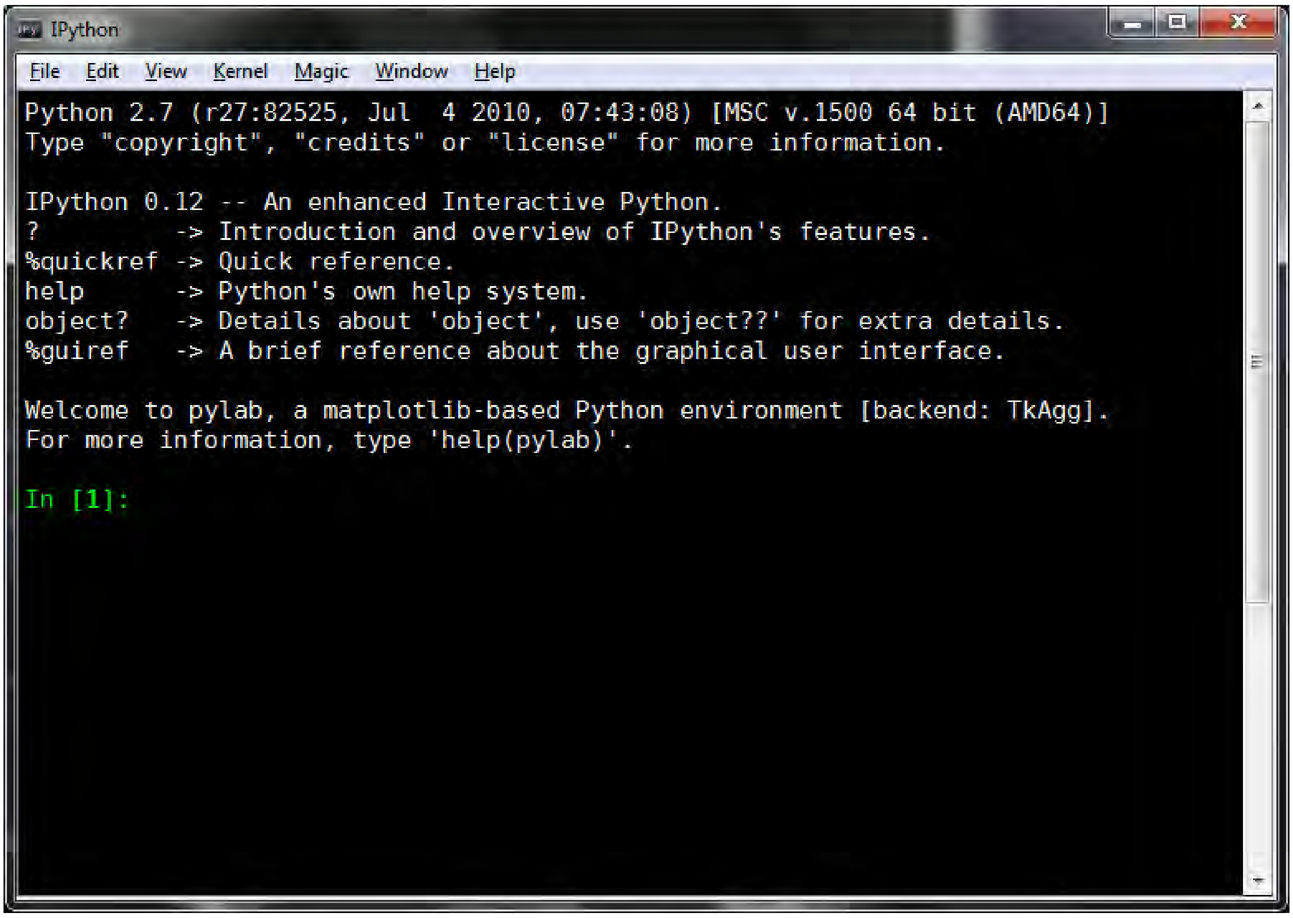
1.4.1.2 Best: QtConsole

IPython comes with its own environment build using the Qt Toolkit. To use this version, it is necessary to install PyQt, PyZMQ and Pygments. Both PyQt and PyZMQ come with installers and so installation is simple.

Pygments must be manually installed. Begin by extracting Pygments-1.4.tar.gz to c:\Python27\. Open a command prompt (cmd.exe), and enter the following two commands:

```
cd c:\Python27\Pygments-1.4
c:\Python27\Python.exe setup.py install
```

Finally, create a shortcut to `c:\Python27\Scripts\iPython-qtconsole.exe --pylab`. The font and colors of the QtConsole can be customized using command line switches such as:



```
Python 2.7 (r27:82525, Jul  4 2010, 07:43:08) [MSC v.1500 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 0.12 -- An enhanced Interactive Python.
?         -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help      -> Python's own help system.
object?   -> Details about 'object', use 'object??' for extra details.
%guiref   -> A brief reference about the graphical user interface.

Welcome to pylab, a matplotlib-based Python environment [backend: TkAgg].
For more information, type 'help(pylab)'.

In [1]:
```

Figure 1.3: IPython environment running pylab inside QtConsole. QtConsole allows for some unique setups, such as displaying figures inline (see figure 1.4).

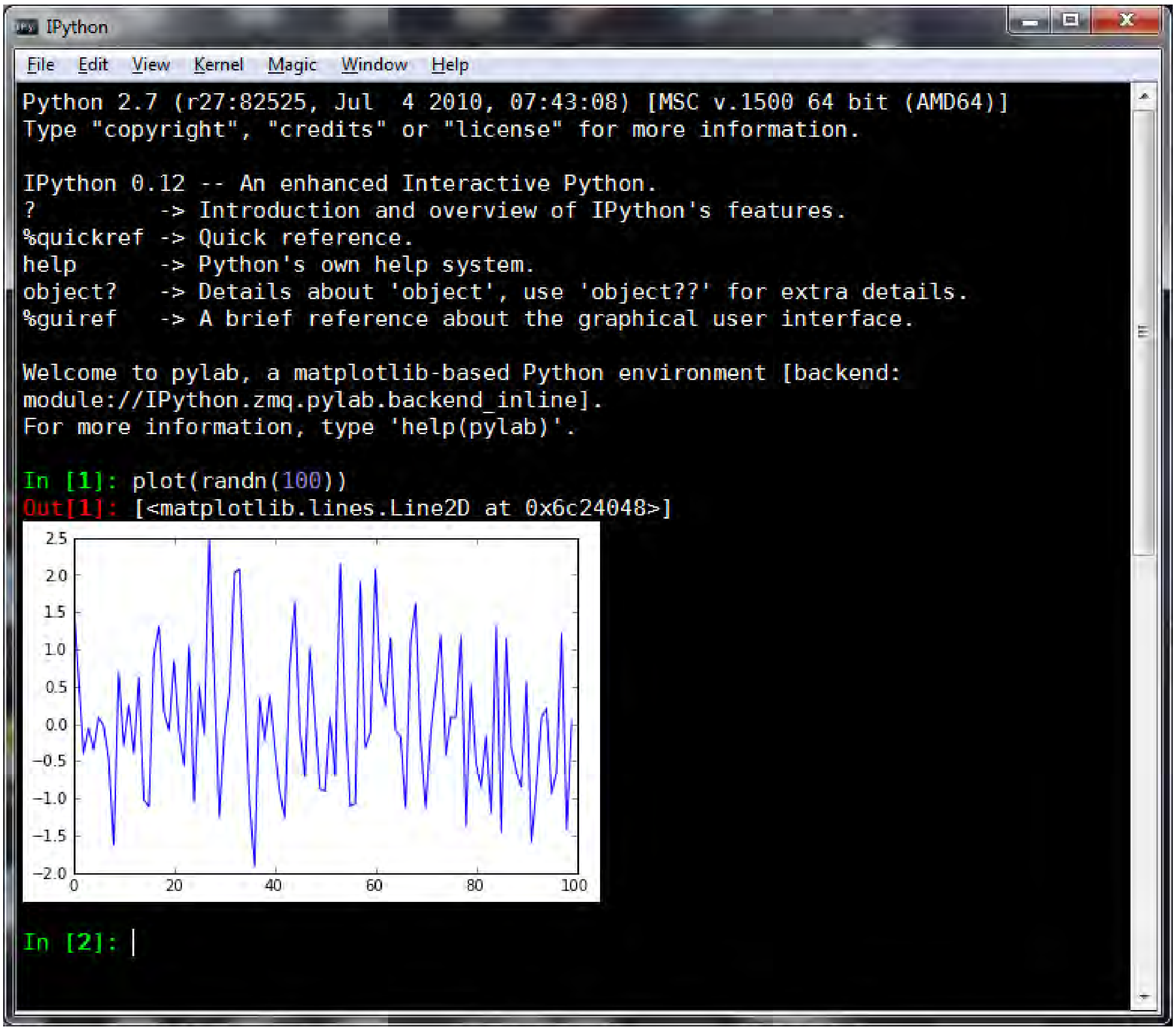
```
c:\Python27\Scripts\iPython-qtconsole.exe --pylab --colors=linux --ConsoleWidget.font_family
    ="Bitstream Vera Sans Mono" --ConsoleWidget.font_size=11
```

One final command line switch which may be useful is to add `=inline` to `--pylab` (so the command has `--pylab=inline`). This will produce graphics which appear inside the QtConsole, rather than in their own window.

1.4.2 Linux

Installing in Linux is very simple. These instruction assume that the base Python (or later) is available through the preferred distribution. At the time of writing, this was true for both Fedora and Ubuntu. If available, you should retrieve the following packaged from your distributions maintained repositories:

- Python-devel
- Python-setuptools (or Python-distribute)
- Python-iPython
- Python-numpy and Python-numpy-f2py
- Python-scipy



```
Python 2.7 (r27:82525, Jul  4 2010, 07:43:08) [MSC v.1500 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 0.12 -- An enhanced Interactive Python.
?         -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help      -> Python's own help system.
object?   -> Details about 'object', use 'object??' for extra details.
%guiref   -> A brief reference about the graphical user interface.

Welcome to pylab, a matplotlib-based Python environment [backend:
module://IPython.zmq.pylab.backend_inline].
For more information, type 'help(pylab)'.

In [1]: plot(randn(100))
Out[1]: [<matplotlib.lines.Line2D at 0x6c24048>]
```

```
In [2]: |
```

Figure 1.4: An example of the IPython QtConsole using the command line switch –pylab=inline, which produces plots inside the console.

- Python-matplotlib
- Python-PyQt4
- Python-zmq
- Python-pygments
- Python-tk

1.4.2.1 Missing or Outdated components

If a component is badly outdated, you should manually install the current version (after uninstalling using package manager in your distribution).

IPython, PyZMQ and Pygments IPython, PyZMQ and Pygments can all be installed using `easy_install`. Run the following commands in a terminal window, omitting any which have maintained versions for your distribution of Linux:

```
sudo easy_install iPython
sudo easy_install pyzmq
sudo easy_install pygments
```

If you have followed the instructions, these should all complete without issue.

Notes:

- If the install of PyZMQ fails, you may need to install or build zeromq and zeromq-devel (see below).

matplotlib Begin by heading to the matplotlib github repository in your browser. There you will find a link which says zip. Click on the link and download the file. Extract the contents of the file, and navigate in the terminal to the directory which contains the extracted files. Build and install matplotlib by running

```
unzip matplotlib-matplotlib-v.1.1.0.411-glcd07a6.zip
cd matplotlib-matplotlib-v.1.1.0.411-glcd07a6
Python setup.py build
sudo Python setup.py install
```

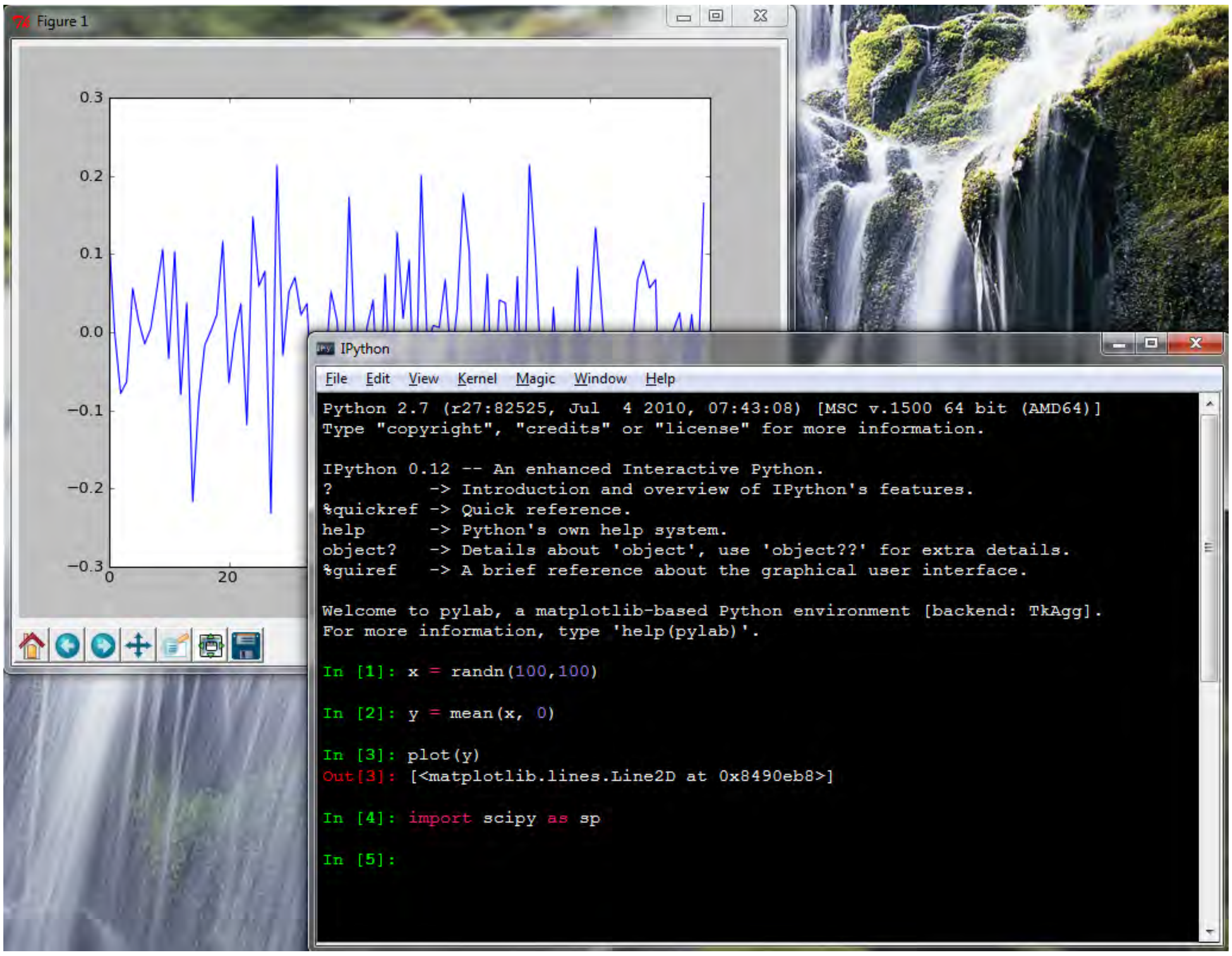
Note: The file name for the matplotlib source will change as it is further developed.

1.4.3 OS X

OS X is similar to Linux. I do not have access to an OS X computer for testing the installation procedure, and so no instructions are included. Instructions for installing Python (or Python 3) on OS X are readily available on the internet, and, once available, the remainder of the install should be similar to that of Linux.

1.5 Testing the Environment

To make sure that you have successfully installed the required components, run IPython using the shortcut previously created on windows, or by running `iPython --pylab` or `iPython-qtconsole --pylab` in a Linux terminal window. Enter the following commands, one at a time (Don't worry about what these mean).



```
Python 2.7 (r27:82525, Jul  4 2010, 07:43:08) [MSC v.1500 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 0.12 -- An enhanced Interactive Python.
?         -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help      -> Python's own help system.
object?   -> Details about 'object', use 'object??' for extra details.
%guiref   -> A brief reference about the graphical user interface.

Welcome to pylab, a matplotlib-based Python environment [backend: TkAgg].
For more information, type 'help(pylab)'.

In [1]: x = randn(100,100)

In [2]: y = mean(x, 0)

In [3]: plot(y)
Out[3]: [<matplotlib.lines.Line2D at 0x8490eb8>]

In [4]: import scipy as sp

In [5]:
```

Figure 1.5: A successful test that matplotlib, IPython, NumPy and SciPy were all correctly installed.

```
>>> x = randn(100,100)
>>> y = mean(x,0)
>>> plot(y)
>>> import scipy as sp
```

If everything was successfully installed, you should see something similar to figure 1.5.

1.6 Python Programming

Python can be programmed using an interactive session, preferably using IPython, or by executing Python scripts, which are simply test files which normally end with the extension .py.

1.6.1 Python and IPython

Most of this introduction focuses on interactive programming, which has some distinct advantages when learning a language. Interactive Python can be initiated using either the Python interpreter directly, by launching `Python.exe` (Windows) or `Python` (Linux). The standard Python interactive console is very basic, and does not support useful features such as tab completion. IPython, and especially the QtConsole version of IPython, transforms the console into a highly productive environment which supports a number of useful features:

- Tab completion - After entering 1 or more characters, pressing the tab button will bring up a list of functions, packages and variables which match have the same beginning. If the list of matches is long, a pager is used. Press 'q' to exit the pager.
- "Magic" function which make tasks such as navigating the local file system (using `%cd ~/directory/`) or running other Python programs (using `%run program.py`) simple. Entering `%magic` inside and IPython session will provide a detailed description of the available functions. Alternatively, `%lsmagic` provides a succinct list of available magic commands.
- Integrated help - When using the QtConsole, calling a function provides a view of the top of the help function. For example, `mean` computes the mean of an array of data. When using the QtConsole, entering `mean(` will produce a view of the top 15 lines or so of the help available for mean.
- Inline figures - The QtConsole can also display figure inline (when using the `--pylab=inline` switch when starting), which produces a neat environment. In some cases this may be desirable.
- The special variable _ contains the last result in the console. This results can be saved to a new variable (in this case, named x) using `x = _`.

1.6.2 Getting Help

Help is available in IPython sessions using `help(`*function*`)`. Some functions (and modules) have very long help files. These can be paged using the command `?`*function* or *function*`?`, and the text can be scrolled using page up and down, and q to quit. `??`*function* or *function*`??` can be used to type the function in the interactive console.

1.6.3 Configuring IPython

The IPython environment can be configured using standard Python scripts located in a configuration directory. On Windows, the start-up directory is located at C:\users\username\.iPython\profile_default\startup, and on Linux it is located at ~/.config/iPython/profile_default/startup. In this directory, create a file names startup.py, containing:

```
# __future__ imports
# division and print_function
import IPython.core.ipapi
ip = IPython.core.ipapi.get()
ip.ex('ip.compile("from __future__ import division", "<input>",  "single") in ip.user_ns')
ip.ex('ip.compile("from __future__ import print_function", "<input>",  "single") in ip.
    user_ns')

# Startup directory
import os
# Replace with actual directory
os.chdir('c:\\dir\\to\\start\\in')
# Linux: os.chdir('/dir/to/start/in/')
```

This code does 2 things. First, it imports 2 “future” features, the print function division, which are useful for numerical programming.

- In Python 2.7, `print` is not standard function and is used like `print 'string to print'`. Python 3.x changes this behavior to be a standard function call, `print('string to print')`. I prefer the latter since it will make the move to 3.x easier, and is more coherent.

- In Python 2.7, division of integers always produces an integer, and the result is truncated, so `9/5=1`. In Python 3.x, division of integers does not produce an integer if the integers are not even multiples, so `9/5=1.8`. Additionally, Python 3.x uses the syntax `9//5` to force integer division with truncation (e.g. `11/5=2.2`, while `11//5=2`).

Second, startup.py sets the startup directory to a location of your choosing.

1.6.4 Running Python programs

While interactive programing is useful for learning a language or quickly developing some simple code, more complex projects require the use of more complete programs. Programs can be run either using the IPython magic work `%run program.py` or by directly launching the Python program using the standard interpreter using `Python program.py` (Windows). The advantage of using the IPython environment is that the variables used in the program can be inspected after the end of the program run, while directly calling Python will run the program and then terminate – and so it is necessary to output any important results to a file so that they can be viewed later.[1]

To test that you can successfully execute a Python program, input the the code in the block below into a text file and save it as `firstprogram.py`.

```
# First Python program
from __future__ import print_function
from __future__ import division
import time

print('Welcome to your first Python program.')
raw_input('Press enter to exit the program.')
print('Bye!')
time.sleep(2)
```

Once you have saved this file, open the console, navigate to the directory you saved the file and run `Python firstprogram.py`. If the program does not run on Windows with an error that states Python cannot be found, you need to add the Python root directory to your path. The path can be located in the Control Panel, under Environment Variables. Finally, run the program in IPython by first launching IPython, and the using `%cd` to change to the location of the program, and finally executing the program using `%run firstprogram.py`.

1.6.4.1 Integrated Development Environments

As you progress in Python, and begin writing more sophisticated programs, you will find that using an Integrated Development Environment (IDE) will increase your productivity. Most contain productivity en-

[1]Programs can also be run in the standard Python interpreter using the command:
`exec(compile(open('filename.py').read(),'filename.py','exec'))`

hancements such as built-in consoles, intellisense (for completing function names) and integrated debugging. Discussion of IDEs is beyond the scope of this text, although I recommend Spyder (free, cross-platform).

1.7 Exercises

1. Install Python.
2. Test the installation using the code in section 1.5.
3. Configure IPython using the start-up script in section 1.6.3.
4. Customize IPython-QtConsole using a font or color scheme. More customizations can be found by running `ipyton-qtconsole -h`.
5. Explore tab completion in IPython by entering `a`<*TAB*> to see the list of functions which start with a and are loaded by pylab. Next try `i`<*TAB*>, which will produce a list longer than the screen – press q to exit the pager.
6. Install Spyder. For optimal performance, Spyder requires a number of additional packages. These are all available using `easy_install` *packageName*. The additional packages are:
   - pyflakes
   - rope
   - sphinx
   - pylint
   - pep8

Chapter 2

Python 2.7 vs. 3.2 (and the rest)

Python comes in a number of varieties which may be suitable for econometrics, statistics and numerical analysis. This chapter explains why, ultimately 2.7 was chosen for these notes, and highlights some alternatives.

2.1 Python 2.7 vs. 3.2

Python 2.7 is the final version of the Python 2.x line – all future development work will focus on Python 3.2. It may seem strange to learn an "old" language. The reasons for using 2.7 are:

- There are more modules available for Python 2.7. While all of the core python modules are available for both Python 2.7 and 3.2, some relevant modules are only available in 2.7, for example, modules which allow Excel files to be read and written to. Over time, many of these modules will be available for Python 3.2+, but they aren't today.
- The language changes relevant for numerical computing are *very* small – and these notes explicitly minimize these so that there should few changes needed to run against Python 3.2+ in the future (ideally none).
- Configuring and installing 2.7 is much easier, especially on Linux.

Learning Python 3.2 has some advantages:

- No need to update in the future
- Some improved out-of-box behavior for numerical applications.

2.2 Intel Math Kernel Library and AMD Core Math Library

Intel's MKL and AMD's CML provide optimized linear algebra routines. They are much faster then simple implementations and are, by default, multithreaded so that a matrix inversion can use all of the processors on your system. They are used by NumPy, although most pre-compiled code does not use them. The exception for Windows are the pre-built NumPy binaries made available by Christoph Gohlke. Directions for building NumPy on Linux with Intel's MKL are available online. It is strongly recommended that you use a

NumPy built using these highly tuned linear algebra routines if performance on matrix algebra operations is important. Alternatively, Enthought Python Distribution (see below) is built with MKL and is available for all Intel platforms (Windows, Linux and OS X).

2.3 Other Variants

Some other variants are worth mentioning.

2.3.1 Enthought Python Distribution

Enthought Python Distribution (EPD) is a collection of a large number of modules for scientific computing (including core Python). It is available for Windows, Linux and OS X. EPD is regularly updated and is currently available for free to members of academic institutions. EPD is also built using MKL, and so matrix algebra performance is very fast.

2.3.2 IronPython

IronPython is a variant which runs on the CLR (Windows .NET). The core modules – NumPy and SciPy – are available for IronPython, and so it is a viable alternative for numerical computing, especially if already familiar with the C# or another .NET language. Other libraries, for example, matplotlib (plotting) are not available, so there are some important caveats.

2.3.3 PyPy

PyPy is a new implementation of Python which uses Just-in-time compilation to accelerate code, especially loops (which are common in numerical computing). It may be anywhere between 2 - 5 times faster than standard Python. Unfortunately, at the time of writing, the core library, NumPy is only partially implemented, and so it is not ready for use. Current plans are to have a version ready by the end of 2012, and if so, PyPy may quickly become the preferred version of Python for numerical computing.

2.A Relevant Differences between Python 2.7 and 3.2

Most difference significant between Python 2.7 and 3.2 are not important for using Python in econometrics, statistics and numerical analysis. I will make three common assumptions which will allow 2.7 and 3.2 to be used interchangeable. These differences are important in stand-alone Python programs. The configuration instructions for IPython will produce similar behavior when run interactively.

2.A.1 `print`

`print` is a function used to display test in the console when running programs. In Python 2.7, `print` is a keyword which behaves differently from other functions. In Python 3.2, `print` behaves like most functions. The standard use in Python 2.7 is

```
print 'String to Print'
```

while in Python 3.2, the standard use is

```
print('String to Print')
```

which resembles calling a function. Python 2.7 contains a version of the Python 3.2 print, which can be used in any program by including

```
from __future__ import print_function
```

at the top of the file. I prefer the 3.2 version of print, and so I assume that all programs will include this statement.

2.A.2 division

Python 3.2 changes the way integers are divided. In Python 2.7, the ratio of two integers was always an integer, and was truncated towards 0 if the result was fractional. For example, in Python 2.7, 9/5 is 1. Python 3.2 gracefully converts the result to a floating point number, and so in Python 3.2, 9/5 is 1.8. When working with numerical data, automatically converting ratios avoids some rare errors. Python 2.7 can use the 3.2 behavior by including

```
from __future__ import division
```

at the top of the program. I assume that all programs will include this statement.

2.A.3 range and xrange

It is often useful to generate a sequence of number for use when iterating over the some data. In Python 2.7, the best practice is to use the keyword xrange to do this, while in Python 3.2, this keyword has been renamed range. Fortunately Python 2.7 contains a function range which is inefficient but compatible with the range function in Python 3.2, and so I will always use range, even where best practices indicate that xrange should be used. No changes are needed in code for use range in both Python 2.7 and 3.2.

Chapter 3

Built-in Data Types

Before diving into Python for analyzing data to running Monte Carlos, it is necessary to understand some basic concepts about the available data types in Python and NumPy. In many ways, this description is necessary since Python is a general purpose programming language which is also well suited to data analysis, econometrics and statistics. This differs from environments such as MATLAB and R which are statistical/numerical packages first, and general purpose programming languages second. For example, the basic numeric type in MATLAB is an array (using double precision, which is useful for *floating point* mathematics), while the numeric basic data type in Python is a 1-dimensional scalar which may be either integer or a double-precision floating point, depending on how the number is formatted when entered.

3.1 Variable Names

Variable names can take many forms, although they can only contain numbers, letters (both upper and lower), and underscores (_). They must begin with a letter or an underscore and are `CaSe SeNsItIve`. Additionally, some words are reserved in Python and so cannot be used for variable names (e.g. `import` or `for`). For example,

```
x = 1.0
X = 1.0
X1 = 1.0
X1 = 1.0
x1 = 1.0
dell = 1.0
dellreturns = 1.0
dellReturns = 1.0
_x = 1.0
x_ = 1.0
```

are all legal and distinct variable names. Note that names which begin or end with an underscore are convey special meaning in Python, and so should be avoided in general. Illegal names do not follow these rules.

```
# Not allowed
x: = 1.0
1X = 1
_x = 1
X-1 = 1
```

```
for = 1
```

Multiple variables can be assigned on the same line using

```
x, y, z = 1, 3.1415, 'a'
```

3.2 Core Native Data Types

3.2.1 Numeric

Simple numbers in Python can be either integers, floats or complex. Integers correspond to either 32 bit or 64-bit integers, depending on whether the python interpreter was compiled 32-bit or 64-bit, and floats are always 64-bit (corresponding to doubles in C/C++). Long integers, on the other hand, do not have a fixed size and so can accommodate numbers which are larger than maximum the basic integer type can handle. Note: This chapter does not cover all Python data types, only those which are most relevant for numerical analysis, econometrics and statistics. The following built-in data types are not described: bytes, bytearray and memoryview.

3.2.1.1 Floating Point (float)

The most important (scalar) data type for numerical analysis is the float. Unfortunately, not all non-complex numeric data types are floats. To input a floating data type, it is necessary to include a . (period, dot) in the expression. This example uses the function type() to determine the data type of a variable.

```
>>> x = 1
>>> type(x)
int

>>> x = 1.0
>>> type(x)
float

>>> x = float(1)
>>> type(x)
float
```

This example shows that using the expression that `x = 1` produces an integer while `x = 1.0` produces a float. Using integers can produce unexpected results and so it is important to ensure values entered manually are floats (e.g. include ".0" when needed).[1]

3.2.1.2 Complex (complex)

Complex numbers are also important for numerical analysis. Complex numbers are created in Python using `j` or the function `complex()`.

```
>>> x = 1.0
>>> type(x)
float
```

[1]Programs which contain `from __future__ import division` will automatically convert integers to floats when dividing.

```
>>> x = 1j
>>> type(x)
complex

>>> x = 2 + 3j
>>> x
(2+3j)

>>> x = complex(1)
>>> x
(1+0j)
```

Note that a+b`j` is the same as `complex(`a`,`b`)`, while `complex(`a`)` is the same as a+0`j`.

3.2.1.3 Integers (int and long)

Floats use an approximation to represent numbers which may contain a decimal portion. The integer data type stores numbers using an exact representation, so that no approximation is needed. The cost of the exact representation is that the integer data type cannot (naturally) express anything that isn't an integer. This renders integers of limited use in most numerical analysis work.

Basic integers can be entered either by excluding the decimal (see float), or explicitly using the `int()` function. The `int()` function can also be used to find the smallest integer (in absolute value) to a floating point number.

```
>>> x = 1
>>> type(x)
int

>>> x = 1.0
>>> type(x)
float

>>> x = int(x)
>>> type(x)
int
```

Integers can range from -2^{31} to $2^{31}-1$. Python contains another type of integer known as a long integers which has essentially no range. Long integers are entered using the syntax `x = 1L` or by calling `long()`. Additionally python will automatically convert integers outside of the standard integer range to long integers.

```
>>> x = 1
>>> x
1

>>> type(x)
int

>>> x = 1L
>>> x
```

```
1L

>>> type(x)
long

>>> x = long(2)
>>> type(x)
long

>>> x = 2 ** 64
>>> x
18446744073709551616L
```

The trailing `L` after the number indicates that it is a long integer, rather than a standard integer.

3.2.2 Boolean (bool)

The Boolean data type is used to represent true and false, using the reserved keywords `True` and `False`. Boolean variables are important for program flow control (see Chapter 12) and are typically created as a result of logical operations (see Chapter 11), although they can be entered directly.

```
>>> x = True
>>> type(x)
bool

>>> x = bool(1)
>>> x
True

>>> x = bool(0)
>>> x
False
```

Non-zero values, in general, evaluate to true when evaluated by `bool()`, although `bool(0)`, `bool(0.0)`, and `bool(None)` are all false.

3.2.3 Strings (str)

Strings are not usually important for *numerical* analysis, although they are frequently encountered when dealing with data files, especially when importing, or when formatting output for human readability (e.g. nice, readable tables of results). Strings are delimited using `''` or `""`. While either single or double quotes are valid for declaring strings, they cannot be mixed (e.g. do not try `'"`) in a single string, except when used to express a quotation.

```
>>> x = 'abc'
>>> type(x)
str

>>> y = '"A quotation!"'
>>> print(y)
"A quotation!"
```

String manipulation is further discussed in Chapter 18.

3.2.3.1 Slicing Strings

Substrings within a string an be accessed using *slicing*. Slicing uses [] to contain the indices of the characters in a string, where the first index is 0, and the last (assuming the string has n letters) and $n-1$. The following table describes the types of slices which are available. The most useful are `str[`i`]`, which will return the character in position i, `str[:`i`]`, which return the characters at the beginning of the string from positions 0 to $i-1$, and `str[`i`:]` which returns the characters at the end of the string from positions i to $n-1$. The table below provides a list of the types of slices which can be used. The second column shows that slicing can use negative indices which essentially index the string backward.

Slice	Behavior,	Slice	Behavior
`str[:]`	Returns all `str`	`str[`$-i$`]`	Returns letter $n-i$
`str[`i`]`	Returns letter i	`str[`$-i$`:]`	Returns letters $n-i,\ldots,n-1$
`str[`i`:]`	Returns letters $i,\ldots,n-1$	`str[:`$-i$`]`	Returns letters $0,\ldots,n-i$
`str[:`i`]`	Returns letters $0,\ldots,i-1$	`str[`$-j$`:`$-i$`:]`	Returns letters $n-j,\ldots,n-i$
`str[`i`:`j`:]`	Returns letters $i,\ldots,j-1$	`str[`j`:`i`:-1]`	Returns letters j,$j-1$,...,$i+1$

```
>>> text = 'Python strings are sliceable.'
>>> text[0]
'P'

>>> text[10]
'i'

>>> L = len(text)
>>> text[L] # Error
IndexError: string index out of range

>>> text[:10]
'Python str'

>>> text[10:]
'ings are sliceable.'
```

3.2.4 Lists (list)

Lists are a built-in data type which requires the other data types to be useful. A list is essentially a collection of other *objects* – floats, integers, complex numbers, strings or even other lists. Lists are essential to Python programming since they are used to store collections of other values. For example, a list of floats can be used to express a vector (although the NumPy data types array and matrix are better suited). Lists also support *slicing* to retrieve one or more elements. Basic lists are constructed using square braces, `[]`, and values are separated using commas, `,`.

```
>>> x = []
>>> type(x)
```

```
builtins.list

>>> x=[1,2,3,4]
>>> x
[1,2,3,4]

# 2-dimensional list (list of lists)
>>> x = [[1,2,3,4], [5,6,7,8]]
>>> x
[[1, 2, 3, 4], [5, 6, 7, 8]]

# Jagged list, not rectangular
>>> x = [[1,2,3,4] , [5,6,7]]
>>> x
[[1, 2, 3, 4], [5, 6, 7]]

# Mixed data types
>>> x = [1,1.0,1+0j,'one',None,True]
>>> x
[1, 1.0, (1+0j), 'one', None, True]
```

These examples show that lists can be regular, nested and can contain any mix of data types. `x = [[1,2,3,4], [5,6,7,8]]` is a 2-dimensional list, where the main elements of x are lists, and the elements of these lists are integers.

3.2.4.1 Slicing Lists

Lists, like strings, can be sliced. Slicing is similar, although lists can be sliced in more ways than strings. The difference arises since lists can be multi-dimensional while strings are always $1 \times n$. Basic list slicing is identical to strings, and operations such as `x[:]`, `x[1:]`, `x[:1]` and `x[-3:]` can all be used. To understand slicing, assume x is a 1-dimensioanl list with n elements and $i \geq 0, j > 0, i < j$. Python using 0-based indices, and so the n elements of x can be thought of as $x_0, x_1, \ldots, x_{n-1}$.

Slice	Behavior,	Slice	Behavior
`x[:]`	Return all x	`x[`i`]`	Return x_i
`x[`i`]`	Return x_i	`x[`$-i$`]`	Return x_{n-i}
`x[`i`:]`	Return $x_i, \ldots x_{n-1}$	`x[`$-i$`:]`	Return $x_{n-i}, \ldots, x_{n-1}$
`x[:`i`]`	Return $x_0, \ldots, x_{i-1}$	`x[:`$-i$`]`	Return $x_0, \ldots, x_{n-i}$
`x[`i`:`j`:]`	Return $x_i, x_{i+1}, \ldots x_{j-1}$	`x[`$-j$`:`$-i$`:]`	Return $x_{n-j}, \ldots, x_{n-i}$

Examples of accessing elements of 1-dimensional lists are presented in the code block below.

```
>>> x = [0,1,2,3,4,5,6,7,8,9]
>>> x[0]
0
>>> x[5]
5
```

```
>>> x[10] # Error
IndexError: list index out of range
>>> x[4:]
[4, 5, 6, 7, 8, 9]
>>> x[:4]
[0, 1, 2, 3]
>>> x[1:4]
[1, 2, 3]
>>> x[-0]
0
>>> x[-1]
9
>>> x[-10:-1]
[0, 1, 2, 3, 4, 5, 6, 7, 8]
```

List can be multidimensional, and slicing can be done directly in higher dimensions. For simplicity, consider slicing a 2D list `x = [[1,2,3,4], [5,6,7,8]]`. If single indexing is used, `x[0]` will return the first (inner) list, and `x[1]` will return the second (inner) list. Since the value returned by `x[0]` is sliceable, the inner list can be directly sliced using `x[0][0]` or `x[0][1:4]`.

```
>>> x = [[1,2,3,4], [5,6,7,8]]
>>> x[0]
[1, 2, 3, 4]
>>> x[1]
[5, 6, 7, 8]
>>> x[0][0]
1
>>> x[0][1:4]
[2, 3, 4]
>>> x[1][-4:-1]
[5, 6, 7]
```

3.2.4.2 List Functions

A number of functions are available for manipulating lists. The most useful are

Function	Method	Description
`list.append(x,` *value*`)`	`x.append(`*value*`)`	Appends *value* to the end of the list.
`len(x)`	–	Returns the number of elements in the list.
`list.extend(x,` *list*`)`	`x.extend(`*list*`)`	Appends the values in *list* to the existing list.
`list.pop(x,` *index*`)`	`x.pop(`*index*`)`	Removes the value in position *index*.
`list.remove(x,` *value*`)`	`x.remove(`*value*`)`	Removes the first occurrence of *value* from the list.
`list.count(x,` *value*`)`	`x.count(`*value*`)`	Counts the number of occurrences of *value* in the list.

```
>>> x = [0,1,2,3,4,5,6,7,8,9]
>>> x.append(0)
>>> x
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0]
>>> len(x)
```

```
11
>>> x.extend([11,12,13])
>>> x
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 11, 12, 13]
>>> x.pop(1)
>>> x
[0, 2, 3, 4, 5, 6, 7, 8, 9, 0, 11, 12, 13]
>>> x.remove(0)
>>> x
[2, 3, 4, 5, 6, 7, 8, 9, 0, 11, 12, 13]
```

3.2.4.3 del

Elements can also be deleted from lists using the keyword `del` in combination with a slice.

```
>>> x = [0,1,2,3,4,5,6,7,8,9]
>>> del x[0]
>>> x
[1, 2, 3, 4, 5, 6, 7, 8, 9]

>>> x[:3]
[1, 2, 3]

>>> del x[:3]
>>> x
[4, 5, 6, 7, 8, 9]

>>> del x[1:3]
>>> x
[4, 7, 8, 9]

>>> del x[:]
>>> x
[]
```

3.2.5 Tuples (tuple)

A tuple is in many ways like a list. A tuple contains multiple pieces of data which comprised of a variety of data types. Aside from using a different syntax to construct a tuple, they are close enough to lists to ignore the difference *except that tuples are immutable*. Immutability means that the elements of tuple cannot change, and so once a tuple is constructed, it is not possible to change an element without reconstructing a new tuple.

Tuples are constructed using parentheses (`()`), rather than square braces (`[]`) of lists. Tuples can be sliced in an identical manner as lists. A list can be converted into a tuple using `tuple()` (Similarly, a tuple can be converted to list using `list()`).

```
>>> x =(0,1,2,3,4,5,6,7,8,9)
>>> type(x)
tuple
```

```
>>> x[0]
0

>>> x[-10:-5]
(0, 1, 2, 3, 4)

>>> x = list(x)
>>> type(x)
list

>>> x = tuple(x)
>>> type(x)
tuple
```

Note that tuples must have a comma when created, so that `x = (2,)` is assign a tuple to x, while `x=(2)` will assign 2 to x. The latter interprets the parentheses as if they are part of a mathematical formula, rather than being used to construct a tuple. `x = tuple([2])` can also be used to create a single element tuple. Lists do not have this issue since square brackets are reserved.

```
>>> x =(2)
>>> type(x)
int

>>> x = (2,)
>>> type(x)
tuple

>>> x = tuple([2])
>>> type(x)
tuple
```

3.2.5.1 Tuple Functions

Tuples are immutable, and so only have the functions `index` and `count`, which behave in an identical manner to their list counterparts.

3.2.6 Xrange (xrange)

A xrange is a useful data type which is most commonly encountered when using a `for` loop. Range are essentially lists of numbers. `xrange(a,b,i)` creates the sequences that follows the pattern $a, a+i, a+2i, \ldots, a+(m-1)i$ where $m = \lceil \frac{b-a}{i} \rceil$. In other words, it find all integers x starting with a such $a \leq x < b$ and where two consecutive values are separated by i. Range can also be called with 1 or two parameters. `xrange(a,b)` is the same as `xrange(a,b,1)` and `xrange(b)` is the same as `xrange(0,b,1)`.

```
>>> x = xrange(10)
>>> type(x)
xrange

>>> print(x)
```

```
xrange(0, 10)

>>> list(x)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

>>> x = xrange(3,10)
>>> list(x)
[3, 4, 5, 6, 7, 8, 9]

>>> x = xrange(3,10,3)
>>> list(x)
[3, 6, 9]

>>> y = range(10)
>>> type(y)
list

>>> y
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

Xrange is not technically a list, which is why the statement `print(x)` returns `xrange(0,10)`. Explicitly converting a range to a list using `list()` produces a list which allows the values to be printed. Technically `xrange` is an iterator which does not actually require the storage space of a list. This is a performance optimization, and is not usually important in numerical applications.

3.2.7 Dictionary (dict)

Dictionaries are encountered far less frequently than then any of the previously described data types in numerical Python. They are, however, commonly used to pass options into other functions such as optimizers, and so familiarity with dictionaries is essential. Dictionaries in Python are similar to the more familiar type in that they are composed of keys (words) and values (definitions). In Python dictionaries keys must be unique strings, and values can contain any valid Python data type. Values in dictionaries are accessed by their place in the list; values in dictionaries are accessed using keys.

```
>>> data = {'key1': 1234, 'key2' : [1,2]}
>>> type(data)
builtins.dict
>>> data['key1']
1234
```

Values associated with an existing key can be updated by making an assignment to the key in the dictionary.

```
>>> data['key1'] = 'xyz'
>>> data['key1']
'xyz'
```

New key-value pairs can be added by defining a new key and assigning a value to it.

```
>>> data['key3'] = 'abc'
>>> data
{'key1': 1234, 'key2': [1, 2], 'key3': 'abc'}
```

Key-value pairs can be deleted using the reserved keyword `del`.

```
>>> del data['key1']
>>> data
{'key2': [1, 2], 'key3': 'abc'}
```

3.2.8 Sets (set, frozenset)

Sets are collections which contain all *unique* elements of a collection. `set` and `frozenset` only differ in that the latter is immutable (and so has higher performance). While sets are generally not important in numerical analysis, they can be very useful when working with messy data – for example, finding the set of unique tickers in a long list of tickers.

3.2.8.1 Set Functions

add,difference,difference_update,intersection,intersection_update,union,remove,

A number of functions are available for manipulating lists. The most useful are

Function	Method	Description
`set.add(x,`*element*`)`	`x.add(`*element*`)`	Appends *element* to a set.
`len(x)`	–	Returns the number of elements in the set.
`set.difference(x,`*set*`)`	`x.difference(`*set*`)`	Returns the elements in x which are not in *set*.
`set.intersection(x,`*set*`)`	`x.intersection(`*set*`)`	Returns the elements of x which are also in *set*.
`set.remove(x,`*element*`)`	`x.remove(`*element*`)`	Removes *element* from the set.
`set.union(x,`*set*`)`	`x.union(`*set*`)`	Returns the set containing all elements of x and *set*.

```
>>> x = set(['MSFT','GOOG','AAPL','HPQ'])
>>> x
set(['GOOG', 'AAPL', 'HPQ', 'MSFT'])

>>> x.add('CSCO')
>>> x
set(['GOOG', 'AAPL', 'CSCO', 'HPQ', 'MSFT'])

>>> y = set(['XOM', 'GOOG'])
>>> x.intersection(y)
set(['GOOG'])

>>> x = x.union(y)
>>> x
set(['GOOG', 'AAPL', 'XOM', 'CSCO', 'HPQ', 'MSFT'])

>>> x.remove('XOM')
set(['GOOG', 'AAPL', 'CSCO', 'HPQ', 'MSFT'])
```

3.3 Python and Memory Management

Python uses a highly optimized memory allocation system which attempts to avoid allocating unnecessary memory. As a result, when one variable is assigned to another (e.g. to `y = x`), these will actually point to the same data in the computer's memory. To verify this, `id()` can be used to determine the unique identification number of a piece of data.[2]

```
>>> x = 1
>>> y = x
>>> id(x)
82970264L

>>> id(y)
82970264L

>>> x = 2.0
>>> id(x)
82970144L

>>> id(y)
82970264L
```

In the above example, the initial assignment of `y = x` produced two variables with the same ID. However, once `x` was changed, its ID changed while the ID of `y` did not, indicating that the data in each variable was stored in different locations. This behavior is very safe yet very efficient, and is common to the basic Python types: int, long, float, complex, string, xrange and tuple.

3.3.1 Example: Lists

Lists are mutable and so assignment does not create a copy – changes to either variable affect both.

```
>>> x = [1, 2, 3]
>>> y = x
>>> y[0] = -10
>>> y
[-10, 2, 3]

>>> x
[-10, 2, 3]
```

Slicing a list creates a copy of the list and *any immutable* types in the list – but not mutable elements in the list.

```
>>> x = [1, 2, 3]
>>> y = x[:]
>>> id(x)
86245960L

>>> id(y)
86240776L
```

[2]The ID numbers on your system will likely differ from those in the code listing.

For example, consider slicing a list of lists.

```
>>> x=[[0,1],[2,3]]
>>> y = x[:]
>>> y
[[0, 1], [2, 3]]

>>> id(x[0])
117011656L

>>> id(y[0])
117011656L

>>> x[0][0]
0.0

>>> y[0][0] = -10.0
>>> y
[[-10.0, 1], [2, 3]]

>>> x
[[-10.0, 1], [2, 3]]
```

When lists are nested or contain other mutable objects (which do not copy), slicing copies the outermost list to a new ID, but the inner lists (or other objects) are still linked. In order to copy nested lists, it is necessary to explicitly call `deepcopy()`, which is in the module `copy`.

```
>>> import copy as cp
>>> x=[[0,1],[2,3]]
>>> y = cp.deepcopy(x)
>>> y[0][0] = -10.0
>>> y
[[-10.0, 1], [2, 3]]

>>> x
[[0, 1], [2, 3]]
```

3.4 Exercises

1. Enter the following into Python, assigning each to a unique variable name:

   (a) `4`

   (b) `3.1415`

   (c) `1.0`

   (d) `2+4j`

   (e) `'Hello'`

   (f) `'World'`

2. What is the type of each variable? Use `type` if you aren't sure.

3. Which of the 6 types can be:

   (a) Added +

   (b) Subtracted -

   (c) Multiplied *

   (d) Divided /

4. What are the types of the output (when an error is not produced) in the above operations?

5. Input the variable `ex = 'Python is an interesting and useful language for numerical computing!'`. Using slices, how could you extract:

   (a) `Python`

   (b) `!`

   (c) `computing`

   (d) `in`
   Note: There are multiple answers for all.

   (e) `!gnitupmoc laciremun rof egaugnal lufesu dna gnitseretni na si nohtyP'` (Reversed)

   (f) `nohtyP`

   (g) `Pto sa neetn n sfllnug o ueia optn!`

6. What are the 2 methods to construct a tuple that has only a single item? How many ways are there to construct a list with a single item?

7. Construct a nested list to hold the matrix

$$\begin{bmatrix} 1 & .5 \\ .5 & 1 \end{bmatrix}$$

   so that item `[i][j]` corresponds to the position in the matrix (Remember that Python uses 0 indexing).

8. Assign the matrix you just created first to `x`, and then assign `y=x`. Change `y[0][0]` to `1.61`. What happens to `x`?

9. Next assign `z=x[:]` using a simple slice. Repeat the same exercise using `y[0][0] = 1j`. What happens to `x` and `z` ? What are the `id`s of `x`, `y` and `z`? What about `x[0]`, `y[0]` and `z[0]`?

10. How could you create `w` from `x` so that `w` can be changed without affecting `x`?

11. Initialize a list containing `4`, `3.1415`, `1.0`, `2+4j`, `'Hello'`, `'World'`. How could you:

    (a) Delete `1.0` if you knew it's position? What if you didn't know its position?

(b) How can the list `[1.0, 2+4j, 'Hello']` be added to the existing list?

(c) How can the list be reversed?

(d) In the extended list, how can you count the occurrence of `'Hello'`?

12. Construct a dictionary with the keyword-value pairs: `alpha` and `1.0`, `beta` and `3.1415`, `gamma` and `-99`. How can the value of `alpha` be retrieved?

13. Convert the final list at the end of problem 11 to a `set`. How is the set different from the list?

Chapter 4

Arrays and Matrices

NumPy provides the most important data types for econometrics, statistics and numerical analysis. The two data types provided by NumPy are the arrays and matrices. Arrays and matrices are closely related, and matrices are essentially a special case of arrays – 2 (and only 2)-dimensional arrays. The differences between arrays and matrices can be summarized as:

- Arrays can have 1, 2, 3 or more dimensions. Matrices always have 2 dimensions. This means that a 1 by n vector stored as an array has 1 dimension and 5 elements, while the same vector stored as a matrix has 2-dimensions where the sizes of the dimensions are 1 and n (in either order).

- Standard mathematical operators on arrays operate *element-by-element*. This is not the case for matrices, where multiplication (`*`) follows the rules of linear algebra. 2-dimensional arrays can be multiplied using the rules of linear algebra using `dot()`. Similarly, the function `multiply()` can be used on two matrices for element-by-element multiplication.

- Arrays are more common than matrices, and so all functions work and are tested with arrays (they *should also* work with matrices, but an occasional strange result may be encountered).

- Arrays can be quickly treated as a matrix using either `asmatrix()` or `mat()` without copying the underlying data.

4.1 Array

Arrays are the base data type in NumPy, and are the most important data type for numerical analysis in Python. In many ways, arrays are similar to lists in that they can be used to help collections of elements. The focus of this section is on arrays which only hold 1 type of data – whether it is float or int – and so all elements must have the same type (See Chapter 20). Additionally, arrays are always rectangular – in other words, if the first row has 10 elements, all other rows must have 10 elements.

Arrays are initialized using lists (or tuples), and calling `array()`. 2-dimensional arrays are initialized using lists of lists (or tuples of tuples, or lists of tuples, etc.), and higher dimensional arrays can be initialized by further nesting lists or tuples.

```
>>> x = [0.0, 1, 2, 3, 4]
>>> y = array(x)
>>> y
```

```
array([0, 1, 2, 3, 4])

>>> type(y)
NumPy.ndarray
```

2 (or higher) dimensional arrays are initialized using nested lists.

```
>>> y = array([[0.0, 1, 2, 3, 4], [5, 6, 7, 8, 9]])
>>> y
array([[ 0.,  1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.,  9.]])

>>> shape(y)
(2L, 5L)

>>> y = array([[[1,2],[3,4]],[[5,6],[7,8]]])
>>> y
array([[[1, 2],
        [3, 4]],

       [[5, 6],
        [7, 8]]])

>>> shape(y)
(2L, 2L, 2L)
```

4.1.1 Array dtypes

Arrays can contain a variety to data types. The most useful is 'float64', which corresponds to the python built-in data type of float (and C/C+ double). By default, calls to `array()` will preserve the type of the input, if possible. If an input contains all integers, it will have a dtype of 'int32' (the built in data type 'int'). If an input contains integers, floats, or a mix of the two, the array's dtype will be float. It is contains a mix of integers, floats and complex types, the array will be complex.

```
>>> x = [0, 1, 2, 3, 4] # Integers
>>> y = array(x)
>>> y.dtype
dtype('int32')

>>> x = [0.0, 1, 2, 3, 4] # 0.0 is a float
>>> y = array(x)
>>> y.dtype
dtype('float64')

>>> x = [0.0 + 1j, 1, 2, 3, 4] # (0.0 + 1j) is a complex
>>> y = array(x)
>>> y
array([ 0.+1.j,  1.+0.j,  2.+0.j,  3.+0.j,  4.+0.j])

>>> y.dtype
dtype('complex128')
```

NumPy attempts to find the smallest data type which can represent the data when constructing an array. It is possible to force NumPy to use a particular dtype by passing another argument, `dtype=`*datetype* to `array()`.

```
>>> x = [0, 1, 2, 3, 4] # Integers
>>> y = array(x)
>>> y.dtype
dtype('int32')

>>> y = array(x, dtype='float64')
>>> y.dtype
dtype('float64')
```

Important: If an array has an integer dtype, trying to place a float into the array results in the float being truncated and stored as an integer. This is dangerous, and so in most cases, arrays should be initialized to contain floats unless a conscious decision is taken to have them contain a different data type.

```
>>> x = [0, 1, 2, 3, 4] # Integers
>>> y = array(x)
>>> y.dtype

dtype('int32')

>>> y[0] = 3.141592
>>> y
array([3, 1, 2, 3, 4])

>>> x = [0.0,1, 2, 3, 4] # 1 Float makes all float
>>> y = array(x)
>>> y.dtype
dtype('float64')

>>> y[0] = 3.141592
>>> y
array([ 3.141592,  1.      ,  2.      ,  3.      ,  4.      ])
```

4.2 Matrix

Matrices are essentially a subset of arrays, and behave in a virtually identical manner. The two important differences are:

- Matrices always have 2 dimensions
- Matrices follow the rules of linear algebra for `*`

1- and 2-dimensional arrays can be copied to a matrix by calling `matrix()` on an array. Alternatively, calling `mat()` or `asmatrix()` provides a faster method where an array can behave like a matrix (without being explicitly converted).

4.3 Arrays, Matrices and Memory Management

Arrays and matrices do not behave like lists – slicing an array does not create a copy. In general, when an array, matrix or list is sliced, the slice will refer to the same memory as original variable – this means changing an element in the slice also changes an element in the original variable.

```
>>> x = array([0.0, 1.0, 2.0])
>>> y = x
>>> x
array([ 0.,  1.,  2.])

>>> y
array([ 0.,  1.,  2.])

>>> id(x)
130165568L

>>> id(y)
130165568L

>>> y[0] = -1.0
>>> y
array([-1.,  1.,  2.])

>>> x
array([-1.,  1.,  2.])
```

`y = x` sets `x` and `y` to the same data, and so changing one changes the other. Next, consider what happens when `y` is a slice of `x`.

```
>>> x = array([[0.0, 1.0],[2.0,3.0]])
>>> y = x[0]
>>> y
array([ 0.,  1.])

>>> y[0] = -1.0
>>> y
array([ -1.,  1.])

>>> x # x changes too
array([[-1.,  1.],
       [ 2.,  3.]])
```

In order to get a new variable when slicing or assigning an array or a matrix, it is necessary to explicitly copy the data. Arrays or matrices can be copied by calling `copy`. Alternatively, they can also be copied by by calling `array()` on arrays, or `matrix()` on matrices.

```
>>> x = array([[0.0, 1.0],[2.0,3.0]])
>>> y = copy(x)
>>> id(x)
130166048L
```

```
>>> id(y)
130165952L

>>> y[0,0] = -10.0
>>> y
array([[-10.,   1.],
       [  2.,   3.]])

>>> x # No change in x
array([[ 0.,  1.],
       [ 2.,  3.]])

>>> z = x.copy()
>>> id(z)
130166432L

>>> w = array(x)
>>> id(w)
130166144L
```

w, x, y and z all have unique IDs are distinct. Changes to one will not affect any of the others.

Finally, assignments from functions which change the value automatically create a copy.

```
>>> x = array([[0.0, 1.0],[2.0,3.0]])
>>> y = x
>>> id(x)
130166816L

>>> id(y)
130166816L

>>> y = x + 1.0
>>> y
array([[ 1.,  2.],
       [ 3.,  4.]])

>>> id(y)
130167008L

>>> y = exp(x)
>>> y
array([[  1.        ,   2.71828183],
       [  7.3890561 ,  20.08553692]])

>>> id(y)
130166912L
```

Even trivial function such as `y = x + 0.0` create a copy of x, and so the only cases where explicit copying is required is when y is directly assigned a slice of x, y is changed, but x should not be.

4.4 Entering Data

Almost all of the data used in are matrices by construction, even if they are 1 by 1 (scalar), K by 1 or 1 by K (vectors). Vectors, both row (1 by K) and column (K by 1), can be entered directly into the command window. The mathematical notation

$$x = [1 \quad 2 \quad 3 \quad 4 \quad 5]$$

is entered as

```
>>> x=array([1.0,2.0,3.0,4.0,5.0])
array([ 1.,  2.,  3.,  4.,  5.])
```

when an array is needed or

```
>>> x=matrix([1.0,2.0,3.0,4.0,5.0])
>>> x
matrix([[ 1.,  2.,  3.,  4.,  5.]])
```

for a matrix. 1-dimensional arrays do not have row or column forms, but matrices do. The column vector,

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

is entered using a set of nested lists

```
>>> x=matrix([[1.0],[2.0],[3.0],[4.0],[5.0]])
>>> x
matrix([[ 1.],
        [ 2.],
        [ 3.],
        [ 4.],
        [ 5.]])
>>> x = array(x)
>>> array([[ 1.,  2.,  3.,  4.,  5.]])
```

The final two line show that converting a column matrix to an array eliminates any notion row and column.

4.5 Entering Matrices

Matrices are just rows of columns. For instance, to input

$$x = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix},$$

enter the matrix one row at a time, each in a list, and then surround the row lists with another list.

```
>>> x = array([[1.0,2.0,3.0],[4.0,5.0,6.0],[7.0,8.0,9.0]])
>>> x
array([[ 1.,  2.,  3.],
       [ 4.,  5.,  6.],
       [ 7.,  8.,  9.]])
```

4.6 Higher Dimension Arrays

Multi-dimensional (N-dimensional) arrays are available for N up to about 30, depending on the size of each matrix dimension. Manually initializing higher dimension arrays is tedious and error prone, and so it is better to use functions such as `zeros((2, 2, 2))` or `empty((2, 2, 2))`. Higher dimensional arrays are useful, e.g. when tracking matrix values through time, such as a time-varying covariance matrices.

4.7 Concatenation

Concatenation is the process by which one vector or matrix is appended to another. Arrays and matrices can be concatenation horizontally or vertically. For instance, suppose

$$x = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } y = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix};$$

and

$$z = \begin{bmatrix} x \\ y \end{bmatrix}.$$

needs to be constructed. This can be accomplished by treating `x` and `y` as elements of a new matrix and using the function `concatenate` using the named parameter `axis` to determine whether the matrices are vertically (`axis = 0`) or horizontally (`axis = 1`) concatenated.

```
>>> x = array([[1.0,2.0],[3.0,4.0]])
>>> y = array([[5.0,6.0],[7.0,8.0]])
>>> z = concatenate((x,y),axis = 0)
>>> z
array([[ 1.,  2.],
       [ 3.,  4.],
       [ 5.,  6.],
       [ 7.,  8.]])

>>> z = concatenate((x,y),axis = 1)
>>> z
array([[ 1.,  2.,  5.,  6.],
       [ 3.,  4.,  7.,  8.]])
```

Concatenating is the code equivalent of block-matrix forms in standard matrix algebra. Alternatively the functions `vstack` and `hstack` can be used to vertically or horizontally stack arrays, respectively.

```
>>> z = vstack((x,y)) # Same as z = concatenate((x,y),axis = 0)
>>> z = hstack((x,y)) # Same as z = concatenate((x,y),axis = 1)
```

4.8 Accessing Elements of Array (Slicing)

Arrays, like lists and tuples, can be sliced. Slicing in arrays is virtually identical to slicing in lists, except that since arrays are explicitly multidimensional and rectangular, slicing in more than 1-dimension is implemented using a different syntax. 1-dimensional arrays can be sliced in an identical manner as lists or tuples. 2 (or higher)-dimensional arrays are sliced using the syntax `[:,:,...,:]` (where the number of dimensions of the arrays determines the size of the slice). The 2-dimensions, first dimension is always the row, and the second is the column.

```
>>> y = array([[0.0, 1, 2, 3, 4],[5, 6, 7, 8, 9]])
>>> y
array([[ 0.,  1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.,  9.]])

>>> y[0,:] # Row 0, all columns
array([ 0.,  1.,  2.,  3.,  4.])

>>> y[:,0] # all rows, column 0
array([ 0.,  5.])

>>> y[0,0:3] # Row 0, columns 0 to 3
array([ 0.,  1.,  2.])

>>> y[0:,3:] # Row 0 and 1, columns 3 and 4
array([[ 3.,  4.],
       [ 8.,  9.]])

>>> y = array([[[1.0,2],[3,4]],[[5,6],[7,8]]])
>>> y[0,:,:] # Panel 0 of 3D y
array([[1, 2],
       [3, 4]])

>>> y[0] # Same as y[0,:,:]
array([[1., 2.],
       [3., 4.]])

>>> y[0,0,:] # Row 0 of panel 0
array([1., 2.])

>>> y[0,1,0] # Panel 0, row 1, column 0
3.0
```

4.8.1 Linear Slicing using `flat`

k-dimensional arrays can be sliced using the `[:,:,...,:]` syntax, or they can be linear sliced. Linear slicing assigns an index to each element of the array, starting with the first (0), the second (1), and so on up to the last ($n-1$). In 2-dimensions, linear slicing works be first counting across rows, and then down columns. To use linear slicing, the method or function `flat` must first be used

```
>>> y = reshape(arange(25.0),(5,5))
>>> y
array([[  0.,   1.,   2.,   3.,   4.],
       [  5.,   6.,   7.,   8.,   9.],
       [ 10.,  11.,  12.,  13.,  14.],
       [ 15.,  16.,  17.,  18.,  19.],
       [ 20.,  21.,  22.,  23.,  24.]])

>>> y[0]
array([ 0.,  1.,  2.,  3.,  4.])

>>> y.flat[0]
0

>>> y[6] # Error
IndexError: index out of bounds

>>> y.flat[6]
6.0

>>> y.flat[:]
array([[   0.,   1.,   2.,   3.,   4.,   5.,   6.,   7.,   8.,   9.,  10.,
          11.,  12.,  13.,  14.,  15.,  16.,  17.,  18.,  19.,  20.,  21.,
          22.,  23.,  24.]])
```

arange and reshape are useful functions are described in later chapters.

Once a vector or matrix has been constructed, it is important to be able to access the elements individually. Data in matrices is stored in *row-major order*. This means elements are indexed by first counting across rows and then down columns. For instance, in the matrix

$$x = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

the first element of x is 1, the second element is 2, the third is 3, the fourth is 4, and so on.

4.9 import and Modules

Python, by default, only has access to a small number of built-in types and functions. The vast majority of functions are located in modules, and before a function can be accessed, the module which contains the function must be imported. For example, when using `ipython --pylab` (or any variants), a large number of modules are automatically imported, including NumPy and matplotlib. This is useful for learning, but care is needed to make sure that the correct module is imported when working in stand-alone python.

`import` can be used in a variety of ways. The simplest is to use `from` *module* `import *`. This will import all functions in *module* and make them immediately available. This method of using `import` can dangerous since if you use it more than once, it is possible for functions to be hidden by later imports. For example,

```
from pylab import *
from numpy import *
```

creates a conflict for `load` which is first imported by pylab (from `matplotlib.pylab.load`), and then imported by NumPy (from `numpy.lib.npyio.load`). A better method is to just import the required functions. This still places functions at the top level of the namespace, but can be used to avoid conflicts.

```
from pylab import load # Will import load only
from numpy import array, matrix # Will not import the load from NumPy
```

The functions `load`, `array` and `matrix` can be directly called. An alternative, and more common, method is to use `import` in the form

```
import pylab
import scipy
import numpy
```

or the closely related alternative

```
import pylab as pl
import scipy as sp
import numpy as np
```

The only difference between these two is that `import numpy` is equivalent to `import numpy as numpy`. When this form of import is used, function will be located below the "as" name. For example, the load provided by NumPy, is located at `np.load`, while the pylab load is `pl.load` – and both can be used where appropriate. While this method is the most general, it does require slightly more typing.

4.10 Calling Functions

Functions calls have different conventions other expressions. The most important difference is that functions can take more than one input and return more than one output. The generic structure of a function call is *out1*, *out2*, *out3*, ... `=` `functionname(`*in1*, *in2*, *in3*, ...`)`. The important aspects of this structure are

- If multiple outputs are returned, but only one output variable is provided, the output will (generally) be a tuple.
- The number of output variables determines how many outputs will be returned. Asking for more outputs than the function provides will result in an error.
- Both inputs and outputs must be separated by commas (,)
- Inputs can be the result of other functions as long only one output is returned. For example, the following are equivalent,

  ```
  >>> y = var(x)
  >>> mean(y)
  ```

  and

  ```
  >>> mean(var(x))
  ```

Required Arguments Most functions have required arguments. For example, consider the definition of `array` from `help(array)`,

```
array(object, dtype=None, copy=True, order=None, subok=False, ndmin=0)
```

Array has 1 required input, `object`, which is usually the list or tuple which contains values to use when creating the array. Required arguments can be determined by inspecting the function signature since all of the input follow the patters *keyword=default* except `object` – required arguments will not have a default value provided. The other arguments can be called in order (`array` accepts at most 2 non-keyword arguments).

```
>>> array([[1.0,2.0],[3.0,4.0]])
array([[ 1.,  2.],
       [ 3.,  4.]])

>>> array([[1.0,2.0],[3.0,4.0]], 'int32')
array([[1, 2],
       [3, 4]])
```

Keyword Arguments All of the arguments to `array` can be called by their keyword, which is listed in the help file definition.

```
array(object=[[1.0,2.0],[3.0,4.0]])
array([[1.0,2.0],[3.0,4.0]], dtype=None, copy=True, order=None, subok=False, ndmin=0)
```

The real advantage of keyword arguments is that they do not have to appear in any order (Note: randomly ordering arguments is not good practice, and this is only an example).

```
>>> array(dtype='complex64', object = [[1.0,2.0],[3.0,4.0]], copy=True)
array([[ 1.+0.j,  2.+0.j],
       [ 3.+0.j,  4.+0.j]], dtype=complex64)
```

Default Arguments Functions have defaults for optional arguments. These are listed in the function definition and appear in the help in *keyword=default* pairs. Returning to `array`, all inputs have default arguments except `object`, which is the only required input.

Multiple Outputs Some functions can have more than 1 output. These functions can be used in a single output mode or in multiple output mode. For example, shape can be used on an array to determine the size of each dimension.

```
>>> x = array([[1.0,2.0],[3.0,4.0]])
>>> s = shape(x)
>>> s
(2L, 2L)
```

Since shape will return as many outputs as there are dimensions, it can be called with 2 outs when the input is a 2-dimensional array.

```
>>> x = array([[1.0,2.0],[3.0,4.0]])
>>> M,N = shape(x)
>>> M
2L
```

```
>>> N
2L
```

Requesting more outputs than are required will produce an error.

```
>>> M,N,P = shape(x) # Error
ValueError: need more than 2 values to unpack
```

Similarly, providing two few output can also produce an error. Consider the case where the argument used with shape is a 3-dimensional array.

```
>>> x = randn(10,10,10)
>>> shape(x)
(10L, 10L, 10L)
>>> M,N = shape(x) # Error
ValueError: too many values to unpack
```

4.11 Exercises

1. Input the following mathematical expressions into Python as both arrays and matrices.

$$u = [1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8]$$

$$v = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \\ 5 \\ 8 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

$$w = \begin{bmatrix} x & x \\ y & y \end{bmatrix}$$

   Note: A column vector must be entered as a 2-dimensional array.

2. What command would pull x out of w? (Hint: `w[?,?]` is the same as x.)

3. What command would pull $\left[x' \; y'\right]'$ out of `w`? Is there more than one? If there are, list all alternatives.

4. What command would pull y out of z? List all alternatives.
5. Explore the options for creating an array using keyword arguments. Create an array containing

$$y = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$$

with combination of keyword arguments in:

   (a) `dtype` in `float`, `float64`, `int32` (32-bit integers), `uint32` (32-bit unsigned integers) and `complex128` (double precision complex numbers).
   (b) `copy` either `True` or `False`.
   (c) `ndim` either `3` or `4`. Use `shape(y)` to see the effect of this argument.

6. Enter $y = [1.6180\ 2.7182\ 3.1415]$ as an array. Define `x = mat(y)`. How is x different from y?

Chapter 5

Basic Math

Note: Python contains a math module that contains functions which operate on built-in scalar data types (e.g. float and complex). This and subsequent chapters assume mathematical functions must operate on arrays and matrices, and so are imported from NumPy.

5.1 Operators

These standard operators are available:

Operator	Meaning	Example	Algebraic
+	Addition	x + y	$x+y$
-	Subtraction	x - y	$x-y$
*	Multiplication	x * y	xy
/	Division (Left divide)	x/y	$\frac{x}{y}$
**	Exponentiation	x**y	x^y

When x and y are scalars, the behavior of these operators is obvious. The only exception occurs when both x and y are integers for division, where x/y returns the smallest integer less than the ratio (e.g. $\lfloor \frac{x}{y} \rfloor$). The simplest method to avoid this problem is to explicitly avoid integers by using 5.0 rather than 5. Alternatively, integers can be explicitly cast to floats before the division.

```
>>> x = 9
>>> y = 5
>>> (type(x), type(y))
(int, int)

>>> x/y
1

>>> float(x)/y
1.8
```

When x and y are arrays or matrices, things are a bit more complex. The examples usually refer to arrays, and except where explicit differences are noted, it is safe to assume that the behavior is identical for 2-dimensional arrays and matrices.

I recommend using the import command from `__future__ import division` in all programs and IPython. The "future" division avoids this issue by always casting division to floating point.

5.2 Broadcasting

Under the normal rules of array mathematics, addition and subtraction are only defined for arrays with the same shape or between an array and a scalar. For example, there is no obvious method to add a 5-element vector and a 5 by 4 matrix. NumPy uses a technique called broadcasting to allow mathematical operations on arrays (and matrices) which would not be compatible under the normal rules of array mathematics. Arrays can be used in element-by-element mathematics if x is broadcastable to y.

Suppose x is an m-dimensional array with dimensions $d = [d_1, d_2 \ldots d_m]$, and y is an n-dimensional array with dimensions $f = [f_1, f_2 \ldots f_n]$ where $m \geq n$. Formally, the rules of broadcasting are:

1. If $m > n$, then treat y as a m-dimensional array with size $g = [1, 1, \ldots, 1, f_1, f_2 \ldots f_n]$ where the number of 1s prepended is $m - n$. The elements are $g_i = 1$ for $i = 1, \ldots m - n$ and $g_i = f_{i-m+n}$ for $i > m - i$.

2. For $i = 1, \ldots, m$, $\max\left(d_i, g_i\right) / \min\left(d_i, g_i\right) \in \left\{1, \max\left(d_i, g_i\right)\right\}$.

The first rule is simply states that if one array has fewer dimensions, it is treated as having the same number of dimensions as the larger array by prepending 1s. The second rule states that arrays will only be broadcastable is either (a) they have the same dimension along axis i or (b) one has dimension 1 along axis i. When 2 arrays are broadcastable, the dimension of the output array is simply $\max\left(d_i, g_i\right)$ for $i = 1, \ldots n$.

Consider the following examples:

x	y	Broadcastable	Output Size	x Operation	y Operation
Any	Scalar	✓	Same as x	`x`	`tile(y,shape(x))`
$m, 1$	$1, n$ or n	✓	m, n	`tile(x,(1,n))`	`tile(y,(m,1))`
$m, 1$	$n, 1$	×			
m, n	$1, n$ or n	✓	m, n	`x`	`tile(y,(m,1))`
$m, n, 1$	$1, 1, p$ or $1, p$ or p	✓	m, n, p	`tile(x,(1,1,p))`	`tile(y,(m,n,1))`
m, n, p	$1, 1, p$ or $1, p$ or p	✓	m, n, p	`x`	`tile(y,(m,n,1))`
$m, n, 1$	$p, 1$	×			
$m, 1, p$	$1, n, 1$ or $n, 1$	✓	m, n, p	`tile(x,(1,n,1))`	`tile(y,(m,1,p))`

One simple method to visualize broadcasting is to use an add and subtract operation where the addition causes the smaller array to be broadcast, and then the subtract removes the values in the larger array. In this example, x is 3 by 5, so y must be either scalar or a 5-element array to be broadcastable. When y is a 3-element array (and so matches the *leading* dimension), an error occurs.

```
>>> x = reshape(arange(15),(3,5))
>>> x
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14]])
>>> y = 1
>>> x + y - x
array([[5, 5, 5, 5, 5],
       [5, 5, 5, 5, 5],
       [5, 5, 5, 5, 5]])

>>> y = arange(5)
>>> y
array([0, 1, 2, 3, 4])

>>> x + y - x
array([[0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4]])

>>> y = arange(3)
>>> y
array([0, 1, 2])

>>> x + y - x # Error
ValueError: operands could not be broadcast together with shapes (3,5) (3)
```

5.3 Array and Matrix Addition (+) and Subtraction (−)

Subject to broadcasting restrictions, addition and subtraction works in the standard way element-by-element.

5.4 Array Multiplication (*)

The standard multiplication operator differs for variables with type `array` and `matrix`. For arrays * is *element-by-element* multiplication and arrays must be broadcastable. For matrices, * is matrix multiplication as defined by linear algebra, and there is *no broadcasting*.

Conformable arrays can be multiplied according to the rules of matrix algebra using the function `dot()`. For simplicity, assume `x` is N by M and `y` is K by L. `dot(x,y)` will produce the array N by L array `z[i,j] = =dot( x[i,:], y[:,j])` where dot on 1-dimensional arrays is the usual vector dot-product. The behavior of `dot()` is described as:

		y	
		Scalar	Array
x	Scalar	Any $z = xy$	Any $z_{ij} = xy_{ij}$
	Array	Any $z_{ij} = yx_{ij}$	Inside Dimensions Match $z_{ij} = \sum_{k=1}^{M} x_{ik}y_{kj}$

These rules conform to the standard rules of matrix multiplication. `dot()` can also be used on higher dimensional arrays, and is useful if `x` is T by M by N and y is N by P to produce an output matrix which is

T by M by P, where each of the M by P (T in total) have the form `dot(x[i],y)`.

5.5 Matrix Multiplication (*)

If x is N by M and y is K by L and both are non-scalar matrices, `x*y` requires $M = K$. Similarly, `y*x` requires $L = N$. If x is scalar and y is a matrix, then `z=x*y` produces `z(i,j)=x*y(i,j)`.
Suppose `z=x * y` where both x and y are matrices:

		y	
		Scalar	Matrix
x	Scalar	Any	Any
		$z = xy$	$z_{ij} = xy_{ij}$
	Matrix	Any	Inside Dimensions Match
		$z_{ij} = yx_{ij}$	$z_{ij} = \sum_{k=1}^{M} x_{ik} y_{kj}$

Note: These conform to the standard rules of matrix multiplication.

`multiply()` provides element-by-element multiplication of matrices. Suppose `z=multiply(x,y)` where x and y are matrices:

		y	
		Scalar	Array
x	Scalar	Any	Any
		$z = xy$	$z_{ij} = xy_{ij}$
	Array	Any	Both Dimensions Match
		$z_{ij} = yx_{ij}$	$z_{ij} = x_{ij} y_{ij}$

Multiply will use broadcasting if necessary, and so matrices are effectively treated as 2-dimensional arrays.

5.6 Array and Matrix Division (/)

Division is always element-by-element, and the rules of broadcasting are used.

5.7 Array Exponentiation (**)

Array exponentiation operates element-by-element, and the rules of broadcasting are used.

5.8 Matrix Exponentiation (**)

Matrix exponentiation differs from array exponentiation, and can only be used on square matrices. When x is a square matrix and y is an integer, and `z=x*x*...*x` (y times). Python does not support non-integer values for y, although x^p can be defined (in linear algebra) using eigenvalues and eigenvectors for a subset of all matrices.

5.9 Parentheses

Parentheses can be used in the usual way to control the order in which mathematical expressions are evaluated, and can be nested to create complex expressions. See section 5.11 on Operator Precedence for more information on the order mathematical expressions are evaluated.

5.10 Transpose

Matrix transpose is expressed using either the `transpose()` function, or the shortcut `.T`. For instance, if `x` is an M by N matrix, `transpose(x)`, `x.transpose()` and `x.T` are all its transpose with dimensions N by M. In practice, using the `.T` will improve readability of code. Consider

```
>>> x = randn(2,2)
>>> xpx1 = x.T * x
>>> xpx2 = x.transpose() * x
>>> xpx3 = transpose(x) * x
```

Transpose has no effect on 1-dimensaional arrays. In 2-dimensions, transpose switches indices so that if `z=x.T`, `z[j,i]` is that same as `x[i,j]`. In higher dimensions, transpose reverses the order or the indices. For example, if `x` has 3 dimensions and `z=x.T`, then `x[i,j,k]` is the same as `z[k,j,i]`. Transpose takes an optional second argument, which can be used to manually determine the order of the axes after the transposition.

5.11 Operator Precedence

Computer math, like standard math, has operator precedence which determined how mathematical expressions such as

```
2**3+3**2/7*13
```

are evaluated. Best practice is to always use parentheses to avoid ambiguity in the order or operations. The order of evaluation is:

Operator	Name	Rank
`( )`	Parentheses	1
`**`	Exponentiation	2
`+,-`	Unary Plus, Unary Minus	3
`*, /, %`	Multiply, Divide, Modulo	3
`+,-`	Addition and Subtraction	4
<, <=, >, >=	Comparison operators	5
`==, !=`	Equality operators	6
`=,+=,-=,/=,*=,**=`	Assignment Operators	7
`is`, `is not`	Identity Operators	8
`in`, `not in`	Membership Operators	9
`and` , `or`, `not`	Logical Operators	10

In the case of a tie, operations are executed left-to-right. For example, `x**y**z` is interpreted as `(x**y)**z`. This table has omitted some operators available in Python (bit-wise) which are not useful (in general) in numerical analysis.

Note: Unary operators are + or - operations that apply to a single element. For example, consider the expression `(-4)`. This is an instance of a unary - since there is only 1 operation. `(-4)**2` produces 16. `-4**2` produces -16 since ** has higher precedence than unary negation and so is interpreted as `-(4**2)`. `-4 * -4` produces 16 since it is interpreted as `(-4) * (-4)`, since unary negation has higher precedence than multiplication.

5.12 Exercises

1. Using the arrays entered in exercise 1 of chapter 4, compute the values of $u + v'$, $v + u'$, vu, uv and xy (where the multiplication is as defined as linear algebra).

2. Repeat exercise 1 treating the inputs as matrices.

3. Which of the arrays in exercise 1 are broadcastable with:

$$a = [3\ 2],$$

$$b = \begin{bmatrix} 3 \\ 2 \end{bmatrix},$$

$$c = [3\ 2\ 1\ 0],$$

$$d = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}.$$

4. Is `x/1` legal? If not, why not. What about `1/x`?

5. Compute the values `(x+y)**2` and `x**2+x*y+y*x+y**2`. Are they the same when x and y are arrays? What if they are matrices?

6. Is `x**2+2*x*y+y**2` the same as any of the above?

7. When will `x**y` for matrices be the same as `x**y` for vectors?

8. For conformable arrays, is `a*b+a*c` the same as `a*b+c`? If so, show it, if not, how can the second be changed so they are equal.

9. Suppose a command `x**y*w+z` was entered. What restrictions on the dimensions of `w`, `x`, `y` and `z` must be true for this to be a valid statement?

10. What is the value of `-2**4`? What about `(-2)**4`? What about `-2*-2*-2*-2`?

Chapter 6

Basic Functions

6.1 Generating Arrays and Matrices

linspace

`linspace`(l,u,n) generates a set of n points uniformly spaced between l, a lower bound (inclusive) and u, an upper bound (inclusive).

```
>>> x = linspace(0, 10, 11)
>>> x
array([  0.,   1.,   2.,   3.,   4.,   5.,   6.,   7.,   8.,   9.,  10.])
```

logspace

`logspace`(l,u,n) produces a set of logarithmically spaced points between 10^l and 10^u. It is identical to `10**linspace(l,u,n)`.

arange

`arange`$(l,u,$`s`$)$ a set of points spaced by s between l, a lower bound (inclusive) and u, an upper bound (exclusive). `arange` can be used with a single parameter, so that `arange(n)` is equivalent to `arange(0,n,1)`. `arange` will return integer data type if all inputs are integer.

```
>>> x = arange(11)
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10])

>>> x = arange(11.0)
array([  0.,   1.,   2.,   3.,   4.,   5.,   6.,   7.,   8.,   9.,  10.])

>>> x = arange(4, 10, 1.25)
array([ 4.  ,  5.25,  6.5 ,  7.75,  9.  ])
```

meshgrid

`meshgrid` is a useful function for broadcasting two vectors into grids when plotting functions in 3 dimensions.

```
>>> x = arange(5)
>>> y = arange(3)
>>> X,Y = meshgrid(x,y)
>>> X
array([[0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4]])

>>> Y
array([[0, 0, 0, 0, 0],
       [1, 1, 1, 1, 1],
       [2, 2, 2, 2, 2]])
```

r_

r_ is a convenience function which generates 1-dimensional arrays from slice notation. While r_ is highly flexible, the most common use it r_[*start* : *end* : *stepOrCount*] where *start* and *end* are the start end end points, and *stepOrCount* can be either a step size, if a real value, or a count, if complex.

```
>>> r_[0:10:1] # arange equiv
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

>>> r_[0:10:.5] # arange equiv
array([ 0. ,  0.5,  1. ,  1.5,  2. ,  2.5,  3. ,  3.5,  4. ,  4.5,  5. ,
        5.5,  6. ,  6.5,  7. ,  7.5,  8. ,  8.5,  9. ,  9.5])

>>> r_[0:10:5j] # linspace equiv, includes end point
array([  0. ,   2.5,   5. ,   7.5,  10. ])
```

Note that r_ is different from typical functions in that it is used with [] and *not* ().

c_

c_ is virtually identical to r_ except that column arrays are generates, which are 2-dimensional (second dimension has size 1)

```
>>> c_[0:5:2]
array([[0],
       [2],
       [4]])

>>> c_[1:5:4j]
array([[ 1.        ],
       [ 2.33333333],
       [ 3.66666667],
       [ 5.        ]])
```

Note that c_ is different from typical functions in that it is used with [] and *not* ().

ix_

ix_(a, b) constructs an n-dimensional open mesh from n 1-dimensional lists or arrays. The output of ix_ is an n-element tuple containing 1-dimensional arrays. The primary use of ix_ is to simplify selecting slabs inside a matrix.

```
>>> x = reshape(arange(25.0),(5,5))
>>> x
array([[  0.,   1.,   2.,   3.,   4.],
       [  5.,   6.,   7.,   8.,   9.],
       [ 10.,  11.,  12.,  13.,  14.],
       [ 15.,  16.,  17.,  18.,  19.],
       [ 20.,  21.,  22.,  23.,  24.]])

>>> x[ix_([2,3],[0,1,2])] # Rows 2 & 3, cols 0, 1 and 2
array([[ 10.,  11.,  12.],
       [ 15.,  16.,  17.]])
```

mgrid

mgrid is very similar to meshgrid but behaves like r_ and c_ in that it takes slices as input, and uses real to denote step size and complex to denote number of values. The output is an $n+1$ dimensional vector where the first index of the output indexes the meshes.

```
>>> mgrid[0:3,0:2:.5]
array([[[ 0. ,  0. ,  0. ,  0. ],
        [ 1. ,  1. ,  1. ,  1. ],
        [ 2. ,  2. ,  2. ,  2. ]],

       [[ 0. ,  0.5,  1. ,  1.5],
        [ 0. ,  0.5,  1. ,  1.5],
        [ 0. ,  0.5,  1. ,  1.5]]])

>>> mgrid[0:3:3j,0:2:5j]
array([[[ 0. ,  0. ,  0. ,  0. ,  0. ],
        [ 1.5,  1.5,  1.5,  1.5,  1.5],
        [ 3. ,  3. ,  3. ,  3. ,  3. ]],

       [[ 0. ,  0.5,  1. ,  1.5,  2. ],
        [ 0. ,  0.5,  1. ,  1.5,  2. ],
        [ 0. ,  0.5,  1. ,  1.5,  2. ]]])
```

ogrid

ogrid is identical to mgrid except that the arrays returned are always 1-dimensional. ogrid output is generally more appropriate for looping code, while mgrid is usually more appropriate for vectorized code. When the size of the arrays is large, then ogrid uses much less memory.

```
>>> ogrid[0:3,0:2:.5]
[array([[ 0.],
       [ 1.],
       [ 2.]]), array([[ 0. ,  0.5,  1. ,  1.5]])]
>>> ogrid[0:3:3j,0:2:5j]
[array([[ 0. ],
       [ 1.5],
       [ 3. ]]),
 array([[ 0. ,  0.5,  1. ,  1.5,  2. ]])]
```

6.2 Rounding

around, round

around rounds to the nearest integer, or to a particular decimal place when called with two arguments.

```
>>> x= randn(3)
array([ 0.60675173, -0.3361189 , -0.56688485])

>>> around(x)
array([ 1.,  0., -1.])

>>> around(x, 2)
array([ 0.61, -0.34, -0.57])
```

around can also be used as a method on an ndarray – except that the method is named round. For example, x.round(2) is identical to around(x, 2). The change of names is needed since there is a built-in function round which is not aware of arrays.

floor

floor rounds to the next smallest integer (negative values are rounded away from 0).

```
>>> x= randn(3)
array([ 0.60675173, -0.3361189 , -0.56688485])

>>> floor(x)
array([ 0., -1., -1.])
```

ceil

ceil rounds to the next largest integer (negative values are rounded towards 0).

```
>>> x= randn(3)
array([ 0.60675173, -0.3361189 , -0.56688485])

>>> ceil(x)
array([ 1., -0., -0.])
```

Note that the values returned are still floating points and so -0. is the same as 0..

6.3 Mathematics

sum, cumsum

sum sums all elements in an array. By default, it will sum all elements in the array, and so the second argument is normally used to provide the axis to use (e.g. 0 to sum down columns, 1 for across rows). cumsum provides the cumulative sum of the values in the array, and is also usually used with the second argument to indicate the axis to use.

```
>>> x= randn(3,4)
>>> x
array([[-0.08542071, -2.05598312,  2.1114733 ,  0.7986635 ],
       [-0.17576066,  0.83327885, -0.64064119, -0.25631728],
       [-0.38226593, -1.09519101,  0.29416551,  0.03059909]])

>>> sum(x) # all elements
-0.62339964288008698

>>> sum(x, 0) # Down rows, 4 elements
array([-0.6434473 , -2.31789529,  1.76499762,  0.57294532])

>>> sum(x, 1) # Across columns, 3 elements
array([ 0.76873297, -0.23944028, -1.15269233])

>>> cumsum(x,0) # Down rows
array([[-0.08542071, -2.05598312,  2.1114733 ,  0.7986635 ],
       [-0.26118137, -1.22270427,  1.47083211,  0.54234622],
       [-0.6434473 , -2.31789529,  1.76499762,  0.57294532]])
```

sum and cumsum can both be used as function or as methods. When used as methods, the fist input is the axis so that sum(x,0) is the same as x.sum(0).

prod, cumprod

prod and cumprod work identically to sum and cumsum, except that the produce and cumulative product are returned. prod and cumprod can be called as function or methods.

diff

diff computes the finite difference on an vector (also array), and so return n-1 element when used on an n element vector. diff operates on the last axis by default, and so diff(x) operates across columns and returns x[:,1:size(x,1)]-x[:,:size(x,1)-1] for a 2-dimensional array. diff takes an optional keyword argument axis so that diff(x, axis=0) will operate across rows. diff can also be used to produce higher order differences (e.g. double difference).

```
>>> x= randn(3,4)
>>> x
array([[-0.08542071, -2.05598312,  2.1114733 ,  0.7986635 ],
       [-0.17576066,  0.83327885, -0.64064119, -0.25631728],
       [-0.38226593, -1.09519101,  0.29416551,  0.03059909]])
```

```
>>> diff(x) # Same as diff(x,1)
-0.62339964288008698

>>> diff(x, axis=0)
array([[-0.09033996,  2.88926197, -2.75211449, -1.05498078],
       [-0.20650526, -1.92846986,  0.9348067 ,  0.28691637]])

>>> diff(x, 2, axis=0) # Double difference, collumn-by-column
array([[-0.11616531, -4.81773183,  3.68692119,  1.34189715]])
```

exp

exp returns the element-by-element exponential (e^x) for an array.

log

log returns the element-by-element natural logarithm ($\ln(x)$) for an array.

log10

log10 returns the element-by-element base-10 logarithm ($\log_{10}(x)$) for an array.

sqrt

sqrt returns the element-by-element square root ($\sqrt{x}$) for an array.

square

square returns the element-by-element square (x^2) for an array.

absolute

absolute returns the element-by-element absolute value for an array. For complex values inputs, $|a+bi| = \sqrt{a^2+b^2}$.

sign

sign returns the element-by-element sign function which is defined as 0 if $x=0$, and $x/|x|$ otherwise.

6.4 Complex Values

real

real returns the real elements of a complex array. real can be called either as a function real(x) or as an attribute x.real.

`imag`

`imag` returns the complex elements of a complex array. `imag` can be called either as a function `imag(x)` or as an attribute `x.imag`.

`conj`, `conjugate`

`conj` returns the element-by-element complex conjugate for a complex array. `conj` can be called either as a function `conj(x)` or as a method `x.conj()`. `conjugate` is identical to `conj`.

6.5 Set Functions

`unique`

`unique` returns the unique elements in an array. It only operates on the entire array. An optional second argument can be returned which contains the original indices of the unique elements.

```
>>> x = repeat(randn(3),(2))
array([ 0.11335982,  0.11335982,  0.26617443,  0.26617443,  1.34424621,
        1.34424621])

>>> unique(x)
array([ 0.11335982,  0.26617443,  1.34424621])

>>> y,ind = unique(x, True)
>>> ind
array([0, 2, 4], dtype=int64)

>>> x.flat[ind]
array([ 0.11335982,  0.26617443,  1.34424621])
```

`in1d`

`in1d` returns a Boolean array with the same size as the first input array indicating the elements which are also in a second array.

```
>>> x = arange(10.0)
>>> y = arange(5.0,15.0)
>>> in1d(x,y)
array([False, False, False, False, False,  True,  True,  True,  True,  True], dtype=bool)
```

`intersect1d`

`intersect1d` is similar to `in1d`, except that it returns the elements rather than a Boolean array, and only unique elements are returned. It is equivalent to `unique(x.flat[in1d(x,y)])`.

```
>>> x = arange(10.0)
>>> y = arange(5.0,15.0)
>>> intersect1d(x,y)
array([ 5.,  6.,  7.,  8.,  9.])
```

union1d

union1d returns the unique set of elements in 2 arrays.

```
>>> x = arange(10.0)
>>> y = arange(5.0,15.0)
>>> union1d(x,y)
array([  0.,   1.,   2.,   3.,   4.,   5.,   6.,   7.,   8.,   9.,  10.,
        11.,  12.,  13.,  14.])
```

BUG: union1d does not work as described in the help. Arrays are not flattened, so that using arrays with different number of dims produces an error. The solution is to use union1d(x.flat,y.flat). (1.6.1)

setdiff1d

setdiff1d return the set of the elements which are only in the first array array but not in the second array.

```
>>> x = arange(10.0)
>>> y = arange(5.0,15.0)
>>> setdiff1d(x,y)
array([ 0.,  1.,  2.,  3.,  4.])
```

setxor1d

setxor1d returns the set of elements which are in one (and only one) of two arrays.

```
>>> x = arange(10.0)
>>> y = arange(5.0,15.0)
>>> setxor1d(x,y)
array([  0.,   1.,   2.,   3.,   4.,  10.,  11.,  12.,  13.,  14.])
```

6.6 Sorting and Extreme Values

sort

sort sorts the elements of an array. By default, it sorts using the last axis of x. It uses an optional second argument to indicate the axis to use for sorting (i.e. 0 for column-by-column, None for sorting all elements). sort does not alter the input when called as function, unlike the method version of sort.

```
>>> x = randn(4,2)
>>> x
array([[ 1.29185667,  0.28150618],
       [ 0.15985346, -0.93551769],
       [ 0.12670061,  0.6705467 ],
       [ 2.77186969, -0.85239722]])

>>> sort(x)
array([[ 0.28150618,  1.29185667],
       [-0.93551769,  0.15985346],
       [ 0.12670061,  0.6705467 ],
       [-0.85239722,  2.77186969]])
```

```
>>> sort(x, 0)
array([[ 0.12670061, -0.93551769],
       [ 0.15985346, -0.85239722],
       [ 1.29185667,  0.28150618],
       [ 2.77186969,  0.6705467 ]])

>>> sort(x, axis=None)
array([-0.93551769, -0.85239722,  0.12670061,  0.15985346,  0.28150618,
        0.6705467 ,  1.29185667,  2.77186969])
```

`ndarray.sort`, `argsort`

`ndarray.sort` is a method for `ndarray`s which performs an in-place sort. It economizes on memory use, although `x.sort()` is different from `x` after the function, unlike a call to `sort(x)`. `x.sort()` sorts along the last axis by default, and takes the same optional arguments as `sort(x)`. `argsort` returns the indices necessary to produce a sorted array, but does not actually sort the data. It is otherwise identical to `sort`, and can be used either as a function or a method.

```
>>> x= randn(3)
>>> x
array([ 2.70362768, -0.80380223, -0.10376901])

>>> sort(x)
array([-0.80380223, -0.10376901,  2.70362768])

>>> x
array([ 2.70362768, -0.80380223, -0.10376901])

>>> x.sort()
>>> x
array([-0.80380223, -0.10376901,  2.70362768])
```

`max`, `amax`, `argmax`, `min`, `amin`, `argmin`

`max` and `min` return the maximum and minimum values from an array. They take an optional second argument which indicates the axis to use.

```
>>> x= randn(3,4)
>>> x
array([[-0.71604847,  0.35276614, -0.95762144,  0.48490885],
       [-0.47737217,  1.57781686, -0.36853876,  2.42351936],
       [ 0.44921571, -0.03030771,  1.28081091, -0.97422539]])

>>> amax(x)
2.4235193583347918

>>> x.max()
2.4235193583347918
```

```
>>> x.max(0)
array([ 0.44921571,  1.57781686,  1.28081091,  2.42351936])

>>> x.max(1)
array([ 0.48490885,  2.42351936,  1.28081091])
```

max and min can only be used on arrays as methods. When used as a function, amax and amin must be used to avoid conflicts with the built-in functions max and min. This behavior is also seen in around and round. argmax and argmin return the index or indices of the maximum or minimum element(s). They are used in an identical manner to max and min, and can be used either as a function or method.

minimum, maximum

maximum and minimum can be used to compute the maximum and minimum of two arrays which are broadcastable.

```
>>> x = randn(4)
>>> x
array([-0.00672734,  0.16735647,  0.00154181, -0.98676201])

>>> y = randn(4)
array([-0.69137963, -2.03640622,  0.71255975, -0.60003157])

>>> maximum(x,y)
array([-0.00672734,  0.16735647,  0.71255975, -0.60003157])
```

6.7 Nan Functions

NaN function are convenience function which act similarly to their non-NaN versions, only ignoring NaN values (rather than propagating) when computing the function.

nansum

nansum is identical sum, except that NaNs are ignored. nansum can be used to easily generate other NaN-functions, such as nanstd (standard deviation, ignoring nans) since variance can be implemented using 2 sums.

```
>>> x = randn(4)
>>> x[1] = np.nan
>>> x
array([-0.00672734,         nan,  0.00154181, -0.98676201])

>>> sum(x)
nan

>>> nansum(x)
-0.99194753275859726

>>> nansum(x) / sum(logical_not(isnan(x)))
```

nanmax, nanargmax, nanmin, nanargmin

nanmax, nanmin, nanargmax and nanargmin are identical to their non-NaN counterparts, except that NaNs are ignored.

6.8 Exercises

1. Construct each of the following sequences using linspace, arange and r_:

$$0, 1, \ldots, 10$$

$$4, 5, 6, \ldots, 13$$

$$0, .25, .5, .75, 1$$

$$0, -1, -2, \ldots, -5$$

2. Show that logspace(0,2,21) can be constructed using linspace and 10 (and **). Similarly, show how linsapce(2,10,51) can be constructed with logspace and log10.

3. Determine the differences between the rounding by applying round (or around), ceil and floor to

$$y = [0, 0.5, 1.5, 2.5, 1.0, 1.0001, -0.5, -1, -1.5, -2.5]$$

4. Prove the relationship that $\sum_{j=1}^{n} j = n(n+1)/2$ for $0 \leq n \leq 10$ using cumsum and directly using math on an array.

5. randn(20) will generate an array containing draws from a standard normal random variable. If x=randn(20), which element of y=cumsum(x) is the same as sum(x)?

6. cumsum computes the cumulative sum while diff computes the difference. Is diff(cumsum(x)) the same as x? If not, how can a small modification be made to the this statement to recover x?

7. Compute the exp of

$$y = [\ln 0.5 \;\; \ln 1 \;\; \ln e]$$

   Note: You should use log and the constant numpy.e to construct y.

8. What is absolute of 0.0, -3.14, and 3+4j?

9. Suppose $x = [-4 \; 2 \; -9 \; -8 \; 10]$. What is the difference between y = sort(x) and x.sort()?

10. Using the same x as in the previous problem, find the max. Also, using argmax and a slice, retrieve the same value.

11. Show that setdiff1d could be replaced with in1d and intersect1d using $x = [1\;2\;3\;4\;5]$ and $y = [1\;2\;4\;6]$? How could setxor1d be replaced with union1d, intersect1d and in1d?

12. Suppose $y = [\text{nan}\; 2.2\; 3.9\; 4.6\; \text{nan}\; 2.4\; 6.1\; 1.8]$. How can nansum be used to compute the variance or the data? Note: sum(1-isnan(y)) will return the count of non-NaN values.

Chapter 7

Special Matrices

Commands are available to produce a number of useful arrays.

ones

ones generates a array of 1s and is generally called with one argument, a tuple which contains the size of each dimension. ones takes an optional second argument (dtype) which specifies the data type. If omitted, the data type is float.

```
M, N = 5, 5
# Produces a N by M array of 1s
x = ones((M,N))
# Produces a M by M by N 3D array of 1s
x =  ones((M,M,N))
# Produces a M by N array of 1s using 32 bit integers
x =  ones((M,N), dtype='int32')
```

Note: To use the function call above, N and M must have been previously defined (e.g. N,M=10,7). ones_like creates an array with the same size and shape as the input. Calling ones_like(x) is equivalent to calling ones(shape(x),x.dtype)

zeros

zeros produces a array of 0s in the same way ones produces a matrix of 1s, and is useful for initializing a matrix to hold values generated by another procedure. zeros takes an optional second argument (dtype) which specifies the data type. If omitted, the data type is float.

```
# Produces a M by N array of 0s
x = zeros((M,N))
# Produces a M by M by N 3D array of 0s
x = zeros((M,M,N))
# Produces a M by N array of 0s using 64 bit integers
x = zeros((M,N),dtype='int64')
```

zeros_like creates an array with the same size and shape as the input. Calling zeros_like(x) is equivalent to calling zeros(shape(x),x.dtype).

empty

empty produces an empty (uninitialized) array to hold values generated by another procedure. empty takes an optional second argument (dtype) which specifies the data type. If omitted, the data type is float.

```
# Produces a M by N empty array
x = empty((M,N))
# Produces a 4D empty array
x = empty((N,N,N,N))
# Produces a M by N empty array using 32-bit floats (single precision)
x = empty((M,N),dtype='float32')
```

Using empty is slightly faster than calling zeros since it does not assign 0 to all elements of the array – the "empty" array created will be populated with (essential random) values. empty_like creates an array with the same size and shape as the input. Calling empty_like(x) is equivalent to calling empty(shape(x),x.dtype).

eye, identity

eye generates an identity matrix (an array with ones on the diagonal, zeros every where else). An identity matrix is square and so usually only 1 input is needed.

```
In = eye(N)
```

identity is a virtually identical function with similar use, In = identity(N).

7.1 Exercises

1. Produce two arrays, one containing all zeros and one containing only ones, of size 10×5.
2. Multiply (linear algebra) these two arrays in both possible ways.
3. Produce an identity matrix of size 5. Take the exponential of this matrix, element-by-element.
4. How could ones and zeros be replaced with tile?
5. How could eye be replaced with diag and ones?
6. What is the value of y=empty((1,))? Is it the same as any element in y=empty((10,))?

Chapter 8

Array and Matrix Functions

Some functions operate exclusively on array inputs. Some are mathematical in nature, for instance computing the eigenvalues and eigenvectors, while other are functions for manipulating the elements of an array.

8.1 Views

Views are computationally efficient methods to produce objects which behave as other objects without copying data. For example, an array x can always be converted to a matrix using matrix(x), which will copy the elements in x. View "fakes" the call to matrix and only inserts a thin layer so that x viewed as a matrix behaves like a matrix.

view

view can be used to produce a representation of an array, matrix or recarray as another type without copying the data. Using view is faster than copying data into a new class.

```
>>> x = arange(5)
>>> type(x)
numpy.ndarray

>>> x.view(np.matrix)
matrix([[0, 1, 2, 3, 4]])

>>> x.view(np.recarray)
rec.array([0, 1, 2, 3, 4])
```

asmatrix, mat

asmatrix and mat can be used to view an array as a matrix. This view is useful since matrix views will use matrix multiplication by default.

```
>>> x = array([[1,2],[3,4]])
>>> x * x           # Element-by-element
array([[ 1,  4],
       [ 9, 16]])
```

```
>>> mat(x) * mat(x) # Matrix multiplication
matrix([[ 7, 10],
        [15, 22]])
```

Both commands are equivalent to using `view(np.matrix)`. `y = mat(x)` differs from `y = matrix(x)` since the latter will, by default, copy `x` to a new matrix, while the former produces a view.

asarray

`asarray` work in a similar matter as `asmatrix`, only that the view produced is that of `np.ndarray`.

ravel

`ravel` returns a flattened view (1-dimensional) of an array or matrix. `ravel` does not copy the underlying data, and so it is very fast.

```
>>> x = array([[1,2],[3,4]])
>>> x
array([[ 1,  2],
       [ 3,  4]])

>>> x.ravel()
array([1, 2, 3, 4])

>>> x.T.ravel()
array([1, 3, 2, 4])
```

8.2 Shape Information and Transformation

shape

`shape` returns the size of all dimensions or an array or matrix as a tuple. `shape` can be called as a function or an attribute. `shape` can also be used to reshape an array by entering a tuple of sizes. Additionally, the new shape can contain `-1` which indicates to expand along this dimension to satisfy the constraint that the number of elements cannot change.

```
>>> x = randn(4,3)
>>> x.shape
(4L, 3L)

>>> shape(x)
(4L, 3L)

>>> M,N = shape(x)
>>> x.shape = 3,4
>>> x.shape
 (3L, 4L)

>>> x.shape = 6,-1
>>> x.shape
(6L, 2L)
```

reshape

`reshape` transforms an array with one set of dimensions and to one with a different set, preserving the number of elements. `reshape` can transform an M by N array x into an K by L array y as long as $MN = KL$. Note that the number of elements *cannot* change. The most useful call to reshape switches an array into a vector or vice versa. For example

```
>>> x = array([[1,2],[3,4]])
>>> y = reshape(x,(4,1))
>>> y
array([[1],
       [2],
       [3],
       [4]])

>>> z=reshape(y,(1,4))
>>> z
array([[1, 2, 3, 4]])

>>> w = reshape(z,(2,2))
array([[1, 2],
       [3, 4]])
```

The crucial implementation detail of `reshape` is that matrices are stored using row-major notation. Elements in matrices are counted first across and then then down rows. `reshape` will place elements of the old array into the same position in the new array and so after calling `reshape`, $x(1) = y(1)$, $x(2) = y(2)$, and so on.

size

`size` returns the total number of elements in an array or matrix. `size` can be used as a function or an attribute.

```
>>> x = randn(4,3)
>>> size(x)
2

>>> x.size
12
```

ndim

`ndim` returns the size of all dimensions or an array or matrix as a tuple. `ndim` can be used as a function or an attribute .

```
>>> x = randn(4,3)
>>> ndim(x)
```

```
2

>>> x.ndim
2
```

tile

tile, along with reshape, are two of the most useful non-mathematical functions. tile replicates an array according to a specified size vector. To understand how repmat functions, imagine forming an array-composed of blocks. The generic form of tile is tile(X, (M, N)) where X is the matrix to be replicated, M is the number of rows in the new block matrix, and N is the number of columns in the new block matrix. For example, suppose X was a matrix

$$X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

and the block matrix

$$Y = \begin{bmatrix} X & X & X \\ X & X & X \end{bmatrix}$$

was needed. This could be accomplished by manually constructing y using concatenate

```
x = array([[1,2],[3,4]])
z = concatenate((x,x,x))
y = concatenate((z.T,z.T),axis=0)
```

However, tile provides a much easier method to construct y

```
y = tile(x,(2,3))
```

tile has two clear advantages over manual allocation: First, tile can be executed using parameters determined at run-time, such as the number of explanatory variables in a model and second tile can be used for arbitrary dimensions. Manual matrix construction becomes tedious and error prone with as few as 3 rows and columns. repeat is a related function which copies data is a less useful manner.

flatten

flatten works much like ravel, only that is copies the array when producing the flattened version.

flat

flat produces a numpy.flatiter object which is an iterator over a flattened view of an array. Because it is an iterator, it is especially fast.

```
>>> x = array([[1,2],[3,4]])
>>> x.flat
<numpy.flatiter at 0x6f569d0>

>>> x.flat[2]
3
```

```
>>> x.flat[1:4] = -1
>>> x
array([[ 1, -1],
       [-1, -1]])
```

broadcast, broadcast_arrays

`broadcast` can be used to broadcast two broadcastable arrays without actually copying any data. It returns a broadcast object, which works like an iterator.

```
>>> x = array([[1,2,3,4]])
>>> y = reshape(x,(4,1))
>>> b = broadcast(x,y)
>>> b.shape
(4L, 4L)

>>> for u,v in b:
...     print('x: ', u, ' y: ',v)
x:  1  y:  1
x:  2  y:  1
x:  3  y:  1
x:  4  y:  1
x:  1  y:  2
 ... ... ...
```

`broadcast_arrays` works similarly to `broadcast`, except that it copies the broadcast matrices into new arrays. `broadcast_arrays` is generally slower than `broadcast`, and should be avoided if possible.

```
>>> x = array([[1,2,3,4]])
>>> y = reshape(x,(4,1))
>>> b = broadcast_arrays(x,y)
>>> b[0]
array([[1, 2, 3, 4],
       [1, 2, 3, 4],
       [1, 2, 3, 4],
       [1, 2, 3, 4]])

>>> b[1]
array([[1, 1, 1, 1],
       [2, 2, 2, 2],
       [3, 3, 3, 3],
       [4, 4, 4, 4]])
```

vstack, hstack

`vstack`, and `hstack` stack compatible arrays and matrices vertically and horizontally, respectively. Any number of matrices can be stacked by placing the input matrices in a tuple, e.g. `(x,y,z)`.

```
>>> x = reshape(arange(6),(2,3))
>>> y = x
>>> vstack((x,y))
array([[0, 1, 2],
       [3, 4, 5],
       [0, 1, 2],
       [3, 4, 5]])

>>> hstack((x,y))
array([[0, 1, 2, 0, 1, 2],
       [3, 4, 5, 3, 4, 5]])
```

concatenate

`concatenate` generalizes `vstack` and `hsplit` to allow concatenation along any axis.

split, vsplit, hsplit

`vsplit` and `hsplit` split arrays and matrices vertically and horizontally, respectively. Both can be used to split an array into n equal parts or into arbitrary segments, depending on the second argument. If scalar, the matrix is split into n equal sized parts. If a 1 dimensional array, the matrix is split using the elements of the array as break points. For example, if the array was `[2,5,8]`, the matrix would be split into 4 pieces using `[:2]` , `[2:5]`, `[5:8]` and `[8:]`. Both `vsplit` and `hsplit` are special cases of `split`.

```
>>> x = reshape(arange(20),(4,5))
>>> y = vsplit(x,2)
>>> len(y)
2

>>> y[0]
array([[0, 1, 2, 3, 4],
       [5, 6, 7, 8, 9]])

>>> y = hsplit(x,[1,3])
>>> len(y)
3

>>> y[0]
array([[ 0],
       [ 5],
       [10],
       [15]])

>>> y[1]
array([[ 1,  2],
       [ 6,  7],
       [11, 12],
       [16, 17]])
```

delete

`delete` removes values from an array, and is similar to splitting an array, and then concatenating the values which are not deleted. The form of delete is `delete(x,`*rc*, *axis*) where *rc* are the row or column indices to delete, and *axis* is the axis to use (0 or 1 for a 2-dimensional array). If *axis* is omitted, `delete` operated on the flattened array.

```
>>> x = reshape(arange(20),(4,5))
>>> delete(x,1,0) # Same as x[[0,2,3]]
array([[ 0,  1,  2,  3,  4],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])

>>> delete(x,[2,3],1) # Same as x[:,[0,1,4]]
array([[ 0,  1,  4],
       [ 5,  6,  9],
       [10, 11, 14],
       [15, 16, 19]])

>>> delete(x,[2,3]) # Same as hstack((x.flat[:2],x.flat[4:]))
array([ 0,  1,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
       19])
```

squeeze

squeeze removes singleton dimensions from an array. `squeeze` can be called as a function or a method.

```
>>> x = ones((5,1,5,1))
>>> shape(x)
(5L, 1L, 5L, 1L)

>>> y = x.squeeze()
>>> shape(y)
(5L, 5L)

>>> y = squeeze(x)
```

fliplr, flipud

`fliplr` and `flipud` flip arrays in a left-to-right and up-to-down directions, respectively. Since 1-dimensional arrays are neither column nor row vectors, these two functions are only applicable on 2-dimensional (or higher) arrays.

```
>>> x = reshape(arange(4),(2,2))
>>> x
array([[0, 1],
       [2, 3]])

>>> fliplr(x)
array([[1, 0],
       [3, 2]])

>>> flipud(x)
array([[2, 3],
       [0, 1]])
```

diag

diag can produce one of two results depending on the form of the input. If the input is a square matrix, it will return a column vector containing the elements of the diagonal. If the input is a vector, it will return a matrix containing the elements of the diagonal along the vector. Consider the following example:

```
>>> x = matrix([[1,2],[3,4]])
>>> x
matrix([[1, 2],
        [3, 4]])

>>> y = diag(x)
>>> y
array([1, 4])

>>> z = diag(y)
>>> z
array([[1, 0],
       [0, 4]])
```

triu, tril

triu and tril produce upper and lower triangular, respectively.

```
>>> x = matrix([[1,2],[3,4]])
>>> triu(x)
matrix([[1, 2],
        [0, 4]])

>>> tril(x)
matrix([[1, 0],
        [3, 4]])
```

8.3 Linear Algebra Functions

matrix_power

matrix_power raises a square array or matrix to an integer power, and matrix_power(x,n) is identical to x**n.

svd

svd computed the singular value decomposition of a matrix. A singular value decomposition of a matrix X is

$$X = U\Sigma V'$$

where Σ is a diagonal elements, and U and V are unitary matrices (orthonormal if real valued). SVDs are closely related to eigenvalue decompositions when X is a real, positive definite matrix.

cond

cond computes the condition number of a matrix, which measures how close to singular a matrix is. Lower numbers are better conditioned (and further from singular).

```
>>> x = matrix([[1.0,0.5],[.5,1]])
>>> cond(x)
3
>>> x = matrix([[1.0,2.0],[1.0,2.0]]) # Singular
>>> cond(x)
inf
```

slogdet

slogdet computes the sign and log of the absolute value of the determinant. slogdet is useful for computing determinants which may be very large or small to avoid overflow or underflow.

solve

solve solves the system $X\beta = y$ when X is square and invertible so that the solution is exact.

```
>>> X = array([[1.0,2.0,3.0],[3.0,3.0,4.0],[1.0,1.0,4.0]])
>>> y = array([[1.0],[2.0],[3.0]])
>>> solve(X,y)
array([[ 0.625],
       [-1.125],
       [ 0.875]])
```

lstsq

lstsq solves the system $X\beta = y$ when X is n by k, $n > k$ by finding the least squares solution. lstsq returns a 4-element tuple where the first element is β and the second element is the sum of squared residuals. The final two outputs are diagnostic – the third is the rank of X and the fourth contains the singular values of X.

```
>>> X = randn(100,2)
>>> y = randn(100)
>>> lstsq(X,y)
(array([ 0.03414346,  0.02881763]),
 array([ 3.59331858]),
 2,
 array([ 3.045516  ,  1.99327863]))array([[ 0.625],
       [-1.125],
       [ 0.875]])
```

cholesky

`cholesky` computes the Cholesky factor of a positive definite matrix or array. The Cholesky factor is a lower triangular matrix and is defined as C in

$$CC' = \Sigma$$

where Σ is a positive definite matrix.

```
>>> x = matrix([[1,.5],[.5,1]])
>>> y = cholesky(x)
>>> y*y.T
matrix([[ 1. ,  0.5],
        [ 0.5,  1. ]])
```

det

`det` computes the determinant of a square matrix or array.

```
>>> x = matrix([[1,.5],[.5,1]])
>>> det(x)
0.75
```

eig

`eig` computes the eigenvalues and eigenvector of a square matrix. Two output arguments are required in order compute both the eigenvalues and eigenvectors, `val,vec = eig(R)`.

```
>>> x = matrix([[1,.5],[.5,1]])
>>> val,vec = eig(x)
>>> vec*diag(val)*vec.T
matrix([[ 1. ,  0.5],
        [ 0.5,  1. ]])
```

`eigvals` can be used if only eigenvalues are needed.

eigh

`eigh` computes the eigenvalues and eigenvector of a square, symmetric matrix. Two output arguments are required in order compute both the eigenvalues and eigenvectors, `val,vec = eigh(R)`. `eigh` is faster than `eig` since it exploits the symmetry of the input. `eigvalsh` can be used if only eigenvalues are needed from a square, symmetric.

inv

`inv` computes the inverse of a matrix. `inv(R)` can alternatively be computed using `x^(-1)`.

```
>>> x = matrix([[1,.5],[.5,1]])
>>> xInv = inv(x)
>>> x*xInv
matrix([[ 1.,  0.],
        [ 0.,  1.]])
```

`kron`

`kron` computes the Kronecker product of two matrices,

$$z = x \otimes y$$

and is written as `z = kron(x,y)`.

`trace`

`trace` computes the trace of a square matrix (sum of diagonal elements) and so `trace(x)` equals `sum(diag(x))`.

`matrix_rank`

`matrix_rank` computes the rank of a matrix using a SVD.

```
>>> x = matrix([[1,.5],[1,.5]])
>>> x
matrix([[ 1. ,  0.5],
        [ 1. ,  0.5]])
>>> matrix_rank(x)
1
```

8.4 Exercises

1. Let `x = arange(12.0)`. Use both `shape` and `reshape` to produce 1×12, 2×6, 3×4,4×3, 6×2 and $2 \times 2 \times 3$ versions or the array. Finally, return `x` to its original size.

2. Let `x = reshape(arange(12.0),(4,3))`. Use `ravel`, `flatten` and `flat` to extract elements 1, 3, . . ., 11 from the array (using a 0 index).

3. Let x be 2 by 2 array, y be a 1 by 1 array, and z be a 3 by 2 array. Construct

$$w = \begin{bmatrix} x & \begin{array}{ccc} y & y & y \\ y & y & y \end{array} \\ z & \begin{array}{c} z' \\ \begin{array}{ccc} y & y & y \end{array} \end{array} \end{bmatrix}$$

   using `hstack`, `vstack`, and `tile`.

4. Let `x = reshape(arange(12.0),(2,2,3))`. What does `squeeze` do to x?

5. How can a diagonal matrix containing the diagonal elements of

$$y = \begin{bmatrix} 2 & .5 \\ .5 & 4 \end{bmatrix}$$

be constructed using `diag`?

6. Using the y array from the previous problem, verify that `cholesky` work by computing the Cholesky factor, and then multiplying to get y again.

7. Using the y array from the previous problem, verify that the sum of the eigenvalues is the same as the trace, and the product of the eigenvalues is the determinant.

8. Using the y array from the previous problem, verify that the inverse of y is equal to $VD^{-1}V'$ where V is the array containing the eigenvectros, and D is a diagonal array containing the eigenvalues.

9. Simulate some data where `x = randn(100,2)`, `e = randn(100,1)`, `B = array([[1],[0.5]])` and $y = x\beta + \epsilon$. Use `lstsq` to estimate β from x and y.

10. Suppose

$$y = \begin{bmatrix} 5 & -1.5 & -3.5 \\ -1.5 & 2 & -0.5 \\ -3.5 & -0.5 & 4 \end{bmatrix}$$

use `matrix_rank` to determine the rank of this array. Verify the results by inspecting the eigenvalues using `eig` and check that the determinant is 0 using `det`.

11. Let `x = randn(100,2)`. Use `kron` to compute

$$I_2 \otimes \Sigma_X$$

where Σ_X is the 2 by 2 covariance matrix of x.

Chapter 9

Importing and Exporting Data

9.1 Importing Data

Importing data ranges from easy for files which contain only numbers difficult, depending on the data size and format. A few principles can simplify this task:

- The file imported should contain numbers *only*, with the exception of the first row which may contain the variable names.
- Use another program, such as Microsoft Excel, to manipulate data before importing.
- Each column of the spreadsheet should contain a single variable.
- Dates should be converted to YYYYMMDD, a numeric format, before importing. This can be done in Excel using the formula:
  `=10000*YEAR(A1)+100*MONTH(A1)+DAY(A1)+(A1-FLOOR(A1,1))`

9.2 CSV and other formatted text files

A number of importers are available for regular (e.g. all rows have the same number of columns) comma-separated value (CSV) data. The choice of which importer to use depends on the complexity and size of the file. Purely numeric files are the simplest to import, although most files which have a repeated structure can be imported (unless they are very large).

9.2.1 `loadtxt`

`loadtxt` (`numpy.lib.npyio.loadtxt`)is a simple, but fast, text importer. The basic use is `loadtxt(`*filename*`)`, which will attempt to load the data in file name as floats. Other useful named arguments include `delim`, which allow the file delimiter to be specified, and `skiprows` which allows one or more rows to be skipped. `loadtxt` requires the data to be numeric and so is only useful for the simplest files.

```
>>> data = loadtxt('FTSE_1984_2012.csv',delimiter=',') # Error
ValueError: could not convert string to float: Date

# Fails since csv has a header
>>> data = loadtxt('FTSE_1984_2012_numeric.csv',delimiter=',') # Error
```

```
ValueError: could not convert string to float: Date

>>> data = loadtxt('FTSE_1984_2012_numeric.csv',delimiter=',',skiprows=1)
>>> data[0]
array([ 4.09540000e+04, 5.89990000e+03, 5.92380000e+03, 5.88060000e+03, 5.89220000e+03,
8.01550000e+08, 5.89220000e+03])
```

9.2.2 genfromtxt

genfromtxt (numpy.lib.npyio.genfromtxt) is a slower, but more robust, importer than loadtxt. genfromtxt is called in an identical matter as loadtxt, but will not fail if a non-numeric type is encountered. Instead, genfromtxt will return a NaN (not-a-number) for fields in the file it cannot read.

```
>>> data = genfromtxt('FTSE_1984_2012.csv',delimiter=',')
>>> data[0]
array([ nan,  nan,  nan,  nan,  nan,  nan,  nan])
>>> data[1]
array([ nan, 5.89990000e+03, 5.92380000e+03, 5.88060000e+03, 5.89220000e+03, 8.01550000e+08,
5.89220000e+03])
```

Tab delimited data can be read in a similar manner using delimiter='\t'.

```
>>> data = genfromtxt('FTSE_1984_2012_numeric_tab.txt',delimiter='\t')
```

9.2.2.1 csv2rec

csv2rec (matplotlib.mlab.csv2rec) is an even more robust – and slower – csv importer which allows for non-numeric data such as dates. It also attempts to find the best data type using for each row.

```
>>> data = csv2rec('FTSE_1984_2012.csv',delimiter=',')
>>> data[0]
(datetime.date(2012, 2, 15), 5899.9, 5923.8, 5880.6, 5892.2, 801550000L, 5892.2)
```

Unlike loadtxt and genfromtxt, which both return an array, csv2rec returns a recarray (numpy.core. .records .recarray, see Chapter 20) which is, in many ways, like a list. csv2rec converted each row of the input file into a datetime (see Chapter 16), followed by 4 floats for open, high, low and close, then a long integer for volume, and finally a float for the adjusted close.

Because the values returned are not an array, it is normally necessary to create an array to store the array.

```
>>> open = data['open']
>>> open
array([ 5899.9,  5905.7,  5852.4, ...,  1095.4,  1095.4,  1108.1])
```

9.3 Reading 97-2003 Excel Files

Reading Excel files in Python is more involved, so unless essential, it is probably simpler to convert the xls to csv. Reading 97-2003 Excel files requires a python package which is not in the core, xlutils, which can be installed using easy_install xlutils.

```
from __future__ import print_function
import xlrd

wb = xlrd.open_workbook('FTSE_1984_2012.xls')
sheetNames = wb.sheet_names()
# Assumes 1 sheet name
sheet = wb.sheet_by_name(sheetNames[0])
excelData = []
for i in xrange(sheet.nrows):
    excelData.append(sheet.row_values(i))

# - 1 since excelData has the header row
open = empty(len(excelData) - 1)
for i in xrange(len(excelData) - 1):
    open[i] = excelData[i+1][1]
```

The listing does a few things. First, it opens the workbook for reading (`xlrd. open_workbook( 'FTSE_1984_ _2012.xls')`), then it gets the sheet names (`wb.sheet_names()`) and opens a sheet (`wb.sheet_by_name(sheetNames[0])`). From the sheet, it gets the number of rows (`sheet.nrows`), and fills a list with the values, row-by-row. Once the data has been read-in, the final block fills an array from the data from opening prices in the list. This is substantially more complicated than converting to a CSV file, although reading Excel files is useful for automated work (e.g. you have no choice but to import from an Excel file since it is produced by some other software).

Python 2.7 vs. 3.2
xlrd is not available for Python 3.

9.4 Reading 2007 & 2010 Excel Files

xlrd only reads 97-2003 files, and so a different package, openpyxl, is needed to read xlsx files created in Office 2007 or later. Unfortunately openpyxl has a different syntax to xlrd, and so a modified reader is needed for xlsx files.

```
from __future__ import print_function
import openpyxl

wb = openpyxl.load_workbook('FTSE_1984_2012.xlsx')
sheetNames = wb.get_sheet_names()
# Assumes 1 sheet name
sheet = wb.get_sheet_by_name(sheetNames[0])
excelData = []
rows = sheet.rows

# - 1 since excelData has the header row
open = empty(len(rows) - 1)
for i in xrange(len(excelData) - 1):
    open[i] = rows[i+1][1].value
```

The strategy with 2007-2010 xlsx files is essentially the same as with 97-2003 files. The main difference is that the command `sheet.rows()` returns a tuple which contains the all of the rows in the selected sheet. Each row is itself a tuple which contains `Cells` (which are a type created by openpyxl), and each cell has a `value` (Cells also have other useful attributes such as `data_type` and methods such as `is_date()`) .

Python 2.7 vs. 3.2

openpyxl is not available for Python 3.

9.5 Reading MATLAB Data Files (.mat)

SciPy enables MATLAB data files to be read. The native file format is the MATLAB data file, or mat file. Data from a mat file can be loaded using `scipy.io.loadmat`. The data is loaded into a dictionary, and so individual variables can be accessed using the keys of the dictionary.

```
from __future__ import print_function
import scipy.io as io

matData = io.loadmat('FTSE_1984_2012.mat')
open = matData['open']
```

9.6 Manually Reading Poorly Formatted Text

Python can be programmed to read virtually any text format since it contains functions for parsing and interpreting arbitrary text containing numeric data. Reading poorly formatted data files is an advanced technique and should be avoided if possible. However, some data is only available in formats where reading in data line-by-line is the only option. For instance, the standard import method fails if the raw data is very large (too large for Excel) and is poorly formatted. In this case, the only possibility is to write a program to read the file line-by-line and to process each line separately.

The file *IBM_TAQ.txt* contains a simple example of data that is difficult to import. This file was downloaded from WRDS and contains all prices for IBM from the TAQ database in the interval January 1,2001 through January 31, 2001. It is too large to use in Excel and has both numbers, dates and text on each line. The following code block shown how the data in this file can be parsed.

```
f = file('IBM_TAQ.txt', 'r')
line = f.readline()
# Burn the first list as a header
line = f.readline()

date = []
time = []
price = []
volume = []
while line:
    data = line.split(',')
    date.append(int(data[1]))
    price.append(float(data[3]))
    volume.append(int(data[4]))
    t = data[2]
    time.append(int(t.replace(':','')))
    line = f.readline()

# Convert to arrays, which are more useful than lists
# for numeric data
date = array(date)
price = array(price)
volume = array(volume)
time = array(time)

allData = array([date,price,volume,time])

f.close()
```

This block of code does a few thing:

- Open the file directly using `file`
- Reads the file line by line using `readline`
- Initializes lists for all of the data
- Rereads the file parsing each line by the location of the commas using `split(',')` to split the line at each comma into a list
- Uses `replace(':','')` to remove colons from the times
- Uses `int()` and `float()` to convert strings to numbers
- Closes the file directly using `close()`

9.7 Stat Transfer

StatTransfer is available on the servers and is capable of reading and writing approximately 20 different formats, including MATLAB, GAUSS, Stata, SAS, Excel, CSV and text files. It allow users to load data in one format and output some or all of the data in another. *StatTransfer* can make some hard-to-manage situations (e.g. poorly formatted data) substantially easier. *StatTransfer* has a comprehensive help file to provide assistance.

9.8 Saving and Exporting Data

Native NumPy Format

A number of options are available for saving data. These include using native npz data files, MATLAB data files, csv or plain text. Multiple numpy arrays can be saved using `savez` (`numpy.savez`).

```
x = arange(10)
y = zeros((100,100))
savez('test',x,y)
data = load('test.npz')
# If no name is given, arrays are generic names arr_1, arr_2, etc
x = data['arr_1']

savez('test',x=x,otherData=y)
data = load('test.npz')
# x=x provides the name x for the data in x
x = data['x']
# otherDate = y saves the data in y as otherData
y = data['otherData']
```

A version which compresses data but is otherwise identical is `savez_compressed`. Compression is very helpful for arrays which have repeated values or are very large.

```
x = arange(10)
y = zeros((100,100))
savez_compressed('test',x=x,otherData=y)
data = load('test.npz')
# x=x provides the name x for the data in x
x = data['x']
# otherDate = y saves the data in y as otherData
y = data['otherData']
```

9.8.1 Writing MATLAB Data Files (.mat)

SciPy enables MATLAB data files to be written. Data can be written using `scipy.io.savemat`, which takes two inputs, a file name and a dictionary containing data, in its simplest form.

```
from __future__ import print_function
import scipy.io

x = array([1.0,2.0,3.0])
y = zeros((10,10))
# Set up the dictionary
saveData = {'x':x, 'y':y}
io.savemat('test',saveData,do_compression=True)
# Read the data back
matData = io.loadmat('test.mat')
```

`savemat` uses the optional argument `do_compression = True`, which compresses the data, and is generally a good idea on modern computers and/or for large datasets.

9.8.2 Exporting Data

Data can be exported to a tab-delimited text files using `savetxt`. By default, `savetxt` produces tab delimited files, although then can be changed using the names argument `delimiter`.

```
x = randn(10,10)
```

```
# Save using tabs
savetxt('tabs.txt',x)
# Save to CSV
savetxt('commas.csv',x,delimiter=',')
# Reread the data
xData = loadtxt('commas.csv',delimiter=',')
```

9.9 Exercises

1. The file *exercise3.xls* contains three columns of data, the date, the return on the S&P 500, and the return on XOM (ExxonMobil). Using Excel, convert the date to YYYYMMDD format and save the file.

2. Save the file as both CSV and tab delimited. Use the three text readers to read the file, and compare the arrays returned.

3. Parse loaded data into three variables, `dates`, `SP500` and `XOM`.

4. Save NumPy, compressed NumPy and MATLAB data files with all three variables. Which files is the smallest?

5. Construct a new variable, `sumreturns` as the sum of `SP500` and `XOM`. Create another new variable, `outputdata` as a horizontal concatenation of `dates` and `sumreturns`.

6. Export the variable `outputdata` to a new CSV file using `savetxt`.

7. (Difficult) Read in *exercise3.xls* directly using xlrd.

8. (Difficult) Save *exercise3.xls* as *exercise3.xlsx* and read in directly using openpyxl.

Chapter 10

`Inf`, NaN and Numeric Limits

Three special expressions are reserved to indicate certain non-numerical "values".

`inf` represents infinity and `inf` is distinct from `-inf`. `inf` can be constructed in a number for ways, for example or `exp(710)`.

`nan` stands for Not a Number. `nan`s are created whenever a function produces a result that cannot be clearly defined as a number or infinity. For instance, `inf/inf` results in `nan`.

All numeric software has limited precision; Python is no different. The easiest way to understand the upper and lower limits, which are 1.7976×10^{308} (see `finfo(float).max`) and -1.7976×10^{308} (`finfo(float).min`). Numbers larger (in absolute value) than these are `inf`. The smallest positive number that can be expressed is 2.2250×10^{-308} (see `finfo(float).tiny`). Numbers between -2.2251×10^{-308} and 2.2251×10^{-308} are numerically 0.

However, the hardest concept to understand about numerical accuracy is the limited relative precision. The relative precision is 2.2204×10^{-16}. This value is returned from the command `finfo(float).eps` and may vary based on the type of CPU and/or the operating system used. Numbers which are outside of a *relative* range of 2.2204×10^{-16} are numerically the same. To explore the role of precision, examine the results of the following:

```
>>> x = 1.0
>>> eps = finfo(float).eps
>>> x = x+eps/2
>>> x == 1
True

>>> x-1
0.0

>>> x = 1 + 2*eps
>>> x == 1
False

>>> x-1
ans = 4.4408920985006262e-16
```

To understand what is meant by *relative* range, examine the following output

```
>>> x=10
>>> x+2*eps
>>> x-10
0
```

In the first example, `eps/2<eps` *when compared to 1* so it has no effect while `2*eps>eps` and so this value is different from 1. In the second example, `2*eps/10<eps`, it has no effect when added. This is a very tricky concept to understand, but failure to understand numeric limits can results in errors in code that appears to be otherwise correct.

The practical lesson is to think about data scaling. Many variables have natural scales which are vastly different, and so rescaling is often necessary to avoid numeric limits.

10.1 Exercises

Let `eps = finfo(float).eps` in the following exercises.

1. What is the value of `log(exp(1000))` both analytically and in Python? Why do these differ?
2. Is `eps/10` different from 0? If `x = 1 + eps/10 - 1`, is `x` different from 0?
3. Is `.1` different from `.1+eps/10`?
4. Is `x = 1.0 * 10**120` (1×10^{120}) different from `y = 1.0 * 10**120 + 1 * 10**102`? (Hint: Test with `x == y`)
5. Why is `x = 10**120` (1×10^{120}) different from `y = 10**120 + 10**102`?
6. Suppose `x = 2.0`. How many times can `x = 1.0 + (x-1.0)/2.0` be run before `x==1` shows `True`? What is the value of `2.0**(-n)` where n is the number of time it can be divided. Is this value surprising?

Chapter 11

Logical Operators and Find

Logical operators are useful when writing batch files or custom functions. Logical operators, when combined with flow control, allow for complex choices to be compactly expressed.

11.1 >, >=, <, <=, ==, !=

The core logical operators are

Symbol	Function	Definition
>	`greater`	Greater than
>=	`greater_equal`	Greater than or equal to
<	`less`	Less than
<=	`less_equal`	Less then or equal to
==	`equal`	Equal to
!=	`not_equal`	Not equal to

Logical operators can be used on scalars, arrays or matrices. All comparisons are done element-by-element and return either `True` or `False`. For instance, suppose `x` and `y` are matrices. `z= x < y` will be a matrix of the same size as `x` and `y` composed of `True` and `False`. Alternatively, if one is scalar, say `y`, then the elements of `z` are `z[i,j] = x[i,j] < y`. Logical operators can be used to access elements of a vector or matrix. For instance, suppose `z = x`L`y` where L is one of the logical operators above such as < or ==. The following table examines the behavior when `x` and/or `y` are scalars or matrices. Suppose `z = x < y`:

		y	
		Scalar	Matrix
x	Scalar	Any $z = x < y$	Any $z_{ij} = x < y_{ij}$
	Matrix	Any $z_{ij} = x_{ij} < y$	Same Dimensions $z_{ij} = x_{ij} < y_{ij}$

Logical operators are used in portions of programs known as flow control (e.g. `if` ... `else` ... blocks) which will be discussed later. It is important to remember that vector or matrix logical operations return vector or matrix output and that flow control blocks *require scalar* logical expressions.

11.2 and, or, not and xor

Logical expressions can be combined using four logical devices,

Keyword	Function	True if . . .
and	logical_and	Both True
or	logical_or	Either or Both True
not	logical_not	Not True
	logical_xor	One True and One False

and and logical_and() both return true if both arguments are true. The keyword and can only be used on scalars, and so is called a short-circuit operator. logical_and() can be used on matrices. The same is true

```
>>> x=matrix([[1,2],[-3,-4]])
>>> y = x > 0
>>> z = x < 0
>>> logical_and(y, z)
matrix([[False, False],
        [False, False]], dtype=bool)

>>> y[0,0] and z[0,0]
Out: False
```

These operators follow the same rules as other logical operators. If used on two matrices, the dimensions must be the same. If used on a scalar and a matrix, the effect is the same as calling the logical device on the scalar and each element of the matrix.

Suppose x and y are logical variables (1s or 0s). and define z=logical_and(x,y):

		y	
		Scalar	Matrix
x	Scalar	Any $z = x \& y$	Any $z_{ij} = x \& y_{ij}$
	Matrix	Any $z_{ij} = x_{ij} \& y$	Same Dimensions $z_{ij} = x_{ij} \& y_{ij}$

11.3 Multiple tests

all and any

The commands all and any take logical data input and are self descriptive. all returns True if all logical elements in an array are 1. If all is called without any additional arguments on an array, it returns True if all elements of the array are logical true and 0 otherwise. any returns logical(True) if any element of an array is True. Both all and any can be also be used along the dimensions of the array using a second argument (or the named argument axis) which indicates the axis of operation, where 0 is column-wise (e.g. is examines all elements in a single row), 1 is row-wise, and so on. When used column- or row-wise, the output is an array with one less dimension than the input, where each element of the output contains the truth value of the operation on the column or row.

```
>>> x = matrix([[1,2][3,4]])
>>> y = x <= 2
>>> y
matrix([[ True,  True],
        [False, False]], dtype=bool)

>>> any(y)
True

>>> any(y,0)
matrix([[ True,  True]], dtype=bool)

>>> any(y,1)
matrix([[ True],
        [False]], dtype=bool)
```

allclose

allclose can be used to compare two arrays, while allowing for a tolerance. This type of function is important when comparing floating point values which may be effectively the same, but not identical.

```
>>> eps = np.finfo(np.float64).eps
>>> eps
2.2204460492503131e-16

>>> x = randn(2)
>>> y = x + eps
>>> x == y
array([False, False], dtype=bool)

>>> allclose(x,y)
True
```

array_equal

array_equal tests if two arrays have the same shape and elements. It is safer than comparing arrays directly since comparing arrays which are not broadcastable produces an error.

array_equiv

array_equiv tests if two arrays are equivalent, even if they do not have the exact same shape. Equivalence is defined as one array being broadcastable to produce the other.

```
>>> x = randn(10,1)
>>> y = tile(x,2)
>>> array_equal(x,y)
False

>>> array_equiv(x,y)
True
```

11.4 Logical Indexing

find

find is an useful function for working with multiple data series. find is not logical itself, but it takes logical inputs and returns indices where the logical statement is true. It is called indices = find (x <= 2) will return indices (0,1,. . .,) so that the elements which are true can be accessed using the slice x.flat[indices]. Note that the flat view is needed since slicing x directly (x[indices] will operate along the first dimension, and so will return rows of a 2-dimensional matrix.

```
>>> x = matrix([[1,2],[3,4]])
>>> y = x <= 2
>>> indices = find(y)
>>> indices
array([0, 1], dtype=int64)

>>> x.flat[indices]
matrix([[1, 2]])

# Wrong output
>>> x[indices]

>>> x = matrix([[1,2],[3,4]]);
>>> y = x <= 4
>>> indices = find(y)
>>> x.flat[indices]
matrix([[1, 2, 3, 4]])

# Produces and error since x has only 2 rows
>>> x[indices] # Error
IndexError: index (2) out of range (0<=index<1) in dimension 0
```

argwhere

argwhere can be used to return an array where a logical condition is met.

```
>>> x = randn(3)
>>> x
array([-0.5910316 ,  0.51475905,  0.68231135])

>>> argwhere(x<0)
array([[0]], dtype=int64)

>>> where(x<-10.0) # Empty array
array([], shape=(0L, 1L), dtype=int64)

>>> x = randn(3,2)
>>> x
array([[ 0.72945913,  1.2135989 ],
       [ 0.74005449, -1.60231553],
       [ 0.16862077,  1.0589899 ]])

>>> argwhere(x<0)
array([[1, 1]], dtype=int64)

>>> x = randn(3,2,4)
>>> argwhere(x<0)
array([[0, 0, 1],
       [0, 0, 2],
       [0, 1, 2],
       [0, 1, 3],
       [1, 0, 2],
       [1, 1, 0],
       [2, 0, 1],
       [2, 1, 0],
       [2, 1, 1],
       [2, 1, 3]], dtype=int64)
```

extract

extract is similar to argwhere except that it returns the values where the condition is true rather then the indices.

```
>>> x = randn(3)
>>> x
array([-0.5910316 ,  0.51475905,  0.68231135])

>>> extract(x<0, x)
array([-0.5910316])

>>> extract(x<-10.0, x) # Empty array
array([], dtype=float64)

>>> x = randn(3,2)
>>> x
array([[ 0.72945913,  1.2135989 ],
       [ 0.74005449, -1.60231553],
       [ 0.16862077,  1.0589899 ]])

>>> extract(x<0,x)
array([-1.60231553])
```

11.5 is*

A number of special purpose logical tests are provided to determine if a matrix has special characteristics. Some operate element-by-element and produce a matrix of the same dimension as the input matrix while other produce only scalars. These functions all begin with is.

isnan	1 if nan	element-by-element
isinf	1 if inf	element-by-element
isfinite	1 if not inf and not nan	element-by-element
isposfin,isnegfin	1 for positive or negative inf	element-by-element
isreal	1 if not complex valued	element-by-element
iscomplex	1 if complex valued	element-by-element
isreal	1 if real valued	element-by-element
is_string_like	1 if argument is a string	scalar
is_numlike	1 if is a numeric type	scalar
isscalar	1 if scalar	scalar
isvector	1 if input is a vector	scalar

There are a number of other special purpose `is*` expressions. For more details, search for is* in help.

```
x=matrix([4,pi,inf,inf/inf])
isnan(x)

matrix([[False, False, False,  True]], dtype=bool)

isinf(x)

matrix([[False, False,  True, False]], dtype=bool)

isfinite(x)

matrix([[ True,  True, False, False]], dtype=bool)
```

Note: `isnan(x)` *isinf(x)* `isfinite(x)` always equals `True` , implying any element falls into one (and only one) of these categories.

11.6 Exercises

1. Using the data file created in Chapter 9, count the number of negative returns in both the S&P 500 and ExxonMobil.

2. For both series, create an indicator variable that takes the value 1 if the return is larger than 2 standard deviations or smaller than -2 standard deviations. What is the average return conditional on falling each range for both returns.

3. Construct an indicator variable that takes the value of 1 when both returns are negative. Compute the correlation of the returns conditional on this indicator variable. How does this compare to the correlation of all returns?

4. What is the correlation when at least 1 of the returns is negative?

5. What is the relationship between `all` and `any`. Write down a logical expression that allows one or the other to be avoided (i.e. write `def myany(x)` and `def myall(y)`).

Chapter 12

Flow Control, Loops and Exception Handling

The previous chapter explored one use of logical variables, selecting elements from a matrix. Logical variables have another important use: flow control. Flow control allows different code to be executed depending on whether certain conditions are met.

12.1 if . . . elif . . . else

if . . . elif . . . else blocks always begin with an if statement immediately followed by a **scalar** logical expression. elif and else are optional and can always be replicated using nested if statements at the expense of more complex logic and deeper nesting. The generic form of an if . . . elif . . . else block is

```
if logical_1:
    Code to run if logical_1
elif  logical_2:
    Code to run if logical_2
elif  logical_3:
    Code to run if logical_3
...
...
else:
    Code to run if all previous logicals are false
```

However, simpler forms are more common,

```
if logical:
    Code to run if logical true
```

or

```
if logical:
    Code to run if logical true
else:
    Code to run if logical false
```

Note: Remember that all *logical*s must be scalar logical values.

A few simple examples

```
>>> x = 5
>>> if x<5:
```

```
...     x += 1
... else:
...     x -= 1

>>> x
4
```

and

```
>>> x = 5;
>>> if x<5:
...    x = x + 1
... elif x>5:
...    x = x - 1
... else:
...    x = x * 2

>>> x
 10
```

These examples have all used simple logical expressions. However, any *scalar* logical expressions, such as `(x<0 or x>1) and (y<0 or y>1)` `(y<0 or y>1)` or `isinf(x) or isnan(x)`, can be used in `if` ... `elif` ... `else` blocks.

12.2 for

`for` loops begin with `for` *item* `in` *iterable*: . The generic structure of a `for` loop is

```
for item in iterable:
    Code to run
```

item is an element from *iterable*, and *iterable* can be anything that is iterable in Python. The most common examples are `xrange` or `range`, lists, tuples, arrays or matrices. The `for` loop will iterate across all items in iterable, beginning with item `0` and continuing until the final item.

```
count = 0
for i in range(100):
    count += i

count = 0
x = linspace(0,500,50)
for i in x: # Python 3: for i in range(0,500,5)
    count += i

count = 0
x = list(arange(-20,21))
for i in x:
    count += i
```

The first loop will iterate over $i = 0, 1, 2, \ldots, 99$. The second loops over the values produced by the function `linspace`, which returns an array, which creates 50 uniform points between 0 and 500, inclusive. The final loops over `x`, a vector constructed from a call to `list(arange(-20,21))`, which produces a list containing the series $-20, -19, \ldots, 0, \ldots 19, 20$. All three – range, arrays, and lists – are iterable. The key to understanding `for` loop behavior is that `for` always iterates over the elements of the *iterable* in the order they are presented (i.e. *iterable*[0], *iterable[1]*, . . .).

Python 2.7 vs. 3.2 Note: This chapter exclusively uses `range` in loops (instead of `xrange`). This is a simplification used so that the same code will run in Python 2.7 and 3.2, although the best practice is to use `xrange` in Python 2.7 loops.

Loops can also be nested:

```
count = 0
for i in range(10):
    for j in range(10):
        count += j
```

and can contain flow control variables:

```
returns = randn(100)
count = 0
for ret in returns:
    if ret<0:
        count += 1
```

This `for` expression can be equivalently expressed using `range`, by using `len` to get the number of items in the iterable:

```
returns = randn(100)
count = 0
for i in range(len(returns)):
    if returns[i]<0:
        count += 1
```

Finally, these ideas can be combined to produce nested loops with flow control.

```
x = zeros((10,10))
for i in range(size(x,0)):
    for j in range(size(x,1)):
        if i<j:
            x[i,j]=i+j;
        else:
            x[i,j]=i-j
```

or loops containing nested loops that are executed based on a flow control statement.

```
x = zeros((10,10))
for i in range(size(x,0)):
    if (i % 2) == 1:
        for j in range(size(x,1)):
            x[i,j] = i+j
    else:
        for j in range(int(i/2)):
            x[i,j] = i-j
```

Note: The iterable variable cannot be changed once inside the loop. Consider, for example,

```
x = range(10) # Python 3: x = range(10)
for i in x:
    print(i)
    print('Length of x:', len(x))
    x = range(5)
```

Produces the output

```
# Output
0
Length of x: 10
1
Length of x: 5
2
Length of x: 5
3
...
8
Length of x: 5
9
Length of x: 5
```

Note that it is not safe to modify the sequence of the *iterable* when looping over it. The means that the *iterable* should not change size, which can occur when using a list and the functions `pop()`, `insert()` or `append()` or the keyword `del`. The loop below would never terminate (except for the `break`) since `L` is being extended each iteration.

```
L = [1, 2]
for i in L:
     print(i)
     L.append(i+2)
     if i>5:
         break
```

Finally, `for` loops can be used with 2 *items* when the *iterable* is wrapped in `enumerate`, which allows the elements of the *iterable* to be directly accessed, as well as their index in the *iterable*.

```
x = linspace(0,100,11)
for i,y in enumerate(x):
    print('i is :', i)
    print('y is :', y)
```

12.2.1 Whitespace

Like `if` ... `elif` ... `else` flow control blocks, `for` loops are whitespace sensitive. The indentation of the line immediately below the `for` statement determines the indentation that all statements in the block must have. The convention is 4 spaces.

12.2.2 break

A loop can be terminated early using `break`. `break` is usually used after an `if` statement to terminate the loop prematurely if some condition has been met.

```
x = randn(1000)
for i in x:
    print(i)
    if i > 2:
        break
```

`break` is more useful in `while` loops.

12.2.3 continue

`continue` can be used to skip an iteration of a loop, immediately returning to the top of the loop using the next item in *iterable*. `continue` is usually used to avoid a level of nesting, such as in the following two examples.

```
x = randn(10)
for i in x:
    if i < 0:
        print(i)

for i in x:
    if i >= 0:
        continue
    print(i)
```

Avoiding excessive levels of indentation is essential in Python programming – 4 is usually considered the maximum – and `continue` can be used to in a `for` loop to avoid one level of indentation.

12.3 while

`while` loops are useful when the number of iterations needed depends on the outcome of the loop contents. `while` loops are commonly used when a loop should only stop if a certain condition is met, such as the change in some parameter is small. The generic structure of a `while` loop is

```
while logical:
    Code to run
```

Two things are crucial when using a `while` loop: first, the *logical* expression should evaluate to true when the loop begins (or the loop will be ignored) and second the inputs to the *logical* expression must be updated inside the loop. If they are not, the loop will continue indefinitely (hit CTRL+C to break an interminable loop). The simplest while loops are (verbose) drop-in replacements for `for` loops:

```
count = 0
i = 1
while i<=10:
    count += i
    i += 1
```

which produces the same results as

```
count=0;
for i in range(0,11):
    count += i
```

while loops should generally be avoided when for loops will do. However, there are situations where no for loop equivalent exists.

```
mu = 1.0
index = 1
while abs(mu) > .0001:
    mu = (mu+randn(1))/index
    index=index+1
```

In the block above, the number of iterations required is not known in advance and since randn is a standard normal pseudo-random number, it may take many iterations until this criteria is met – any finite for loop cannot be guaranteed to meet the criteria.

12.3.1 break

break can be used in a while loop to immediately terminate execution. In general, break should not be used in a while loop – instead the logical condition should be set to False to terminate the loop.

```
condition = True
i = 0
x = randn(1000)
while condition:
    print(x[i])
    i += 1
    if x[i] > 2:
        break
```

It is better to update the logical statement which determines whether the while loop should execute.

```
i = 0
while x[i] <= 2:
    print(x[i])
    i += 1
```

12.3.2 continue

continue can be used in a while loop to skip an iteration of a loop, immediately returning to the top of the loop, which then checks the while condition, and executes the loop if it still true. Use of continue in while loops is also rare.

12.4 try . . . except

Exception handling is an advanced programming technique which can be used to make code more resilient (often at the code of speed). try . . . except blocks are useful for running code which may be dangerous. In most numerical applications, code should be deterministic and so dangerous code can usually be avoided.

When it can't, for example, if reading data from a data source which isn't always available (e.g. a website), then `try` ... `except` can be used to attempt the code, and then do something helpful if the code fails to execute. The generic structure of a `try` ... `except` block is

```
try:
    Dangerous Code
except ExceptionType1:
    Code to run if ExceptionType1 is raised
except ExceptionType2:
    Code to run if ExceptionType1 is raised
...
...
except:
    Code to run if an unlisted exception type is raised
```

12.5 List Comprehensions

List comprehensions are a form of *syntactic sugar* which may simplify code when an iterable object is looped across and the results are saved to a list, possibly conditional on some logical test. Simple list can be used to convert a `for` loop which includes an append into a single line statement.

```
>>> x = arange(5.0)
>>> y = []
>>> for i in xrange(len(x)):
...    y.append(exp(x[i]))
>>> y
[1.0,
 2.7182818284590451,
 7.3890560989306504,
 20.085536923187668,
 54.598150033144236]

>>> z = [exp(x[i]) for i in xrange(len(x))]
>>> z
[1.0,
 2.7182818284590451,
 7.3890560989306504,
 20.085536923187668,
 54.598150033144236]
```

This simple list comprehension saves 2 lines of typing. List comprehensions can also be extended to include a logical test.

```
>>> x = arange(5.0)
>>> y = []
>>> for i in xrange(len(x)):
...    if floor(i/2)==i/2:
...        y.append(x[i]**2)
>>> y
[0.0, 4.0, 16.0]
```

```
>>> z = [x[i]**2 for i in xrange(len(x)) if floor(i/2)==i/2]
>>> z
[0.0, 4.0, 16.0]
```

List comprehensions can also be used to loop over multiple iterable input.

```
>>> x1 = arange(5.0)
>>> x2 = arange(3.0)
>>> y = []
>>> for i in xrange(len(x1)):
...     for j in xrange(len(x2)):
...         y.append(x1[i]*x2[j])
>>> y
[0.0, 0.0, 0.0, 0.0, 1.0, 2.0, 0.0, 2.0, 4.0, 0.0, 3.0, 6.0, 0.0, 4.0, 8.0]

>>> z = [x1[i]*x2[j] for i in xrange(len(x1)) for j in xrange(len(x2))]
[0.0, 0.0, 0.0, 0.0, 1.0, 2.0, 0.0, 2.0, 4.0, 0.0, 3.0, 6.0, 0.0, 4.0, 8.0]

>>> # Only when i==j
>>> z = [x1[i]*x2[j] for i in xrange(len(x1)) for j in xrange(len(x2)) if i==j]
[0.0, 1.0, 4.0]
```

While list comprehensions are powerful methods to compactly express complex operations, they are never essential to Python programming.

Loops make many problems, particularly when combined with flow control blocks, simple and in many cases, possible. Two types of loop blocks are available: for and while. for blocks iterate over a predetermined set of values and while blocks loop as long as some logical expression is satisfied. All for loops can be expressed as while loops although the opposite is not quite true. They are nearly equivalent when break is used, although it is generally preferable to use a while loop than a for loop and a break statement.

12.6 Exercises

1. Write a code block that would take a different path depending on whether the returns on two series are simultaneously positive, both are negative, or they have different signs using an if ... elif ... else block.

2. Simulate 1000 observations from an ARMA(2,2) where ϵ_t are independent standard normal innovations. The process of an ARMA(2,2) is given by

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t$$

   Use the values $\phi_1 = 1.4$, $\phi_2 = -.8$, $\theta_1 = .4$ and $\theta_2 = .8$. **Note**: A T vector containing standard normal random variables can be simulated using `e = randn(T)`. When simulating a process, always simulate more data then needed and throw away the first block of observations to avoid start-up biases. This process is fairly persistent, at least 100 extra observations should be computed.

3. Simulate a GARCH(1,1) process where ϵ_t are independent standard normal innovations. A GARCH(1,1)

process is given by

$$y_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$$

Use the values $\omega = 0.05$, $\alpha = 0.05$ and $\beta = 0.9$, and set $h_0 = \omega / \left(1 - \alpha - \beta\right)$.

4. Simulate a GJR-GARCH(1,1,1) process where ϵ_t are independent standard normal innovations. A GJR-GARCH(1,1) process is given by

$$y_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \gamma y_{t-1}^2 I_{[y_{t-1}<0]} + \beta \sigma_{t-1}^2$$

Use the values $\omega = 0.05$, $\alpha = 0.02$ $\gamma = 0.07$ and $\beta = 0.9$ and set $h_0 = \omega / \left(1 - \alpha - \frac{1}{2}\gamma - \beta\right)$. Note that some form of logical expression is needed in the loop. $I_{[\epsilon_{t-1}<0]}$ is an indicator variable that takes the value 1 if the expression inside the [ ] is true.

5. Simulate a ARMA(1,1)-GJR-GARCH(1,1)-in-mean process,

$$y_t = \phi_1 y_{t-1} + \theta_1 \sigma_{t-1} \epsilon_{t-1} + \lambda \sigma_t^2 + \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega + \alpha \sigma_{t-1}^2 \epsilon_{t-1}^2 + \gamma \sigma_{t-1}^2 \epsilon_{t-1}^2 I_{[\epsilon_{t-1}<0]} + \beta \sigma_{t-1}^2$$

Use the values from Exercise 3 for the GJR-GARCH model and use the $\phi_1 = -0.1$, $\theta_1 = 0.4$ and $\lambda = 0.03$.

6. Find two different methods to use a `for` loop to fill a 5×5 array with $i \times j$ where i is the row index, and j is the column index. One will use `xrange` as the *iterable*, and the other should directly iterate on the rows, and then the columns of the matrix.

7. Using a `while` loop, write a bit of code that will do a bisection search to invert a normal CDF. A bisection search cuts the interval in half repeatedly, only keeping the sub interval with the target in it. Hint: keep track of the upper and lower bounds of the random variable value and use flow control. This problem requires `stats.norm.cdf`.

8. Test out the loop using by finding the inverse CDF of 0.01, 0.5 and 0.975. Verify it is working by taking the absolute value of the difference between the final value and the value produced by `stats.norm.ppf`.

9. Write a list comprehension that will iterate over a 1-dimensional array and extract the negative elements to a list. How can this be done using *only* logical functions (no explicit loop), without the list comprehension (and returning an array)?

Chapter 13

Custom Function and Modules

Python supports a wide range of programming styles including procedural (imperative), object oriented and functional. While object oriented programming and functional programming are powerful programming paradigms, especially in large, complex software, procedural is often much easier to understand, and is often a direct representation of a mathematical formula. The basic idea of procedural programming is to produce a function or set of function (generically) of the form

$$y = f(x).$$

That is, the functions take inputs, and produce outputs – there can be more than one of either.

13.1 Functions

Python functions are very simple to declare and can occur in a variety of locations, including in the same file as the main program or in a standalone module. Functions are declared using the `def` keyword, and the value produced is returned using the `return` keyword. Consider a simple function which returns the square of the input,

$$y = x^2.$$

```
from __future__ import print_function
from __future__ import division

def square(x):
    return x**2

# Call the function
x = 2
y = square(x)
print(x,y)
```

In this example, the same Python file contains the main program – the bottom 3 lines – as well as the function. More complex function can be crafted with multiple inputs.

```
from __future__ import print_function
from __future__ import division
```

```python
def l2distance(x,y):
    return (x-y)**2

# Call the function
x = 3
y = 10
z = l2distance(x,y)
print(x,y,z)
```

They can also be defined using NumPy arrays and matrices.

```python
from __future__ import print_function
from __future__ import division

import numpy as np

def l2_norm(x,y):
    d = x - y
    return np.sqrt(np.dot(d,d))

# Call the function
x = np.random.randn(10)
y = np.random.randn(10)
z = l2_norm(x,y)
print(x-y)
print("The L2 distance is ",z)
```

Similarly, multiple outputs can be returned, usually as a tuple.

```python
from __future__ import print_function
from __future__ import division

import numpy as np

def l1_l2_norm(x,y):
    d = x - y
    return sum(np.abs(d)),np.sqrt(np.dot(d,d))

# Call the function
x = np.random.randn(10)
y = np.random.randn(10)
# Using 1 output returns a tuple
z = l1_l2_norm(x,y)
print(x-y)
print("The L1 distance is ",z[0])
print("The L2 distance is ",z[1])

# Using 2 output returns the values
l1,l2 = l1_l2_norm(x,y)
print("The L1 distance is ",l1)
print("The L2 distance is ",l2)
```

All of these functions have been placed in the same file as the main program. While this is a simple method, it limits reuse. Placing functions in modules allows for reuse in multiple programs, and will be discussed later in this chapter.

13.1.1 Keyword Arguments

Input values in functions are automatically keyword arguments, so that the function can be accessed either by placing the inputs in the order they appear in the function (positional arguments), or by calling the input by their name using *keyword=value*.

```
from __future__ import print_function
from __future__ import division

import numpy as np

def lp_norm(x,y,p):
    d = x - y
    return sum(abs(d)**p)**(1/p)

# Call the function
x = np.random.randn(10)
y = np.random.randn(10)
z1 = lp_norm(x,y,2)
z2 = lp_norm(p=2,x=x,y=y)
print("The Lp distances are ",z1,z2)
```

Because variable names are automatically keywords, it is important to use meaningful variable names when possible, rather than generic variables such as a, b, c or x, y and z. In some cases, x may be a reasonable default, but in the previous example which computed the L_p norm, calling the third input z would be bad idea.

13.1.2 Default Values

Default values are set in the function declaration using the syntax *input=default*.

```
from __future__ import print_function
from __future__ import division

import numpy as np

def lp_norm(x,y,p = 2):
    d = x - y
    return sum(abs(d)**p)**(1/p)

# Call the function
x = np.random.randn(10)
y = np.random.randn(10)
# Inputs with default values can be ignored
l2 = lp_norm(x,y)
l1 = lp_norm(x,y,1)
```

```
print("The l1 and l2 distances are ",l1,l2)
print("Is the default value overridden?", sum(abs(x-y))==l1)
```

Default values should not normally be mutable (e.g. lists or arrays) since they are only initialized the first time the function is called. Subsequent calls will use the same value, which means that the default value could change every time the function is called.

```
from __future__ import print_function
from __future__ import division

import numpy as np

def bad_function(x = zeros(1)):
    print(x)
    x[0] = np.random.randn(1)


# Call the function
bad_function()
bad_function()
bad_function()
```

Each call to `bad_function()` shows that x has a different value – despite the default being 0. The solution to this problem is to initialize mutable objects to `None`, and then the use an `if` to check and initialize.

```
from __future__ import print_function
from __future__ import division

import numpy as np

def good_function(x = None):
    if x == None:
        x = zeros(1)
    print(x)
    x[0] = np.random.randn(1)


# Call the function
good_function()
good_function()
```

Repeated calls to `good_function()` all show x as 0.

13.1.3 Variable Inputs

Most function written as an "end user" have a deterministic number of inputs. However, functions which evaluate other functions often must accept variable numbers of input. Variable inputs can be handled using the `*arguments` or `**keywords` syntax. The `*arguments` syntax will generate a containing all inputs past the specified input list. For example, consider extending the L_p function so that it can accept a set of p values as extra inputs (Note: in practice it would make more sense to accept an array for p).

```
from __future__ import print_function
from __future__ import division

import numpy as np

def lp_norm(x,y,p = 2, *arguments):
    d = x - y
    print('The L' + str(p) + ' distance is :', sum(abs(d)**p)**(1/p))
    out = [sum(abs(d)**p)**(1/p)]

    for p in arguments:
        print('The L' + str(p) + ' distance is :', sum(abs(d)**p)**(1/p))
        out.append(sum(abs(d)**p)**(1/p))

    return tuple(out)


# Call the function
x = np.random.randn(10)
y = np.random.randn(10)
# Inputs with default values can be ignored
lp = lp_norm(x,y,1,2,3,4,1.5,2.5,0.5)
```

The alternative syntax, `**keywords`, generates a dictionary with all keyword inputs which are not in the function signature. One reason for using `**keywords` is to allow a long list of optional inputs without having to have an excessively long function definition, and is how this input mechanism is often encountered when using other code, for example `plot()`.

```
from __future__ import print_function
from __future__ import division

import numpy as np

def lp_norm(x,y,p = 2, **keywords):
    d = x - y
    for key in keywords:
        print('Key :', key, ' Value:', keywords[key])

    return sum(abs(d)**p)

# Call the function
x = np.random.randn(10)
y = np.random.randn(10)
# Inputs with default values can be ignored
lp = lp_norm(x,y,kword1=1,kword2=3.2)
# The p keyword is in the function def, so not in **keywords
lp = lp_norm(x,y,kword1=1,kword2=3.2,p=0)
```

It is possible to use both `*arguments` and `**keywords` in a function definition and their role does not change – `*arguments` is a tuple which contains all extraneous non-keyword inputs, and `**keywords` will contain all extra keyword arguments. Function with both often have the simple signature `y = f(*arguments,` `**keywords)`which allows for a wide range of configuration.

13.1.4 The Docstring

The docstring is one of the most important elements of any function – especially a function written by consumption by others. The docstring is a special string, enclose using triple-quotation marks, either `'''` or `"""`, which is available using `help()`. When `help(fun)` is called, Python looks for the docstring which is placed immediately below the function definition.

```
from __future__ import print_function
from __future__ import division

import numpy as np

def lp_norm(x,y,p = 2):
    ''' The docstring contains any available help for
        the function.  A good docstring should explain the
        inputs and the outputs, provide an example and a list
        of any other related function.
    '''
    d = x - y
    return sum(abs(d)**p)
```

Calling `help(lp_norm)` produces

```
>>> help(lp_norm)
Help on function lp_norm in module __main__:

lp_norm(x, y, p=2)
    The docstring contains any available help for
    the function.  A good docstring should explain the
    inputs and the outputs, provide an example and a list
    of any other related function.
```

Note that the docstring is not a good example. I suggest following the the NumPy guidelines, currently available atNumPy source repository (or search for *numpy docstring*). Also see NumPy example.py These differ from and are more specialized than the standard Python docstring guidelines, and so are more appropriate for numerical code. A better docstring for `lp_norm` would be

```
from __future__ import print_function
from __future__ import division

import numpy as np

def lp_norm(x,y,p = 2):
    r""" Compute the distance between vectors.

    The Lp normed distance is sum(abs(x-y)**p)**(1/p)

    Parameters
    ----------
    x : ndarray
        First argument
    y : ndarray
        Second argument
    p : float, optional
        Power used in distance calcualtion, >=0

    Returns
    -------
    output : scalar
        Returns the Lp normed distance between x and y

    Notes
    -----

    For p>=1, returns the Lp norm described above.  For 0<=p<1,
    returns sum(abs(x-y)**p).  If p<0, p is set to 0.

    Examples
    --------
    >>> x=[0,1,2]
    >>> y=[1,2,3]

    L2 norm is the default

    >>> lp_norm(x,y)

    Lp can be computed using the optional third input

    >>> lp_norm(x,y,1)

    """

    if p<0: p=0
    d = x - y
    dist = sum(abs(d)**p)

    if p<1:
        return dist
    else:
        return dist**(1/p)
```

Convention is to use triple double-quotes in docstrings, with `r"""` used to indicate "raw" strings, which will ignore backslash, rather than treating it like an escape character (use `u"""` if the docstring contains unicode text, which is not usually necessary). A complete docstring may contain, in order:

- Parameters - a description of key inputs
- Returns - a description of outputs
- Other Parameters - a description of seldom used inputs

- Raises - an explanation of any exceptions raised. See Section 12.4.
- See also - a list of related functions
- Notes - details of the algorithms or assumptions used
- References - any bibliographic information
- Examples - demonstrates use form console

13.2 Variable Scope

Variable scope determines which function can access, and possibly modify a variable. Python determines variable scope using two principles: where the variable appears in the file, and whether the variable is inside a function or in the main program. Variables declared inside a function are local variables and are only available to that function. Variables declared outside a function are global variables, and can be accessed but normally not modified. Consider the example which shows that variables at the root of the program which have been declared before a function can be printed by that function.

```python
from __future__ import print_function
from __future__ import division
import numpy as np

a, b, c = 1, 3.1415, 'Python'

def scope():
    print(a)
    print(b)
    print(c)
    # print(d) #Error, d has not be declared yet

scope()
d = np.array(1)

def scope2():
    print(a)
    print(b)
    print(c)
    print(d) # Ok now

scope2()

def scope3():
    a = 'Not a number' # Local variable
    print('Inside scope3, a is ', a)

print('a is ',a)
scope3()
print('a is now ',a)
```

Using the name of a global variable inside a function does not cause any issues outside of the function. In scope3, a is given a different value. That value is specific to the function scope3 and outside of the function, a will have its global value. Generally, global variables can be accessed, but not modified inside a function. The only exception is when a variable is first declared using the keyword `global`.

```python
from __future__ import print_function
from __future__ import division
import numpy as np

a = 1


def scope_local():
    a = -1
    print('Inside scope_local, a is ',a)



def scope_global():
    global a
    a = -10
    print('Inside scope_global, a is ',a)

print('a is ',a)
scope_local()
print('a is now ',a)
scope_global()
print('a is now ',a)
```

One word of caution: a variable name cannot be used as a local and global variable in the same function. Attempting to access the variable as a global (e.g. for printing) and then locally assign the variable produces an error.

13.3 Example: Least Squares with Newey-West Covariance

Estimating cross-section regressions using time-series data is common practice. When regressors are persistent, and errors may not be white noise, standard inference, including White standard errors, are no longer consistent. The most common solution is to use a long-run covariance estimator, and the most common long-run covariance estimator is known as the Newey-West covariance estimator, which uses a Bartlett kernel applied to the autocovariances of the scores. This example produces a function which returns parameter estimates, the estimated asymptotic covariance matrix of the parameters, the variance of the regression error, the R^2, and adjusted R^2 and the fit values (or errors, since actual is equal to fit plus errors). These be computed using a T-vector for the regressand (dependent variable), a T by k matrix for the regressors, an indicator for whether to include a constant in the model (default True), and the number of lags to include in the long-run covariance (default behavior is to automatically determine based on sample size). The steps required to produce the function are:

1. Determine the size of the variables

2. Append a constant, if needed
3. Compute the regression coefficients
4. Compute the errors
5. Compute the covariance or the errors
6. Compute the covariance of the parameters
7. Compute the R^2 and $\bar{R}^2$

The function definition is simple and allows for up to 4 inputs, where 2 have default values: `def olsnw(y, X, constant=True, lags=None):`. The size of the variables is then determined using `size` and the constant is prepended to the regressors, if needed, using `hstack`. The regression coefficients are computed using `lstsq`, and then the Newey-West covariance is computed for both the errors and and scores. The covariance of the parameters is then computed using the NW covariance of the scores. Finally the R^2 and $\bar{R}^2$ are computed. A complete code listing is presented in the appendix to this chapter.

13.4 Modules

The previous examples all included the function in inside the Python file that contained the main program. While this is convenient, especially for writing the function, it hinders use in other code. Modules allow multiple functions to be combined in a single Python file and accessed using `import` *module* and then *module.function* syntax. Suppose a file named core.py contains the following code:

```
r"""Demonstration module
"""

def square(x):
    r"""Returns the square of a scalar input
    """
    return x*x

def cube(x):
    r"""Returns the cube of a scalar input
    """
    return x*x*x
```

The functions `square` and `cube` can be accessed by other files in the same directory using

```
from __future__ import division
from __future__ import print_function
import core

y = -3
print(core.square(y))
print(core.cube(y))
```

The functions in core.py can be imported using any of the standard import methods such as

```
from core import square, cube
```

or

```
from core import *
```

in which case both functions could be directly accessed.

13.4.1 __main__

Normally modules should only have code required for the module to run, and other code should reside in a different function. However, it is possible that a module could be both directly important and also directly runnable. If this is the case, it is important that the directly runnable code should not be executed when the module is imported by other code. This can be accomplished using a special construct `if __name__=="__main__":`before any code that should execute when run as a standalone program. Consider the following simple example in a module named`test.py`.

```
from __future__ import division
from __future__ import print_function

def square(x):
    return x**2

if __name__=="__main__":
    print('Program called directly.')
else:
    print('Program called indirectly using name: ', __name__)
```

Running and importing `test` cause the different paths to be executed.

```
>>> %run test.py
Program called directly.

>>> import test
Program called indirectly using name:  test
```

13.5 PYTHONPATH

While it is simple to reference files in the same current working directory, this behavior is undesirable for code shared between multiple projects. Fortunately the PYTHONPATH allows directories to be added so that they are automatically searched if a matching module cannot be found in the current directory. The current path can be checked by running

```
>>> import sys
>>> sys.path
```

Additional directories can be added at runtime using

```
import sys

# New directory is first to be searched
sys.path.insert(0, 'c:\\path\\to\add')
# New directory is last to be searched
sys.path.append
```

Directories can also be added permanently by adding or modifying the environment variable PYTHONPATH.

On Windows, the System environment variables can be found in My Computer > Properties > Advanced System Settings > Environment Variables. PYTHONPATH should be a System Variable. If it is present, it can be edited, and if not, added. The format of PYTHONPATH is

```
c:\dir1;c:\dir2;c:\dir2\dir3;
```

which will add 3 directories to the path. On Linux, PYTHONPATH is stored in .bash_profile, and it should resemble

```
PYTHONPATH="${PYTHONPATH}:/dir1/:/dir2/:/dir2/dir3/"
export PYTHONPATH
```

after three directories have been added, using : as a separator between directories.

13.6 Packages

Packages are the next level beyond modules, and allow, for example, nested module names (e.g. `numpy.random` which contains `randn`). Packages are also installed in the local package library, and can be compiled into optimized Python byte code, which makes loading modules faster (but does not make code run faster). Building a package is beyond the scope of these notes, but there are many resources on the internet with instructions for building packages.

13.7 Python Coding Conventions

There are a number of common practices which can be adopted to produce Python code which looks more like code found in other modules:

1. Use 4 spaces to indent blocks – avoid using tab, except when an editor automatically converts tabs to 4 spaces
2. Avoid more than 4 levels of nesting, if possible
3. Limit lines to 79 characters. The \ symbol can be used to break long lines
4. Use blank lines to separate functions or logical sections in a function.
5. Use ASCII model in text editors, not UTF-8
6. One import per line
7. Avoid `from` *module* `import *` (for any module). Use either `from` *module* `import` *func1, func2* or `import` *module* `as` *shortname*.
8. Follow the NumPy guidelines for documenting functions

More suggestions can be found in PEP8.

13.8 Exercises

1. Write a function which takes an array with T elements contains categorical data (e.g. 1,2,3), and returns a T by C array of indicator variables where C is the number of unique values of the categorical variable, and each column of the output is a indicator variable (0 or 1) for whether the input data belonged to that category. For example, if $x = [1\ 2\ 1\ 1\ 2]$, then the the output is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The function should provide a second output containing the categories (e.g. [1 2] in the example).

2. Write a function which takes a T by K array X, a T by 1 array y, and a T by T array Ω are computes the GLS parameter estimates. The function definition should be

```python
def gls(X, y, Omega = None
```

and if Ω is not provided, an identity matrix should be used.

3. Write a function which will compute the partial correlation. Lower partial correlation is defined as

$$\frac{\sum_{\mathcal{S}}\left(r_{i,1}-\bar{r}_{1,\mathcal{S}}\right)\left(r_{i,2}-\bar{r}_{2,\mathcal{S}}\right)}{\sum_{\mathcal{S}}\left(r_{j,1}-\bar{r}_{1,\mathcal{S}}\right)^2\sum_{\mathcal{S}}\left(r_{k,2}-\bar{r}_{2,\mathcal{S}}\right)^2}$$

where $\mathcal{S}$ is the set where $r_{1,i}$ and $r_{2,i}$ are both less than their (own) quantile q. Upper partial correlation uses returns greater than quantile q. The function definition should have definition

```python
def partial_corr(x, y=None, quantile = 0.5, tail = 'Lower')
```

and should take either a T by K array for x, or T by 1 arrays for x and y. If x is T by K, then y is ignored and the partial correlation should be computed pairwise. `quantile` determines the quantile to use for the cut off. Note: if $\mathcal{S}$ is empty or has 1 element, nan should be returned. `tail` is either `'Lower'` or `'Upper'`, and determined whether the lower or upper tail is used. The function should return both the partial correlation matrix (K by K), and the number of observations used in computing the partial correlation.

13.A Listing of econometrics.py

The complete code listing of `econometrics`, which contains the function `olsnw`, is presented below.

```python
from __future__ import print_function
from __future__ import division

from numpy import dot, mat, asarray, mean, size, shape, hstack, ones, ceil, \
    zeros, arange
from numpy.linalg import inv, lstsq
```

```python
def olsnw(y, X, constant=True, lags=None):
    r""" Estimation of a linear regression with Newey-West covariance

    Parameters
    ----------
    y : array_like
        The dependant variable (regressand).  1-dimensional with T elements.
    X : array_like
        The independant variables (regressors). 2-dimensional with sizes T
        and K.  Should not include a constant.
    constant: bool, optional
        If true (default) includes model includes a constant.
    lags: int or None, optional
        If None, the number of lags is set to 1.2*T**(1/3), otherwise the
        number of lags used in the covariance estimation is set to the value
        provided.

    Returns
    -------
    b : ndarray, shape (K,) or (K+1,)
        Parameter estimates.  If constant=True, the first value is the
        intercept.
    vcv : ndarray, shape (K,K) or (K+1,K+1)
        Asymptotic covariance matrix of estimated parameters
    s2 : float
        Asymptotic variance of residuals, computed using Newey-West variance
        estimator.
    R2 : float
        Model R-square
    R2bar : float
        Adjusted R-square
    e : ndarray, shape (T,)
        Array containing the model errors

    Notes
    -----
    The Newey-West covariance estimator applies a Bartlett kernel to estimate
    the long-run covariance of the scores.  Setting lags=0 produces White's
    Heteroskedasticity Robust covariance matrix.

    See also
    --------
    np.linalg.lstsq

    Example
    -------
    >>> X = randn(1000,3)
    >>> y = randn(1000,1)
    >>> b,vcv,s2,R2,R2bar = olsnw(y, X)

    Exclude constant:

    >>> b,vcv,s2,R2,R2bar = olsnw(y, X, False)

    Specify number of lags to use:

    >>> b,vcv,s2,R2,R2bar = olsnw(y, X, lags = 4)

    """


    T = y.size
    if size(X, 0) != T:
        X = X.T

    T,K = shape(X)
    if constant:
        X = copy(X)
        X = hstack((ones((T,1)),X))
        K = size(X,1)

    if lags==None:
        lags = int(ceil(1.2 * float(T)**(1.0/3)))

    # Parameter estimates and errors

    out = lstsq(X,y)
    b = out[0]
    e = y - dot(X,b)

    # Covariance of errors
    gamma = zeros((lags+1))
    for lag in xrange(lags+1):
        gamma[lag] = dot(e[:T-lag],e[lag:]) / T

    w = 1 - arange(0,lags+1)/(lags+1)
    w[0] = 0.5
    s2 = dot(gamma,2*w)

    # Covariance of parameters
    Xe = mat(zeros(shape(X)))
    for i in xrange(T):
        Xe[i] = X[i] * float(e[i])

    Gamma = zeros((lags+1,K,K))
    for lag in xrange(lags+1):
        Gamma[lag] = Xe[lag:].T*Xe[:T-lag]
```

```
Gamma = Gamma/T

S = Gamma[0].copy()
for i in xrange(1,lags+1):
    S = S + w[i]*(Gamma[i]+Gamma[i].T)

XpX = dot(X.T,X)/T
XpXi = inv(XpX)
vcv = mat(XpXi)*S*mat(XpXi)/T
vcv = asarray(vcv)

# R2, centered or uncentered
if constant:
    R2 = dot(e,e)/dot(y-mean(y),y-mean(y))
else:
    R2 = dot(e,e)/dot(y,y)

R2bar = 1-R2*(T-1)/(T-K)
R2 = 1 - R2

return b,vcv,s2,R2,R2bar,e
```

Chapter 14

Probability and Statistics Functions

This chapter is divided into two main parts, one for NumPy and one for SciPy. Both packages contain important functions for simulation, probability distributions and statistics.

NumPy

14.1 Simulating Random Variables

14.1.1 Core Random Number Generators

NumPy random number generators are all stored in the module `numpy.random`. These can be imported with using `import numpy as np` and then calling `np.random.rand()`, for example, or by importing `import numpy.random as rnd` and using `rnd.rand()`.[1]

`rand`, `random_sample`

`rand` and `random_sample` are uniform random number generators which are identical except that `rand` takes a variable number of integer inputs – one for each dimension – while `random_sample` takes a n-element tuple. `rand` is a convenience function for `random_sample`.

```
>>> x = rand(3,4,5)
>>> y = random_sample((3,4,5))
```

`randn`, `standard_normal`

`randn` and `standard_normal` are standard normal random number generators. `randn`, like `rand`, takes a variable number of integer inputs, and `standard_normal` takes an n-element tuple. Both can be called with no arguments to generate a single standard normal (e.g. `randn()`). `randn` is a convenience function for `standard_normal`.

```
>>> x = randn(3,4,5)
>>> y = standard_normal((3,4,5))
```

[1]Other import methods can also be used, such as `from numpy.random import rand` and then calling `rand()`

randint, random_integers

randint and random_integers are uniform integer random number generators which take 3 inputs, low, high and size. Low is the lower bound of the integers generated, high is the upper and size is an n-element tuple. randint and random_integers differ in that randint generates integers exclusive of the value in high (as most Python functions), while random_integers includes the value in high.

```
>>> x = randint(0,10,(100))
>>> x.max() # Is 9 since range is [0,10)
9

>>> y = random_integers(0,10,(100))
>>> y.max() # Is 10 since range is [0,10]
10
```

14.1.2 Random Array Functions

shuffle

shuffle randomly reorders the elements of an array *in place*.

```
>>> x = arange(10)
>>> shuffle(x)
>>> x
array([4, 6, 3, 7, 9, 0, 2, 1, 8, 5])
```

permutation

permutation returns randomly reordered elements of an array.

```
>>> x = arange(10)
>>> permutation(x)
array([2, 5, 3, 0, 6, 1, 9, 8, 4, 7])
>>> x
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
```

14.1.3 Select Random Number Generators

NumPy provides a large selection of random number generators for specific distribution. All take between 0 and 2 required input which are parameters of the distribution, plus a tuple containing the size of the output. All random number generators are in the module numpy.random.

Bernoulli

There is no Bernoulli generator. Instead use `1 - (rand()>p)` to generate a single draw or `1 - (rand(10,10)>p)` to generate an array.

beta

beta(a,b) generates a draw from the Beta(a, b) distribution. beta(a,b,(10,10)) generates a 10 by 10 array of draws from a Beta(a, b) distribution.

binomial

`binomial(n,p)` generates a draw from the Binomial(n, p) distribution. `binomial(n,p,(10,10))` generates a 10 by 10 array of draws from the Binomial(n, p) distribution.

chisquare

`chisquare(nu)` generates a draw from the χ^2_ν distribution, where ν is the degree of freedom. `chisquare(nu,(10,10))` generates a 10 by 10 array of draws from the χ^2_ν distribution.

exponential

`exponential()` generates a draw from the Exponential distribution with scale parameter $\lambda = 1$. `exponential( lambda, (10,10))` generates a 10 by 10 array of draws from the Exponential distribution with scale parameter λ.

f

`f(v1,v2)` generates a draw from the distribution F_{ν_1,ν_2}distribution where ν_1 is the numerator degree of freedom and ν_2 is the denominator degree of freedom. `f(v1,v2,(10,10))` generates a 10 by 10 array of draws from the F_{ν_1,ν_2}distribution.

gamma

`gamma(a)` generates a draw from the Gamma(α, 1) distribution, where α is the shape parameter. `gamma(a, theta, (10,10))` generates a 10 by 10 array of draws from the Gamma(α, θ) distribution where θ is the scale parameter.

laplace

`laplace()` generates a draw from the Laplace (Double Exponential) distribution with centered at 0 and unit scale. `laplace(loc, scale, (10,10))` generates a 10 by 10 array of Laplace distributed data with location `loc` and scale `scale`. Using `laplace(loc, scale)` is equivalent to calling `loc + scale*laplace()`.

lognormal

`lognormal()` generates a draw from a Log-Normal distribution with $\mu = 0$ and $\sigma = 1$. `lognormal(mu, sigma, (10,10))` generates a 10 by 10 array or Log-Normally distributed data where the underlying Normal distribution has mean parameter μ and scale parameter σ.

multinomial

`multinomial(n, p)` generates a draw from a multinomial distribution using n trials and where each outcome has probability p, a k-element array where $\Sigma_{i=1}^{k} p = 1$. The output is a k-element array containing the number of successes in each category. `multinomial(n, p, (10,10))` generates a 10 by 10 by k array of multinomially distributed data with n trials and probabilities p.

`multivariate_normal`

`multivariate_normal(mu, Sigma)` generates a draw from a multivariate Normal distribution with mean μ (k-element array) and covariance Σ (k by k array). `multivariate_normal(mu, Sigma, (10,10))` generates a 10 by 10 by k array of draws from a multivariate Normal distribution with mean μ and covariance Σ.

`negative_binomial`

`negative_binomial(n, p)` generates a draw from the Negative Binomial distribution where n is the number of failures before stopping and p is the success rate. `negative_binomial(n, p, (10, 10))` generates a 10 by 10 array of draws from the Negative Binomial distribution where n is the number of failures before stopping and p is the success rate.

`normal`

`normal()` generates draws from a standard Normal (Gaussian). `normal(mu, sigma)` generates draws from a Normal with mean μ and standard deviation σ. `normal(mu, sigma, (10,10))` generates a 10 by 10 array of draws from a Normal with mean μ and standard deviation σ. `normal(mu, sigma)` is equivalent to `mu + sigma * rand()` or `mu + sigma * standard_normal()`.

`poisson`

`poisson()` generates a draw from a Poisson distribution with $\lambda = 1$. `poisson(lambda)` generates a draw from a Poisson distribution with expectation λ. `poisson(lambda, (10,10))` generates a 10 by 10 array of draws from a Poisson distribution with expectation λ.

`standard_t`

`standard_t(nu)` generates a draw from a Student's t with shape parameter ν. `standard_t(nu, (10,10))` generates a 10 by 10 array of draws from a Student's t with shape parameter ν.

`uniform`

`uniform()` generates a uniform random variable on $(0, 1)$. `uniform(low, high)` generates a uniform on (l, h). `uniform(low, high, (10,10))` generates a 10 by 10 array of uniforms on (l, h).

14.2 Simulation and Random Number Generation

Computer simulated random numbers are usually constructed from very complex, but ultimately deterministic functions. Because they are not actually random, simulated random numbers are generally described to as pseudo-random. All pseudo-random numbers in NumPy use one core random number generator based on the Mersenne Twister, a generator which can produce a very long series of pseudo-random data before repeating (up to $2^{19937} - 1$ non-repeating values).

RandomState

RandomState is the class used to control the random number generators. Multiple generators can be initialized by RandomState.

```
>>> gen1 = np.random.RandomState()
>>> gen2 = np.random.RandomState()
>>> gen1.uniform() # Generate a uniform
0.6767614077579269

>>> state1 = gen1.get_state()
>>> gen1.uniform()
0.6046087317893271

>>> gen2.uniform() # Different, since gen2 has different seed
0.04519705909244154

>>> gen2.set_state(state1)
>>> gen2.uniform() # Same uniform as gen1 produces after assigning state
0.6046087317893271
```

14.2.1 State

Pseudo-random number generators track a set of values known as the state. The state is usually a vector which has the property that if two instances of the same pseudo-random number generator have the same state, the sequence of pseudo-random numbers generated will be identical. The state in NumPy can be read using numpy.random.get_state and can be restored using numpy.random.set_state.

```
>>> st = get_state()
>>> randn(4)
array([ 0.37283499,  0.63661908, -1.51588209, -1.36540624])

>>> set_state(st)
>>> randn(4)
array([ 0.37283499,  0.63661908, -1.51588209, -1.36540624])
```

The two sequences are identical since they the state is the same when randn is called. The state is a 5-element tuple where the second element is a 625 by 1 vector of unsigned 32-bit integers. In practice the state should only be stored using get_state and restored using set_state.

get_state

get_state() gets the current state of the random number generator, which is a 5-element tuple. It can be called as a function, in which case it gets the state of the default random number generator, or as a method on a particular instance of RandomState().

set_state

set_state(state) sets the state of the random number generator. It can be called as a function, in which case it sets the state of the default random number generator, or as a method on a particular instance of

RandomState(). set_state should generally only be called using a state tuple returned by get_state.

14.2.2 Seed

numpy.random.seed is a more useful function for initializing the random number generator, and can be used in one of two ways. seed() will initialize (or reinitialize) the random number generator using some actual random data provided by the operating system.[2] seed(*s*) takes a vector of values (can be scalar) to initialize the random number generator at particular state. seed(*s*) is particularly useful for producing simulation studies which are reproducible. In the following example, calls to seed() produce different random numbers, since these reinitialize using random data from the computer, while calls to seed(0) produce the same (sequence) of random numbers.

```
>>> seed()
>>> randn(1)
array([ 0.62968838])

>>> seed()
>>> randn(1)
array([ 2.230155])

>>> seed(0)
>>> randn(1)
array([ 1.76405235])

>>> seed(0)
>>> randn(1)
array([ 1.76405235])
```

NumPy always calls seed() when the first random number is generated. As a result. calling randn(1) across two "fresh" sessions will not produce the same random number.

seed

seed(value) uses *value* to seed the random number generator. seed() takes actual random data from the operating system (e.g. /dev/random on Linux, or CryptGenRandom in Windows).

14.2.3 Replicating Simulation Data

It is important to have reproducible results when conducting a simulation study. There are two methods to accomplish this:

1. Call seed() and then st = get_state(), and save st to a file which can then be loaded in the future when running the simulation study.

2. Call seed(*s*) at the start of the program (where *s* is a constant).

Either of these will allow the same sequence of random numbers to be used.

[2]All modern operating systems collect data that is effectively random by collecting noise from device drivers and other system monitors.

Warning: Do not *over-initialize* the pseudo-random number generators. The generators should be initialized once per session and then allowed to produce the pseudo-random sequence. Repeatedly re-initializing the pseudo-random number generators will produce a sequence that is decidedly less random than the generator was designed to provide.

14.2.3.1 Considerations when Running Simulations on Multiple Computers

Simulation studies are ideally suited to parallelization. Parallel code does make reproducibility more difficult. There are 2 methods which can ensure that a parallel study is reproducible.

1. Have a single process produce all of the random numbers, where this process has been initialized using one of the two methods discussed in the previous section. Formally this can be accomplished by pre-generating all random numbers, and then passing these into the simulation code as a parameter, or equivalently by pre-generating the data and passing the state into the function. Inside the simulation function, the random number generator will be set to the state which was passed as a parameter. The latter is a better option if the amount of data per simulation is large.

2. Seed each parallel worker independently, and then return then save the state inside the simulation function. The state should be returned and saved along with the simulation results. Since the state is saved for each simulation, it is possible to use the same state if repeating the simulation using, for example, a different estimator.

14.3 Statistics Functions

mean

`mean` computes the average of an array. An optional second argument provides the axis to use (default is to use entire array). `mean` can be used either as a function or as a method on an array.

```
>>> x = arange(10.0)
>>> x.mean()
4.5

>>> mean(x)
4.5

>>> x= reshape(arange(20.0),(4,5))
>>> mean(x,0)
array([  7.5,   8.5,   9.5,  10.5,  11.5])

>>> x.mean(1)
array([  2.,   7.,  12.,  17.])
```

median

`median` computed the median value in an array. An optional second argument provides the axis to use (default is to use entire array).

```
>>> x= randn(4,5)
>>> x
array([[-0.74448693, -0.63673031, -0.40608815,  0.40529852, -0.93803737],
       [ 0.77746525,  0.33487689,  0.78147524, -0.5050722 ,  0.58048329],
       [-0.51451403, -0.79600763,  0.92590814, -0.53996231, -0.24834136],
       [-0.83610656,  0.29678017, -0.66112691,  0.10792584, -1.23180865]])

>>> median(x)
-0.45558017286810903

>>> median(x, 0)
 array([-0.62950048, -0.16997507,  0.18769355, -0.19857318, -0.59318936])
```

Note that when an array or axis dimension contains an even number of elements (n), `median` returns the average of the 2 inner elements.

std

`std` computes the standard deviation of an array. An optional second argument provides the axis to use (default is to use entire array). `std` can be used either as a function or as a method on an array.

var

`var` computes the variance of an array. An optional second argument provides the axis to use (default is to use entire array). `var` can be used either as a function or as a method on an array.

corrcoef

`corrcoef(x)` computes the correlation between the rows of a 2-dimensional array x. `corrcoef(x, y)` computes the correlation between two 1- dimensional vectors. An optional keyword argument `rowvar` can be used to compute the correlation between the columns of the input – this is `corrcoef(x, rowvar=False)` and `corrcoef(x.T)` are identical.

```
>>> x= randn(3,4)
>>> corrcoef(x)
array([[ 1.        ,  0.36780596,  0.08159501],
       [ 0.36780596,  1.        ,  0.66841624],
       [ 0.08159501,  0.66841624,  1.        ]])

>>> corrcoef(x[0],x[1])
array([[ 1.        ,  0.36780596],
       [ 0.36780596,  1.        ]])

>>> corrcoef(x, rowvar=False)
array([[ 1.        , -0.98221501, -0.19209871, -0.81622298],
       [-0.98221501,  1.        ,  0.37294497,  0.91018215],
       [-0.19209871,  0.37294497,  1.        ,  0.72377239],
       [-0.81622298,  0.91018215,  0.72377239,  1.        ]])

>>> corrcoef(x.T)
```

```
array([[ 1.        , -0.98221501, -0.19209871, -0.81622298],
       [-0.98221501,  1.        ,  0.37294497,  0.91018215],
       [-0.19209871,  0.37294497,  1.        ,  0.72377239],
       [-0.81622298,  0.91018215,  0.72377239,  1.        ]])
```

cov

`cov(x)` computes the covariance of an array x. `cov(x,y)` computes the covariance between two 1-dimensional vectors. An optional keyword argument `rowvar` can be used to compute the covariance between the columns of the input – this is `cov(x, rowvar=False)` and `cov(x.T)` are identical.

histogram

`histogram` can be used to compute the histogram (empirical frequency, using k bins) of a set of data. An optional second argument provides the number of bins. If omitted, k =10 bins are used. `histogram` returns two outputs, the first with a k-element vector containing the number of observations in each bin, and the second with the $k+1$ endpoints of the k bins.

```
>>> x = randn(1000)
>>> count, binends = histogram(x)
>>> count
array([  7,  27,  68, 158, 237, 218, 163,  79,  36,   7])

>>> binends
array([-3.06828057, -2.46725067, -1.86622077, -1.26519086, -0.66416096,
       -0.06313105,  0.53789885,  1.13892875,  1.73995866,  2.34098856,
        2.94201846])

>>> count, binends = histogram(x, 25)
```

histogram2d

`histogram2d(x,y)` computes a 2-dimensional histogram for 1-dimensional vectors. An optional second argument provides the number of bins to use.

SciPy

SciPy provides an extended range of random number generators, probability distributions and statistical tests.

```
import scipy
import scipy.stats as stats
```

14.4 Continuous Random Variables

SciPy contains a large number of functions for working with continuous random variables. Each function resides in its own class (e.g. `norm` for Normal or `gamma` for Gamma), and classes expose methods for random number generation, computing the PDF, CDF and inverse CDF, fitting parameters using MLE, and computing various moments. The methods are listed below, where *dist* is a generic placeholder for the distribution name in SciPy. While the functions available for continuous random variables vary in their inputs, all take 3 generic arguments:

1. `*args` a set of distribution specific non-keyword arguments. These must be entered in the order listed in the class docstring. For example, when using a F-distribution, two arguments are needed, one for the numerator degree of freedom, and one for the denominator degree of freedom.

2. `loc` a location parameter, which determines the center of the distribution.

3. `scale` a scale parameter, which determine the scaling of the distribution. For example, if z is a standard normal, then sz is a scaled standard normal.

dist.`rvs`

Pseudo-random number generation. Generically, `rvs` is called using *dist*`.rvs(*args, loc=0, scale=1, size=size)` where `size` is an n-element tuple containing the size of the array to be generated.

dist.`pdf`

Probability density function evaluation for an array of data (element-by-element). Generically, `pdf` is called using *dist*`.pdf(x, *args, loc=0, scale=1)` where `x` is an array that contains the values to use when evaluating PDF.

dist.`logpdf`

Log probability density function evaluation for an array of data (element-by-element). Generically, `logpdf` is called using *dist*`.logpdf(x, *args, loc=0, scale=1)` where `x` is an array that contains the values to use when evaluating log PDF.

dist.`cdf`

Cumulative distribution function evaluation for an array of data (element-by-element). Generically, `cdf` is called using *dist*`.cdf(x, *args, loc=0, scale=1)` where `x` is an array that contains the values to use when evaluating CDF.

dist.`ppf`

Inverse CDF evaluation (also known as percent point function) for an array of values between 0 and 1. Generically, `ppf` is called using *dist*`.ppf(p, *args, loc=0, scale=1)` where `p` is an array with all elements between 0 and 1 that contains the values to use when evaluating inverse CDF.

dist.`fit`

Estimate shape, location, and scale parameters from data by maximum likelihood using an array of data. Generically, `fit` is called using *dist*`.fit(data, *args, floc=0, fscale=1)` where `data` is a data array used to estimate the parameters. `floc` forces the location to a particular value (e.g. `floc=0`). `fscale` similarly forces the scale to a particular value (e.g. `fscale=1`) . It is necessary to use `floc` and/or `fscale` when computing MLEs if the distribution does not have a location and/or scale. For example, the gamma distribution is defined using 2 parameters, often referred to as shape and scale. In order to use ML to estimate parameters from a gamma, `floc=0` must be used.

dist.median

Returns the median of the distribution. Generically, `median` is called using *dist*`.median(*args, loc=0, scale=1)`.

dist.mean

Returns the mean of the distribution. Generically, `mean` is called using *dist*`.mean(*args, loc=0, scale=1)`.

dist.moment

nth non-central moment evaluation of the distribution. Generically, `moment` is called using *dist*`.moment(r, *args, loc=0, scale=1)` where `r` is the order of the moment to compute.

dist.varr

Returns the variance of the distribution. Generically, `var` is called using *dist*`.var(*args, loc=0, scale=1)`.

dist.std

Returns the standard deviation of the distribution. Generically, `std` is called using *dist*`.std(*args, loc=0, scale=1)`.

14.4.1 Example: gamma

The gamma distribution is used as an example. The gamma distribution takes 1 shape parameter a (a is the only element of `*args`), which is set to 2 in all examples.

```
>>> gamma = stats.gamma
>>> gamma.mean(2), gamma.median(2), gamma.std(2), gamma.var(2)
 (2.0, 1.6783469900166608, 1.4142135623730951, 2.0)

>>> gamma.moment(2,2) - gamma.moment(1,2)**2 # Variance
2

>>> gamma.cdf(5, 2), gamma.pdf(5, 2)
(0.95957231800548726, 0.033689734995427337)

>>> gamma.ppf(.95957232, 2)
5.0000000592023914

>>> log(gamma.pdf(5, 2)) - gamma.logpdf(5, 2)
0.0

>>> gamma.rvs(5, size=(2,2))
array([[ 2.60426534,  3.28844939],
       [ 1.2592476 ,  2.29415338]])
```

```
>>> gamma.fit(gamma.rvs(5, size=(1000)), floc = 0) # a, 0, shape
(5.614220461484499, -0.13375877842240613, 0.92353448409408001)
```

14.4.2 Important Distributions

SciPy provides classes for a large number of distribution. The most important in econometrics are listed in the table below, along with any required arguments (shapeq parameters). All classes can be used with the keyword arguments `loc` and `scale` to set the location and scale, respectively. The default location is 0 and the default scale is 1. Setting `loc` to something other than 0 is equivalent to adding `loc` to the random variable. Similarly setting `scale` to something other than 0 is equivalent to multiplying the variable by `scale`.

Distribution Name	SciPy Name	Required Arguments	Notes
Normal	`norm`		Use `loc` to set mean (μ), `scale` to set std. dev. (σ)
Beta(a, b)	`beta`	a: `a`, b: `b`	
Cauchy	`cauchy`		
χ^2_ν	`chi2`	ν: `df`	
Exponential(λ)	`expon`		Use `scale` to set shape parameter (λ)
Exponential Power	`exponpow`	shape: `b`	Nests normal when `b=2`, Laplace when `b=1`
F(ν_1, ν_2)	`f`	ν_1: `dfn`, ν_2: `dfd`	
Gamma(a, b)	`gamma`	a: `a`	Use `scale` to set scale parameter (b)
Laplace, Double Exponential	`laplace`		Use `loc` to set mean (μ), `scale` to set std. dev. (σ)
Log Normal(μ, σ^2)	`lognorm`	σ: `s`	Use `scale` to set μ, as `scale=exp(mu)`
Student's-t_ν	`t`	ν: `df`	

14.4.3 Frozen Random Variable Object

Random variable objects can be used in one of two ways:

1. Calling the class along with any shape, location and scale parameters, simultaneously with the method. For example `gamma(1, scale=2).cdf(1)`.
2. Initializing the class with any shape, location and scale arguments and assigning a variable name. Using the assigned variable name with the method. For example:

```
>>> g = scipy.stats.gamma(1, scale=2)
>>> g.cdf(1)
0.39346934028736652
```

The second method is known as using a frozen random variable object. If the same distribution (with fixed parameters) is repeatedly used, frozen objects can be used to save typing, and potentially improve speed since frozen objects avoid re-initializing the class.

14.5 Select Statistics Functions

mode

`mode` computes the mode of an array. An optional second argument provides the axis to use (default is to use entire array). Returns two outputs: the first contains the values of the mode, the second contains the number of occurrences.

```
>>> x=randint(1,11,1000)
>>> stats.mode(x)
(array([ 4.]), array([ 112.]))
```

moment

moment computed the rth central moment for an array. An optional second argument provides the axis to use (default is to use entire array).

```
>>> x = randn(1000)
>>> moment = stats.moment
>>> moment(x,2) - moment(x,1)**2
0.94668836546169166

>>> var(x)
0.94668836546169166

>>> x = randn(1000,2)
>>> moment(x,2,0) # axis 0
array([ 0.97029259,  1.03384203])
```

skew

skew computes the skewness of an array. An optional second argument provides the axis to use (default is to use entire array).

```
>>> x = randn(1000)
>>> skew = stats.skew
>>> skew(x)
0.027187705042705772

>>> x = randn(1000,2)
>>> skew(x,0)
array([ 0.05790773, -0.00482564])
```

kurtosis

kurtosis computes the *excess* kurtosis (actual kurtosis minus 3) of an array. An optional second argument provides the axis to use (default is to use entire array). Setting the keyword argument fisher=False will compute the actual kurtosis.

```
>>> x = randn(1000)
>>> kurtosis = stats.kurtosis
>>> kurtosis(x)
 -0.2112381820194531

>>> kurtosis(x, fisher=False)
2.788761817980547

>>> kurtosis(x, fisher=False) - kurtosis(x) # Must be 3
```

```
3.0

>>> x = randn(1000,2)
>>> kurtosis(x,0)
array([-0.13813704, -0.08395426])
```

`pearsonr`

`pearsonr` computes the Pearson correlation between two 1-dimensional vectors. It also returns the 2-tailed p-value for the null hypothesis that the correlation is 0.

```
>>> x = randn(10)
>>> y = x + randn(10)
>>> pearsonr = stats.pearsonr
>>> corr, pval = pearsonr(x, y)
>>> corr
0.40806165708698366

>>> pval
0.24174029858660467
```

`spearmanr`

`spearmanr` computes the Spearman correlation (rank correlation). It can be used with a single 2-dimensional array input, or 2 1-dimensional arrays. Takes an optional keyword argument `axis` indicating whether to treat columns (0) or rows (1) as variables. If the input array has more than 2 variables, returns the correlation matrix. If the input array as 2 variables, returns only the correlation between the variables.

```
>>> x = randn(10,3)
>>> spearmanr = stats.spearmanr
>>> rho, pval = spearmanr(x)
>>> rho
array([[ 1.        , -0.02087009, -0.05867387],
       [-0.02087009,  1.        ,  0.21258926],
       [-0.05867387,  0.21258926,  1.        ]])

>>> pval
array([[ 0.        ,  0.83671325,  0.56200781],
       [ 0.83671325,  0.        ,  0.03371181],
       [ 0.56200781,  0.03371181,  0.        ]])

>>> rho, pval = spearmanr(x[:,1],x[:,2])
>>> corr
-0.020870087008700869

>>> pval
0.83671325461864643
```

kendalltau

kendalltau computed Kendall's τ between 2 1-dimensonal arrays.

```
>>> x = randn(10)
>>> y = x + randn(10)
>>> kendalltau = stats.kendalltau
>>> tau, pval = kendalltau(x,y)
>>> tau
0.46666666666666673

>>> pval
0.06034053974834707
```

linregress

linregress estimates a linear regression between 2 1-dimensional arrays. It takes two inputs, the independent variables (regressors) and the dependent variable (regressand). Models always include a constant.

```
>>> x = randn(10)
>>> y = x + randn(10)
>>> linregress = stats.linregress
>>> slope, intercept, rvalue, pvalue, stderr = linregress(x,y)
>>> slope
1.6976690163576993

>>> rsquare = rvalue**2
>>> rsquare
0.59144988449163494

>>> x.shape = 10,1
>>> y.shape = 10,1
>>> z = hstack((x,y))
>>> linregress(z) # Alternative form, [x y]
(1.6976690163576993,
 -0.79983724584931648,
 0.76905779008578734,
 0.0093169560056056751,
 0.4988520051409559)
```

14.6 Select Statistical Tests

normaltest

normaltest tests for normality in an array of data. An optional second argument provides the axis to use (default is to use entire array). Returns the test statistic and the p-value of the test. This test is a small sample modified version of the Jarque-Bera test statistic.

kstest

kstest implements the Kolmogorov-Smirnov test. Requires two inputs, the data to use in the test and the distribution, which can be a string or a frozen random variable object. If the distribution is provided as a string, and if it requires shape parameters, these are passed in the third argument using a tuple containing all parameters, in order.

```
>>> x = randn(100)
>>> kstest = stats.kstest
>>> stat, pval = kstest(x, 'norm')
>>> stat
0.11526423481470172

>>> pval
0.12963296757465059

>>> ncdf = stats.norm().cdf # No () on cdf to get the function
>>> kstest(x, ncdf)
 (0.11526423481470172, 0.12963296757465059)

>>> x = gamma.rvs(2, size = 100)
>>> kstest(x, 'gamma', (2,)) # (2,) contains the shape parameter
(0.079237623453142447, 0.54096739528138205)

>>> gcdf = gamma(2).cdf
>>> kstest(x, gcdf)
(0.079237623453142447, 0.54096739528138205)
```

ks_2samp

ks_2samp implements a 2-sample version of the Kolmogorov-Smirnov test. It is called ks_2samp(x,y) where both inputs are 1-dimensonal arrays, and returns the test statistic and p-value for he null that the distribution of x is the same as that of y.

shapiro

shapiro implements the Shapiro-Wilk test for normality on a 1-dimensional array of data. It returns the test statistic and p-value for the null of normality.

14.7 Exercises

1. For each of the following distributions, simulate 1000 pseudo-random numbers:

   (a) $N(0,1^2)$

   (b) $N\left(3,3^2\right)$

   (c) $Unif(0,1)$

   (d) $Unif(-1,1)$

(e) $Gamma(1,2)$

(f) $LogN\left(.08, .2^2\right)$

2. Use `kstest` to compute the p-value for each set of simulated data.

3. Use `seed` to re-initialize the random number generator.

4. Use `get_state` and `set_state` to to produce the same set of pseudo-random numbers.

5. Write a custom function that will take a T vector of data and returns the mean, standard deviation, skewness and kurtosis (not excess) as a 4-element array.

6. Generate a 100 by 2 array of normal data with covariance matrix

$$\begin{array}{cc} 1 & -.5 \\ -.5 & 1 \end{array}$$

   and compute the Pearson and Spearman correlation and Kendall's τ.

7. Compare the analytical median of a Gamma$(1,2)$ with that of 10,000 simulated data points. (You will need a `hist` , which is discussed in the graphics chapter to finish this problem.)

8. For each of the sets of simulated data in exercise 1, plot the sorted CDF values to verify that these lie on a 45^o line. (You will need `plot` , which is discussed in the graphics chapter to finish this problem.)

Chapter 15

Optimization

The optimization toolbox contains a number of routines to the find extremum of a user-supplied objective function. Most of these implement a form of the Newton-Raphson algorithm which uses the gradient to find the **minimum** of a function. **Note**: The optimization routines can *only* find minima. However, if f is a function to be maximized, $-f$ is a function with the minimum at located the same point as the maximum of f.

A custom function that returns the function value at a set of parameters – for example a log-likelihood or a GMM quadratic form – is required use one of the optimizers must be constructed. All optimization targets must have the parameters as the first argument. For example consider finding the minimum of x^2. A function which allows the optimizer to work correctly has the form

```
def optim_target1(x):
    return x**2
```

When multiple parameters (a parameter vector) are used, the objective function must take the form

```
def optim_target2(params):
    x, y = params

    return x**2-3*x+3+y*x-3*y+y**2
```

Optimization targets can have additional inputs that are not parameters of interest such as data or hyper-parameters.

```
def optim_target3(params,hyperparams):
    x, y = params
    c1, c2, c3=hyperparams

    return x**2+c1*x+c2+y*x+c3*y+y**2
```

This form is useful when optimization targets require at least two inputs: parameters and data. Once an optimization target has been specified, the next step is to use one of the optimizers find the minimum.

SciPy contains a large number of optimizers.

```
import scipy.optimize as opt
```

15.1 Unconstrained Optimization

A number of functions are available for unconstrained optimization using derivative information. Each uses a different algorithm to determine the best direction to move and the best step size to take in the direction. The basic structure of all of the unconstrained optimizers is

```
optimizer(f, x0)
```

where *optimizer* is one of `fmin_bfgs`, `fmin_cg`, `fmin_ncg` or `fmin_powell`, `f` is a callable function and `x0` is an initial value used to start the algorithm. All of the unconstrained optimizers take the following keyword arguments, except where noted:

Keyword	Description	Note
`fprime`	Function returning derivative of `f`. Must take same inputs as `f`	(1)
`args`	Tuple containing extra parameters to pass to `f`	
`gtol`	Gradient norm for terminating optimization	(1)
`norm`	Order of norm (e.g. inf or 2)	(1)
`epsilon`	Step size to use when approximating f'	(1)
`maxiter`	Integer containing the maximum number of iterations	
`disp`	Boolean indicating whether to print convergence message	
`full_output`	Boolean indicating whether to return additional output	
`retall`	Boolean indicating whether to return results for each iteration.	
`callback`	User supplied function to call after each iteration.	

(1) Except `fmin`, `fmin_powell`.

`fmin_bfgs`

`fmin_bfgs` is a classic optimizer which uses derivative information in the 1st derivative to estimate the second derivative, which is known as the BFGS algorithm (after the initials of the creators). This is probably the first choice when trying an optimization problem. A function which returns the first derivative of the problem can be provided. if not provided, it is numerically approximated. The basic use of `fmin_bfgs` for optimizing `optim_target1` is shown below.

```
>>> opt.fmin_bfgs(optim_target1, 2)
Optimization terminated successfully.
         Current function value: 0.000000
         Iterations: 2
         Function evaluations: 12
         Gradient evaluations: 4
array([ -7.45132576e-09])
```

This is a very simple function to minimize and the solution is accurate to 8 decimal places. `fmin_bfgs` can also use first derivative information, which is provided using a function which *must have the same inputs are the optimization target.* In this simple example, $f'(x) = 2x$.

```
def optim_target1_grad(x):
    return 2*x
```

The derivative information is used through the keyword argument fprime. Using analytic derivatives may improve accuracy of the solution is will require fewer function evaluations to find the solution.

```
>>> opt.fmin_bfgs(optim_target1, 2, fprime = optim_target1_grad)
Optimization terminated successfully.
         Current function value: 0.000000
         Iterations: 2
         Function evaluations: 4
         Gradient evaluations: 4
array([  2.71050543e-20])
```

Multivariate optimization problems are defined using an array for the starting values, but are otherwise identical.

```
>>> opt.fmin_bfgs(optim_target2, array([1.0,2.0]))
Optimization terminated successfully.
         Current function value: 0.000000
         Iterations: 3
         Function evaluations: 20
         Gradient evaluations: 5
array([ 1.        ,  0.99999999])
```

Additional inputs are padded through to the optimization target using the keyword argument args and a tuple containing the input arguments in the correct order. Note that since there is a single additional input, the comma is necessary in (hyperp,) to let Python know that this is a tuple.

```
>>> hyperp = array([1.0,2.0,3.0])
>>> opt.fmin_bfgs(optim_target3, array([1.0,2.0]), args=(hyperp ,))
Optimization terminated successfully.
         Current function value: -0.333333
         Iterations: 3
         Function evaluations: 20
         Gradient evaluations: 5
array([ 0.33333332, -1.66666667])
```

Derivative functions can be produced in a similar manner, although the derivative of a scalar function with respect to an n-element vector is an n-element vector. It is important that the derivative (or gradient) returned has the same order as the input parameters. Note that the inputs must have both be present, even if not needed, and in the same order.

```
def optim_target3_grad(params,hyperparams):
    x, y = params
    c1, c2, c3=hyperparams

    return array([2*x+c1+y,x+c3+2*y])
```

Using the analytical derivative reduces the number of function evaluations and produces the same result.

```
>>> opt.fmin_bfgs(optim_target3, array([1.0,2.0]), fprime=optim_target3_grad, args=(hyperp ,))
Optimization terminated successfully.
         Current function value: -0.333333
         Iterations: 3
         Function evaluations: 5
```

```
        Gradient evaluations: 5
array([ 0.33333333, -1.66666667])
```

fmin_cg

`fmin_cg` uses a nonlinear conjugate gradient method to minimize a function. A function which returns the first derivative of the problem can be provided. if not provided, it is numerically approximated.

```
>>> opt.fmin_cg(optim_target3, array([1.0,2.0]), args=(hyperp ,))
Optimization terminated successfully.
        Current function value: -0.333333
        Iterations: 7
        Function evaluations: 59
        Gradient evaluations: 12
array([ 0.33333334, -1.66666666])
```

fmin_ncg

`fmin_ncg` use a Newton conjugate gradient method. `fmin_ncg` also requires a function which can compute the first derivative of the optimization target, and can also take a function which returns the second derivative of the optimization target. It not provided, the hessian will be numerically approximated.

```
>>> opt.fmin_ncg(optim_target3, array([1.0,2.0]), optim_target3_grad, args=(hyperp,))
Optimization terminated successfully.
        Current function value: -0.333333
        Iterations: 5
        Function evaluations: 6
        Gradient evaluations: 21
        Hessian evaluations: 0
array([ 0.33333333, -1.66666666])
```

The hessian can optionally be provided to `fmin_ncg` using the keyword argument `fhess`. The hessian returns $\partial^2 f/\partial x \partial x'$, which is an n by n array of derivatives. In this simple problem, the hessian does not depend on the hyper-parameters, although the Hessian function *must* take the same inputs are the optimization target.

```
def optim_target3_hess(params,hyperparams):
    x, y = params
    c1, c2, c3=hyperparams

    return(array([[2, 1],[1, 2]]))
```

Using an analytical Hessian can reduce the number of function evaluations. While in theory an analytical Hessian should produce better results, it may not improve convergence, especially is for some parameter values the Hessian is nearly singular (for example, near a saddle point which is not a minimum).

```
>>> opt.fmin_ncg(optim_target3, array([1.0,2.0]), optim_target3_grad, \
... fhess = optim_target3_hess, args=(hyperp ,))
Optimization terminated successfully.
        Current function value: -0.333333
        Iterations: 5
```

```
         Function evaluations: 6
         Gradient evaluations: 5
         Hessian evaluations: 5
array([ 0.33333333, -1.66666667])
```

In addition to the keyword argument outlined in the main table, `fmin_ncg` can take the following additional arguments.

Keyword	Description	Note
`fhess_p`	Function returning second derivative of `f` times a vector p. Must take same inputs as `f`	Only `fmin_ncg`
`fhess`	Function returning second derivative of `f`. Must take same inputs as `f`	Only `fmin_ncg`
`avestol`	Average relative error to terminate optimizer.	Only `fmin_ncg`

15.2 Derivative-free Optimization

Derivative free optimizers do not use derivative information and so can be used in a wider variety of problems such as functions which are not continuously differentiable. Derivative free optimizers can also be used for functions which are continuously differentiable as an alternative to the derivative methods, although they are likely to be slower. Derivative free optimizers take some alternative keyword arguments.

Keyword	Description	Note
`xtol`	Change in x to terminate optimization	
`ftol`	Change in function to terminate optimization	
`maxfun`	Maximum number of function evaluations	
`direc`	Initial direction set, same size as x_0 by m	Only `fmin_powell`

`fmin`

`fmin` uses a simplex algorithm to minimize a function. The optimization in a simplex algorithm is often described as an amoeba which crawls around on the function surface expanding and contracting while looking for lower points. The method is derivative free, and so optimization target need not be continuously differentiable, for example the “tick” loss function used in estimation of quantile regression.

```
def tick_loss(quantile, data, alpha):
    e = data - quantile

    return dot((alpha - (e<0)),e)
```

The tick loss function is used to estimate the median by using $\alpha = 0.5$. This loss function is not continuously differential and so regular optimizers

```
>>> data = randn(1000)
>>> opt.fmin(tick_loss, 0, args=(data, 0.5))
Optimization terminated successfully.
```

```
         Current function value: -0.333333
         Iterations: 48
         Function evaluations: 91
array([ 0.33332751, -1.66668794])
>>> median(data)
-0.0053901030307567602
```

The estimate is close to the sample median as expected.

fmin_powell

`fmin_powell` used Powell's method, which is derivative free, to minimize a function. It is an alternative to `fmin` which uses a different algorithm.

```
>>> data = randn(1000)
>>> opt.fmin_powell(tick_loss, 0, args=(data, 0.5))
Optimization terminated successfully.
         Current function value: 396.760642
         Iterations: 1
         Function evaluations: 17
array(-0.004659496638722056)
```

`fmin_powell` converged quickly and requires far fewer function calls.

15.3 Constrained Optimization

Constrained optimization is frequently encountered in economic problems where parameters are only meaningful in some particular range – for example, a variance must be weakly positive. The relevant class constrained optimization problems can be formulated

$$\begin{array}{cl}
\min_{\theta} f(\theta) & \text{subject to} \\
g(\theta) = 0 & \text{(equality)} \\
h(\theta) \geq 0 & \text{(inequality)} \\
\theta_L \leq \theta \leq \theta_H & \text{(bounds)}
\end{array}$$

where the bounds constraints are redundant if the optimizer allows for general inequality constraints since if a scalar x satisfies $x_L \leq x \leq x_H$, then $x - x_L \geq 0$ and $x_H - x \geq 0$. The optimizers in SciPy allow for different subsets of these configurations.

fmin_slsqp

`fmin_slsqp` is the most general constrained optimizer and allows for equality, inequality and bounds constraints. While bounds are redundant, constraints which take the form of bounds should be implemented using bounds since this provides more information directly to the optimizer. Constraints are provided either as list of callable functions or as a single function which returns an array. The latter is simpler if there are multiple constraints, especially if the constraints can be easily calculated using linear algebra. Functions which compute the derivative of the optimization target, the derivative of the equality constraints, and the the derivative of the inequality constraints can be optionally provided. If not provided, these are numerically approximated.

As an example, consider the problem of optimizing a CRS Cobb-Douglas utility function of the form $U(x_1, x_2) = x_1^{\lambda} x_2^{1-\lambda}$ subject to a budget constraint $p_1 x_1 + p_2 x_2 \leq 1$. This is a nonlinear function subject to a linear constraint (note that is must also be that case that $x_1 \geq 0$ and $x_2 \geq 0$). First, specify the optimization target

```
def utility(x, p, alpha):
    # Minimization, not maximization so -1 needed
    return -1.0 * (x[0]**alpha)*(x[1]**(1-alpha))
```

There are three constraints, $x_1 \geq 0$, $x_2 \geq 0$ and the budget line. All constraints must take the form of ≥ 0 constraint, to that the budget line can be reformulated as $1 - p_1 x_1 - p_2 x_2 \geq 0$. Note that the arguments in the constraint must be identical to those of the optimization target, which is why in this case the utility function takes prices as an input, which are not needed, and the constraint takes α which does not affect the budget line.

```
def utility_constraints(x, p, alpha):
    return array([x[0], x[1], 1 - p[0]*x[0] - p[1]*x[1]])
```

The optimal combination of good can be computed using `fmin_slsqp` once the starting values and other inputs for the utility function and budget constraint are constructed.

```
>>> p = array([1.0,1.0])
>>> alpha = 1.0/3
>>> x0 = array([.4,.4])
>>> opt.fmin_slsqp(utility, x0, f_ieqcons=utility_constraints, args=(p, alpha))
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -0.529133683989
            Iterations: 2
            Function evaluations: 8
            Gradient evaluations: 2
array([ 0.33333333,  0.66666667])
```

`fmin_slsqp` can also take functions which compute the gradient of the optimization target, as well as the gradients of the constraint functions (both inequality and equality). The gradient of the optimization function should return a n-element vector, one for each parameter of the problem.

```
def utility_grad(x, p, alpha):
    grad = zeros(2)
    grad[0] = -1.0 * alpha * (x[0]**(alpha-1))*(x[1]**(1-alpha))
    grad[1] = -1.0 * (1-alpha) * (x[0]**(alpha))*(x[1]**(-alpha))
    return grad
```

The gradient of the constraint function returns a m by n array where m is the number of constraints. When both equality and inequality constraints are used, the number of constraints will me m_{eq} and m_{in} which will generally not be the same.

```
def utility_constraint_grad(x, p, alpha):
    grad = zeros((3,2)) # 3 constraints, 2 variables
    grad[0,0] = 1.0
    grad[0,1] = 0.0
    grad[1,0] = 0.0
    grad[1,1] = 1.0
    grad[2,0] = -p[0]
    grad[2,1] = -p[1]
    return grad
```

The two gradient functions can be passed using keyword arguments.

```
>>> opt.fmin_slsqp(utility, x0, f_ieqcons=utility_constraints, args=(p, alpha), \
... fprime = utility_grad, fprime_ieqcons = utility_constraint_grad)
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -0.529133683989
            Iterations: 2
            Function evaluations: 2
            Gradient evaluations: 2
array([ 0.33333333,  0.66666667])
```

Like in other problems, gradient information reduces the number of iterations and/or function evaluations needed to find the optimum.

fmin_slsqp also accepts bounds constraints. Since two of the three constraints where simply $x_1 \geq 0$ and $x_2 \geq 0$, these can be easily specified as a bound. Bounds are given as a list of tuples, where there is a tuple for each variable with an upper and lower bound. It is not always possible to use np.inf as the upper bound, even if there is no implicit upper bound since this may produce a nan. In this example, 2 was used as the upper bound since it was outside of the possible range given the constraint. Using bounds also requires reformulating the budget constraint to only include the budget line.

```
def utility_constraints_alt(x, p, alpha):
    return array([1 - p[0]*x[0] - p[1]*x[1]])
```

Bounds are used with the keyword argument bounds.

```
>>> opt.fmin_slsqp(utility, x0, f_ieqcons=utility_constraints_alt, args=(p, alpha), \
... bounds = [(0.0,2.0),(0.0,2.0)])
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -0.529133683989
            Iterations: 2
            Function evaluations: 8
            Gradient evaluations: 2
array([ 0.33333333,  0.66666667])
```

The use of non-linear constraints can be demonstrated by formulating the dual problem, that of cost minimization subject to achieving a minimal amount of utility. In this alternative formulation, the optimization problems becomes

$$\min_{x_1,x_2} p_1x_1 + p_2x_2 \text{ subject to } U(x_1,x_2) \geq \bar{U}$$

```
def total_expenditure(x,p,alpha,Ubar):
    return dot(x,p)

def min_utility_constraint(x,p,alpha,Ubar):
    x1,x2 = x
    u=x1**(alpha)*x2**(1-alpha)
    return array([u - Ubar]) # >= constraint, must be array, even if scalar
```

The objective and the constraint are used along with a bounds constraint to solve the constrained optimization problem.

```
>>> x0 = array([1.0,1.0])
>>> p = array([1.0,1.0])
>>> alpha = 1.0/3
>>> Ubar = 0.529133683989
>>> opt.fmin_slsqp(total_expenditure, x0, f_ieqcons=min_utility_constraint, \
...     args=(p, alpha, Ubar), bounds =[(0.0,2.0),(0.0,2.0)])
Optimization terminated successfully.    (Exit mode 0)
            Current function value: 0.999999999981
            Iterations: 6
            Function evaluations: 26
            Gradient evaluations: 6
Out[84]: array([ 0.33333333,  0.66666667])
```

As expected, the solution is the same.

fmin_tnc

fmin_tnc supports only bounds constraints.

fmin_l_bfgs_b

fmin_l_bfgs_b supports only bounds constraints.

fmin_cobyla

fmin_cobyla supports only inequality constraints, which must be provided as a list of functions. Since it supports general inequality constraints, bounds constraints are included as a special case, although these must be included in the list of constraint functions.

```
def utility_constraints1(x, p, alpha):
    return x[0]

def utility_constraints2(x, p, alpha):
    return x[1]

def utility_constraints3(x, p, alpha):
    return (1 - p[0]*x[0] - p[1]*x[1])
```

Note that fmin_cobyla takes a list rather than an array for the starting values. Using an array produces a warning, but otherwise works.

```
>>> p = array([1.0,1.0])
>>> alpha = 1.0/3
>>> x0 = array([.4,.4])
>>> cons = [utility_constraints1, utility_constraints2, utility_constraints3]
>>> opt.fmin_cobyla(utility, x0, cons, args=(p, alpha), rhoend=1e-7)
array([ 0.33333326,  0.66666674])
```

15.3.1 Reparameterization

Many constrained optimization problems can be converted into an unconstrained program by reparameterizing from the space of unconstrained variables into the space where parameters must reside. For example, the constraints in the utility function optimization problem require $0 \le x_1 \le 1/p_1$ and $0 \le x_2 \le 1/p_2$. Additionally the budget constrain must be satisfied so that if $x_1 \in [0, 1/p_1]$, $x_2 \in [0, (1 - p_1 x_1)/p_2]$. These constraints can be implemented using a "squasher" function which maps x_1 into its domain, and x_2 into its domain and is one-to-one and onto (i.e. a bijeciton). For example,

$$x_1 = \frac{1}{p_1}\frac{e^{z_1}}{1+e^{z_1}},\quad x_2 = \frac{1-p_1x_1}{p_2}\frac{e^{z_2}}{1+e^{z_2}}$$

will always satisfy the constraints, and so the constrained utility function can be mapped to an unconstrained problem, which can then be optimized using an unconstrained optimizer.

```
def reparam_utility(z,p,alpha,printX = False):
    x = exp(z)/(1+exp(z))
    x[0] = (1.0/p[0]) * x[0]
    x[1] = (1-p[0]*x[0])/p[1] * x[1]
    if printX:
        print(x)
    return -1.0 * (x[0]**alpha)*(x[1]**(1-alpha))
```

The unconstrained utility function can be minimized using `fmin_bfgs`. Note that the solution returned is in the transformed space, and so a special call to `reparam_utility` is used to print the actual values of x at the solution (which are virtually identical to those found using the constrained optimizer).

```
>>> x0 = array([.4,.4])
>>> optX = opt.fmin_bfgs(reparam_utility, x0, args=(p,alpha))
Optimization terminated successfully.
         Current function value: -0.529134
         Iterations: 24
         Function evaluations: 104
         Gradient evaluations: 26
>>> reparam_utility(optX, p, alpha, printX=True)
[ 0.33334741  0.66665244]
```

15.4 Scalar Function Minimization

SciPy provides a number of scalar function minimizers. These are very fast since additional techniques are available for solving scalar problems which are not applicable when the parameter vector has more than 1 element. A simple quadratic function will be used to illustrate the scalar solvers. Scalar function minimizers do not require starting values, but may require bounds for the search.

```
def optim_target5(x, hyperparams):
    c1,c2,c3 = hyperparams

    return c1*x**2 + c2*x + c3
```

`fminbound`

`fminbound` finds the minimum of a scalar function between two bounds.

```
>>> hyperp = array([1.0, -2.0, 3])
>>> opt.fminbound(optim_target5, -10, 10, args=(hyperp,))
1.0000000000000002
>>> opt.fminbound(optim_target5, -10, 0, args=(hyperp,))
-5.3634455116374429e-06
```

`golden`

`golden` uses a golden section search algorithm to find the minimum of a scalar function. It can optionally be provided with bracketing information which can speed up the solution.

```
>>> hyperp = array([1.0, -2.0, 3])
>>> opt.golden(optim_target5, args=(hyperp,))
0.999999992928981
>>> opt.golden(optim_target5, args=(hyperp,), brack=[-10.0,10.0])
0.9999999942734483
```

`brent`

`brent` uses Brent's method to find the minimum of a scalar function.

```
>>> opt.brent(optim_target5, args=(hyperp,))
0.99999998519
```

15.5 Nonlinear Least Squares

Non-linear least squares (NLLS) is similar to general function minimization. In fact, a generic function minimizer can (attempt to) minimize a NLLS problem. The main difference is that the optimization target returns a vector of errors rather than the sum of squared errors.

```
def nlls_objective(beta, y, X):
    b0 = beta[0]
    b1 = beta[1]
    b2 = beta[2]

    return y - b0 - b1 * (X**b2)
```

A simple non-linear model is used to demonstrate `leastsq`, the NLLS optimizer in SciPy.

$$y_i = 10 + 2x_i^{1.5} + e_i$$

where x and e are i.i.d. standard normal random variables.

```
>>> X = 10 *rand(1000)
>>> e = randn(1000)
>>> y = 10 + 2 * X**(1.5) + e
>>> beta0 = array([10.0,2.0,1.5])
>>> opt.leastsq(nlls_objective, beta0, args = (y, X))
(array([ 10.08885711,   1.9874906 ,   1.50231838]), 1)
```

`leastsq` returns a tuple containing the solution, which is very close to the true values, as well as a flag indicating whether convergence was achieved. `leastsq` takes many of the same additional keyword arguments as other optimizers, including `full_output`, `ftol`, `xtol`, `gtol`, `maxfev` (same as `maxfun`). It has the additional keyword argument:

Keyword	Description	Note
`Ddun`	Function to compute the Jacobian of the problem. Element i,j should be $\partial e_i/\partial \beta_j$	
`col_deriv`	Direction to use when computing Jacobian numerically	
`epsfcn`	Step to use in numerical Jacobian calculation.	
`diag`	Scalar factors for the parameters. Used to rescale if scale is very different.	
`factor`	used to determine the initial step size.	Only `fmin_powell`

15.6 Exercises

1. The MLE for μ in a normal random variable is the sample mean. Write a function which takes a scalar parameter μ (1st argument) and a T vector of data and computes the negative of the log-likelihood, assuming the data is random and the variance is 1. Minimize the function (starting from something other than the same mean) using `fmin_bfgs` and `fmin`.

2. Extend to previous example where the first input is a 2-element vector containing μ and σ^2, and compute the negative log-likelihood. Use `fmin_slsqp` along with a a lower bound of 0 for σ^2.

3. Repeat the exercise in problem 2, except using reparameterization so that σ is input (and then squared).

4. Verify that the OLS β is the MLE by writing a function which takes 3 inputs: K vector β,T by K array X and T by 1 array y, and computes the negative log-likelihood for these values. Minimize the function using `fmin_bfgs` starting at the OLS estimates of β.

Chapter 16

Dates and Times

Date and time manipulation is provided by a built-in Python module `datetime`. This chapter assumes that `datetime` has been imported using `import datetime`.

16.1 Creating Dates and Times

Dates are created using `date` using years, months and days and times are created using `time` using hours, minutes, seconds and microseconds.

```
>>> import datetime as dt
>>> yr = 2012; mo = 12; dd = 21
>>> dt.date(yr, mo, dd)
datetime.date(2012, 12, 21)

>>> hr = 12; mm = 21; ss = 12; ms = 21
>>> dt.time(hr, mm, ss, ms)
dt.time(12,21,12,21)
```

Dates created using `date` no not allow times, and dates which require a time stamp can be created using `datetime`, which borrow the inputs from `date` and `time` , in order.

```
>>> dt.datetime(yr, mo, dd, hr, mm, ss, ms)
datetime.datetime(2012, 12, 21, 12, 21, 12, 21)
```

16.2 Dates Mathematics

Date-times and dates (but not times, and only with the same type) can be subtracted to produce a `timedelta`, which consists of three values, days, seconds and microseconds. Time deltas can also be added to dates and times compute different dates – although `date` types will ignore any information in the time delta hour or millisecond fields.

```
>>> d1 = dt.datetime(yr, mo, dd, hr, mm, ss, ms)
>>> d2 = dt.datetime(yr + 1, mo, dd, hr, mm, ss, ms)
>>> d2-d1
datetime.timedelta(365)
```

```
>>> d2 + dt.timedelta(30,0,0)
datetime.datetime(2014, 1, 20, 12, 21, 12, 20)

>>> dt.date(2012,12,21) + dt.timedelta(30,12,0)
datetime.date(2013, 1, 20)
```

If accurately times stamps is important, `date` types can be promoted to `datetime` using `combine`.

```
>>> d3 = dt.date(2012,12,21)
>>> dt.datetime.combine(d3, dt.time(0))
datetime.datetime(2012, 12, 21, 0, 0)
```

Values in `dates`, `times` and `datetimes` can be modified using `replace`, which takes keyword arguments.

```
>>> d3 = dt.datetime(2012,12,21,12,21,12,21)
>>> d3.replace(month=11,day=10,hour=9,minute=8,second=7,microsecond=6)
datetime.datetime(2012, 11, 10, 9, 8, 7, 6)
```

Chapter 17

Graphics

Matplotlib contains a complete graphics library for producing high-quality graphics using Python. Matplotlib contains both number of high level functions which produce particular types of figures, for example a simple line plot or a bar chart, as well as a low level set of functions for creating new types of charts. Matplotlib is primarily a 2D plotting library, although it also 3D plotting which is sufficient for most applications. This chapter covers the basics of producing plots using Python and matplotlib. It only scratches the surface of the capabilities of matplotlib, and more information is available on the matplotlib website or in some books dedicated to producing print quality graphics using matplotlib.

17.1 2D Plotting

Throughout this chapter, the following modules have been imported.

```
>>> import numpy as np
>>> import matplotlib.pyplot as plt
>>> import scipy.stats as stats
```

Other modules will be included only when needed for a specific graphic.

17.1.1 Line Plots

The most basic, and often most useful 2D graphic is a line plot. Line plots are produced using `plot`, which in its simplest form, takes a single input containing a 1-dimensional array.

```
>>> y = np.random.randn(100)
>>> plt.plot(y)
```

The output of this command is presented in panel (a) of figure 17.1. A more complex use of `plot` includes a format string which has 1 to 3 elements, a color, represented suing a letter (e.g. g for green), a marker symbol which is either a letter of a symbol (e.g. s for square, ^ for triangle up), and a line style, which is always a symbol or series of symbols. In the next example, `'g--'` indicates green (g) and dashed line (–).

```
>>> plt.plot(y,'g--')
```

Format strings may contain:

Color		Marker		Line Style	
Blue	b	Point	.	Solid	-
Green	g	Pixel	,	Dashed	--
Red	r	Circle	o	Dash-dot	-.
Cyan	c	Square	s	Dotted	:
Magenta	m	Diamond	D		
Yellow	y	Thin diamond	d		
Black	k	Cross	x		
White	w	Plus	+		
		Star	*		
		Hexagon	H		
		Alt. Hexagon	h		
		Pentagon	p		
		Triangles	^, v, <, >		
		Vertical Line			
		Horizontal Line	_		

The default behavior is to use a blue solid line with no marker (unless there is more than one line, in which case the colors will alter, in order, through those in the Colors column, skipping white). Format strings can contain 1 or more or the three categories of formatting information. For example, `kx--` would produce a black dashed line with crosses marking the points, `*:` would produce a dotted line with the default color using stars to mark points and `yH` would produce a solid yellow line with a hexagon marker.

When one array is provided, the default x-axis values 1,2, ... are used. `plot(x,y)` can be used to plot specific x values against y values. Panel (c) shows he results of running the following code.

```
>>> x = np.cumsum(np.random.rand(100))
>>> plt.plot(x,y,'r-')
```

While format strings are useful for quickly adding meaningful colors or line styles to a plot, they only expose a small number of the customizations available. The next example shows how keyword arguments can be used to add many useful customizations to a plot. Panel (d) contains the plot produced by the following code.

```
>>> plt.plot(x,y,alpha = 0.5, color = '#FF7F00', \
...     label = 'Line Label', linestyle = '-.', \
...     linewidth = 3, marker = 'o', markeredgecolor = '#000000', \
...     markeredgewidth = 2, markerfacecolor = '#FF7F00', \
...     markersize=30)
```

Note that in the previous example, \ is used to indicate to the Python interpreter that a statement is spanning multiple lines.

Keyword	Description
alpha	Alpha (transparency) of the plot. Default is 1 (no transparency)
color	Color description for the line.[1]
label	Label for the line. Used when creating legends
linestyle	A line style symbol
linewidth	A positive integer indicating the width of the line
marker	A marker shape symbol or character
markeredgecolor	Color of the edge (a line) around the marker
markeredgewidth	Width of the edge (a line) around the marker
markerfacecolor	Face color of the marker
markersize	A positive integer indicating the size of the marker

Many more keyword arguments are available for a plot. The full list can be found in the docstring or by running the following code. The functions `getp` and `setp` can be used to get the list of properties for a line (or any matplotlib object). `setp` can also be used to set a particular property.

```
>>> h = plot(randn(10))
>>> matplotlib.artist.getp(h)
    agg_filter = None
    alpha = None
    animated = False
    antialiased or aa = True
    axes = Axes(0.125,0.1;0.775x0.8)
    children = []
    clip_box = TransformedBbox(Bbox(array([[ 0.,  0.],        [ 1...
    clip_on = True
    clip_path = None
    color or c = b
    contains = None
    dash_capstyle = butt
    dash_joinstyle = round
    data = (array([ 0.,  1.,  2.,  3.,  4.,  5.,  6.,  7.,  8...
    drawstyle = default
    figure = Figure(652x492)
    fillstyle = full
    gid = None
    label = _line0
    linestyle or ls = -
    linewidth or lw = 1.0
    marker = None
    markeredgecolor or mec = b
    markeredgewidth or mew = 0.5
    markerfacecolor or mfc = b
    markerfacecoloralt or mfcalt = none
    markersize or ms = 6
    markevery = None
    path = Path([[ 0.         -0.27752688]  [ 1.          0.3...
    picker = None
    pickradius = 5
    rasterized = None
    snap = None
    solid_capstyle = projecting
    solid_joinstyle = round
    transform = CompositeGenericTransform(TransformWrapper(Blended...
    transformed_clip_path_and_affine = (None, None)
    url = None
    visible = True
    xdata = [ 0.  1.  2.  3.  4.  5.]...
    xydata = [[ 0.         -0.27752688]  [ 1.          0.376091...
    ydata = [-0.27752688  0.37609185 -0.24595304  0.28643729  ...
    zorder = 2

>>> matplotlib.artist.setp(h, 'alpha')
alpha: float (0.0 transparent through 1.0 opaque)

>>> matplotlib.artist.setp(h, 'color')
color: any matplotlib color

>>> matplotlib.artist.setp(h, 'linestyle')
linestyle: [ ''-'' ''--'' ''-.'' '':'' ''None'' '' '' '''' ]
and any drawstyle in combination with a linestyle, e.g. ''steps--''.

>>> matplotlib.artist.setp(h, 'linestyle', '--')
```

17.1.2 Scatter Plots

Scatter plots are little more than a line plot without the line and with markers. `scatter` produces a scatter plot between 2 1-dimensional arrays. All examples use a set of simulated normal data with unit variance and correlation of 50%. The output of the basic `scatter` command is presented in figure 17.2, panel (a).

```
>>> z = np.random.randn(100,2)
>>> z[:,1] = 0.5*z[:,0] + np.sqrt(0.5)*z[:,1]
>>> x=z[:,1]
>>> y=z[:,1]
>>> plt.scatter(x,y)
```

Scatter plots can also be modified using keyword arguments. The most important are included in the next example, and have identical meaning to those used in the line plot examples. The effect of these keyword arguments can be see in panel (b).

```
>>> plt.scatter(x,y, s = 60,  c = '#FF7F00', marker='s', \
...     alpha = .5, label = 'Scatter Data')
```

One interesting use of `scatter` is to make add a 3rd dimension to the plot by including an array of size data. This allows the size of the shapes to convey extra information. The use of variable size data is illustrated in the code below, which produced the scatter plot in panel (c).

```
>>> s = np.exp(np.exp(np.exp(np.random.rand(100))))
>>> s = 200 * s/np.max(s)
```

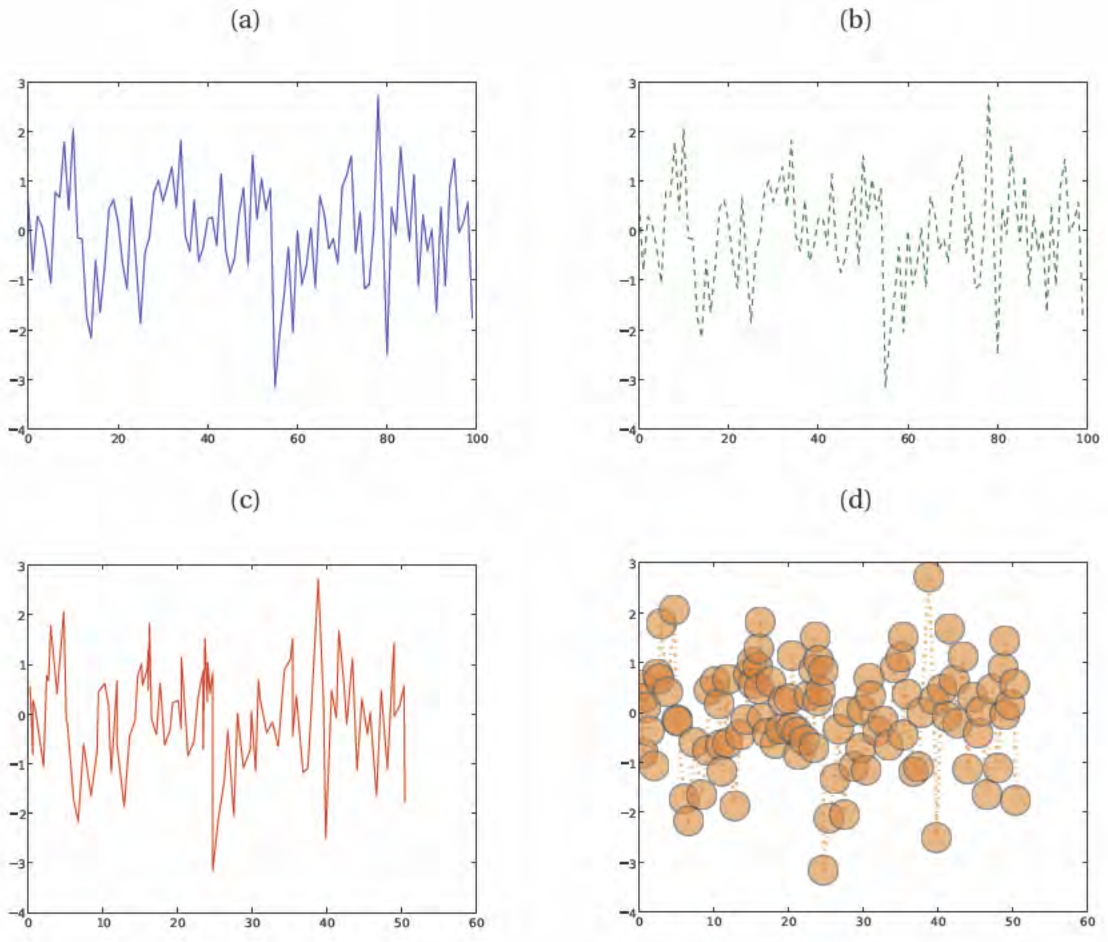


Figure 17.1: Line plots produced using `plot`.

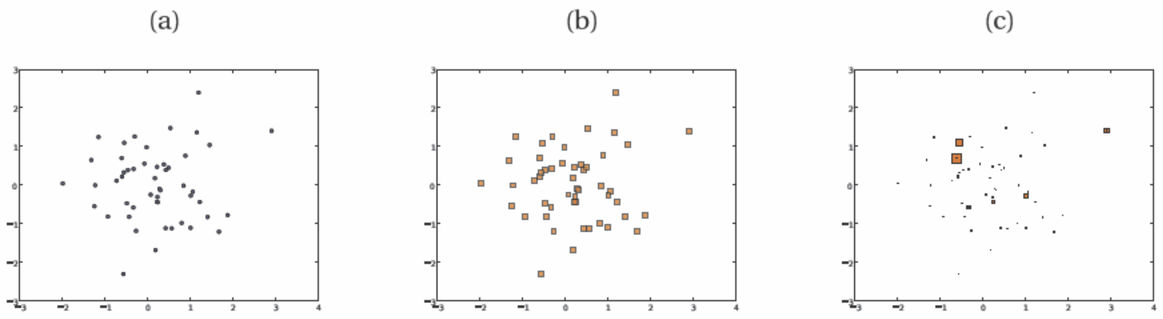


Figure 17.2: Scatter plots produced using `scatter`.

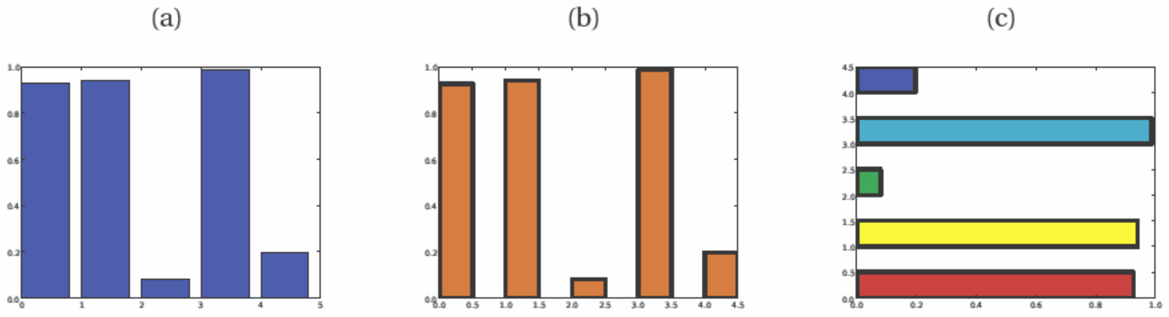


Figure 17.3: Bar charts produced using `bar` and `barh`.

```
>>> s[s<1]=1
>>> plt.scatter(x,y, s = s, c = '#FF7F00', marker='s', \
...     label = 'Scatter Data')
```

17.1.3 Bar Charts

Bar charts are produced using 2 1-dimensional arrays and `bar`. The first specifies the left ledge of the bars and the second the bar lengths. The next code segment produced the bar chart in panel (a) of figure 17.3.

```
>>> y = np.random.rand(5)
>>> x = np.arange(5)
>>> plt.bar(x,y)
```

Bar charts take keyword arguments to alter colors and bar width. Panel (b) contains the output of the following code.

```
>>> plt.bar(x,y, width = 0.5, color = '#FF7F00', \
...     edgecolor = '#000000', linewidth = 5)
```

Finally, `barh` can be used instead of `bar` to produce a horizontal bar chart. The next code snippet produces the horizontal bar chart in panel (c), and demonstrates the use of a list of colors which can be used to produce bars where color information is meaningful, in addition to length.

```
>>> colors = ['#FF0000','#FFFF00','#00FF00','#00FFFF','#0000FF']
>>> plt.barh(x, y, height = 0.5, color = colors, \
...     edgecolor = '#000000', linewidth = 5)
```

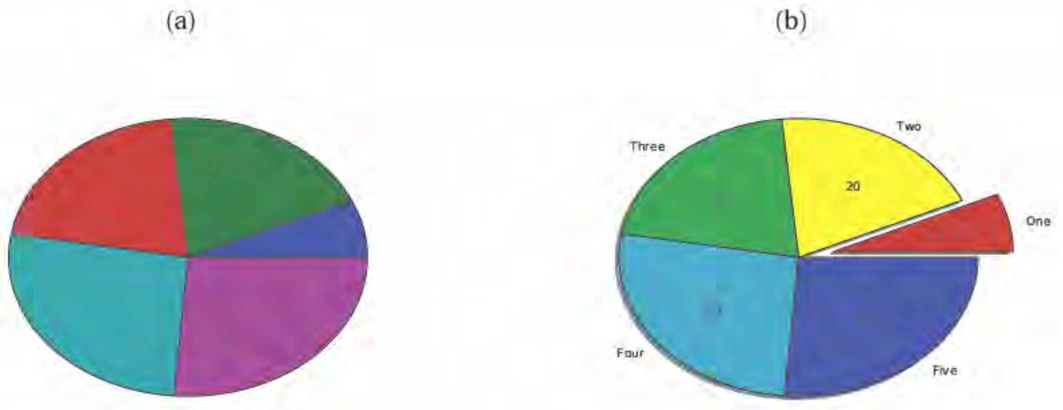


Figure 17.4: Pie charts produced using pie.

17.1.4 Pie Charts

Pie charts can be produced using pie and a 1-dimensional array of data (the data can have any values, and does not need to sum to 1). The basic use of pie is illustrated below. The figure produced is in panel (a) of figure 17.4.

```
>>> y = np.random.rand(5)
>>> y = y/sum(y)
>>> y[y<.05] = .05
>>> plt.pie(y)
```

Pie charts can be modified using a large number of keyword arguments, including labels and custom colors. Exploded views of a pie chart can be produced by providing a vector of distances to the keyword argument explode. Note that autopct = '%2.0f' is using an old style format string to format the numeric labels. The results of running this code is shown in panel (b).

```
>>> explode = np.array([.2,0,0,0,0])
>>> colors = ['#FF0000','#FFFF00','#00FF00','#00FFFF','#0000FF']
>>> labels = ['One','Two','Three','Four','Five']
>>> plt.pie(y, explode = explode, colors = colors, \
...     labels = labels, autopct = '%2.0f', shadow = True)
```

17.1.5 Histograms

Histograms can be produced using hist. A basic histogram produced using the the code below is presented in figure XX, panel (a). This example sets the number of bins used in producing the histogram using the keyword argument bins.

```
>>> x = np.random.randn(1000)
>>> plt.hist(x, bins = 30)
```

Histograms can be modified using keyword arguments. In the next example, cumulative=True produces the cumulative histogram. The output of this code is presented in figure (b).

```
>>> plt.hist(x, bins = 30, cumulative=True, color='#FF7F00')
```

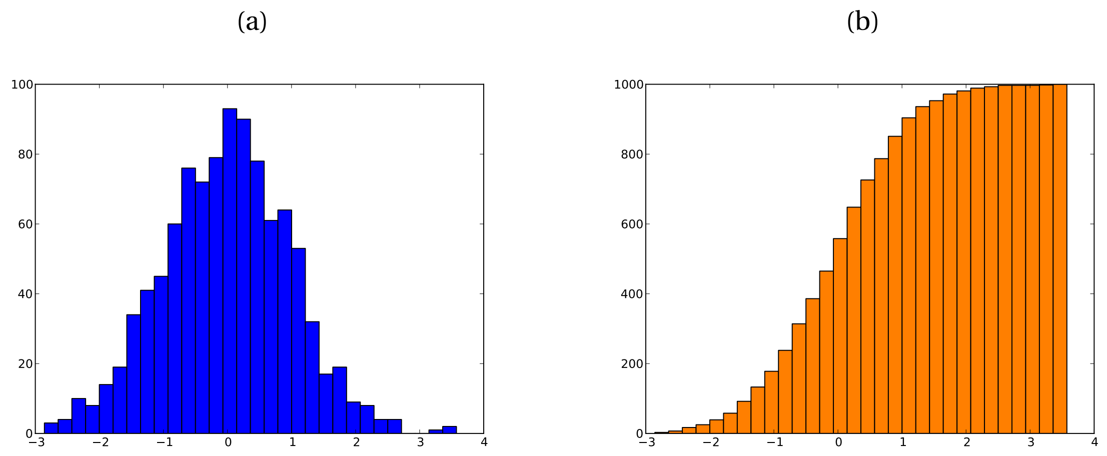


Figure 17.5: Histograms produced using hist.

17.2 Advanced 2D Plotting

17.2.1 Multiple Plots

In some scenarios it is advantageous to have multiple plots or charts in a single figure. Implementing this is simple using figure and then add_subplot. Figure is used to initialize the figure window. Subplots can then be added the the figure using a grid notation with m rows and n columns 1 is the upper left, 2 is the the right of 1, and so on until the end of a row, where the next element is below 1. For example, the plots in a 3 by 2 subplot have indices

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

add_subplot is called using the notation add_subplot(mni) or add_subplot(m,n,i) where m is the number of rows, n is the number of columns and i is the index of the subplot. Note that subplots require the subplot axes to be called as a method from figure. Figure XX contains the output of the code below. Note that the next code block is sufficient long that it isn't practical to run interactively. Also note that plt.show() is used to force an update to the window to ensure that all plots and charts are visible. Figure 17.6 contains the result running the code below.

```
fig = plt.figure()
# Add the subplot to the figure
# Panel 1
ax = fig.add_subplot(2,2,1)
y = np.random.randn(100)
plt.plot(y)
ax.set_title('1')

# Panel  2
y = np.random.rand(5)
x = np.arange(5)
```

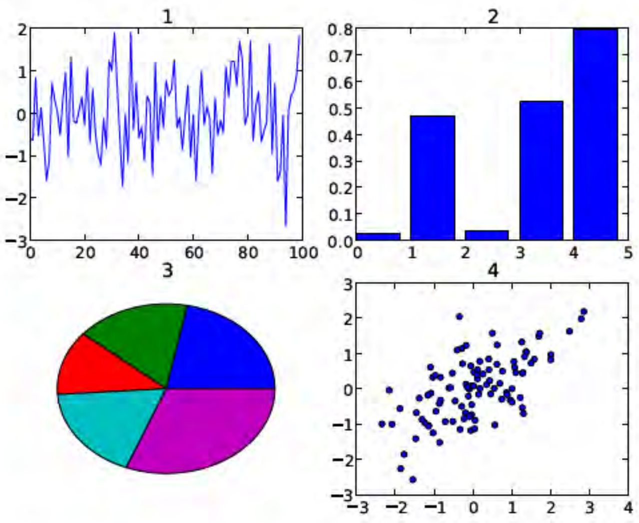


Figure 17.6: A figure containing a 2 by 2 subplot produced using `add_subplot`.

```
ax = fig.add_subplot(2,2,2)
plt.bar(x,y)
ax.set_title('2')

# Panel 3
y = np.random.rand(5)
y = y/sum(y)
y[y<.05] = .05
ax = fig.add_subplot(2,2,3)
plt.pie(y)
ax.set_title('3')

# Panel 4
z = np.random.randn(100,2)
z[:,1] = 0.5* z[:,0] + np.sqrt(0.5) * z[:,1]
x=z[:,1]
y=z[:,1]
ax = fig.add_subplot(2,2,4)
plt.scatter(x,y)
ax.set_title('4')
plt.draw()
```

17.2.2 Multiple Plots on the Same Axes

Occasionally two different types of plots are needed in the same axes, for example, plotting a histogram and a PDF. Multiple plots can be added to the same axes by plotting the first one (e.g a histogram) and then calling `hold(True)` to "hold" the contents of the axes (rather than overdrawing), and then plotting any remaining data. In general it is a good idea to call `hold(False)` when finished.

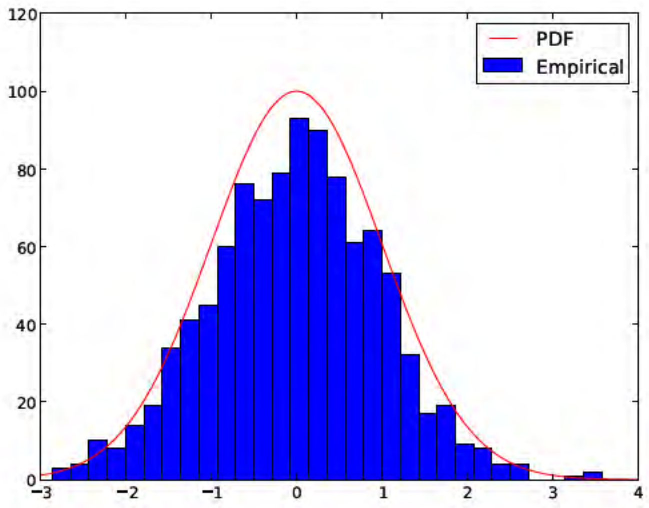


Figure 17.7: A figure containing a histogram and a line plot on the same axes using `hold`.

The code in the example begins by initializing a figure window and adding axes. A histogram is then added to the axes, `hold` is called, and then a Normal PDF is plotted. `legend()` is called to produce a legend using the labels provided in the potting commands. `get_xlim` and `get_ylim` are used to get the limits of the axis after adding the histogram. These points are used when computing the PDF, and finally `set_ylim` is called to increase the axis height so that the PDF is against the top of the chart. Figure 17.7 contains the output of these commands.

```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.hist(x, bins = 30,label = 'Empirical')
xlim = ax.get_xlim()
ylim = ax.get_ylim()
pdfx = np.linspace(xlim[0],xlim[1],200)
pdfy = stats.norm.pdf(pdfx)
pdfy = pdfy / pdfy.max() * ylim[1]
plt.hold(True)
plt.plot(pdfx,pdfy,'r-',label = 'PDF')
ax.set_ylim((ylim[0],1.2*ylim[1]))
plt.legend()
hold(False)
```

17.2.3 Adding a Title and Legend

Titles are added with `title` and legends are added with `legend`. `legend` requires that lines have labels, which is why 3 calls are made to `plot` – each series has its own label. Executing the next code block produces a the image in figure 17.8, panel (a).

```
>>> x = np.cumsum(np.random.randn(100,3), axis = 0)
>>> plt.plot(x[:,0],'b-',label = 'Series 1')
>>> plt.hold(True)
```

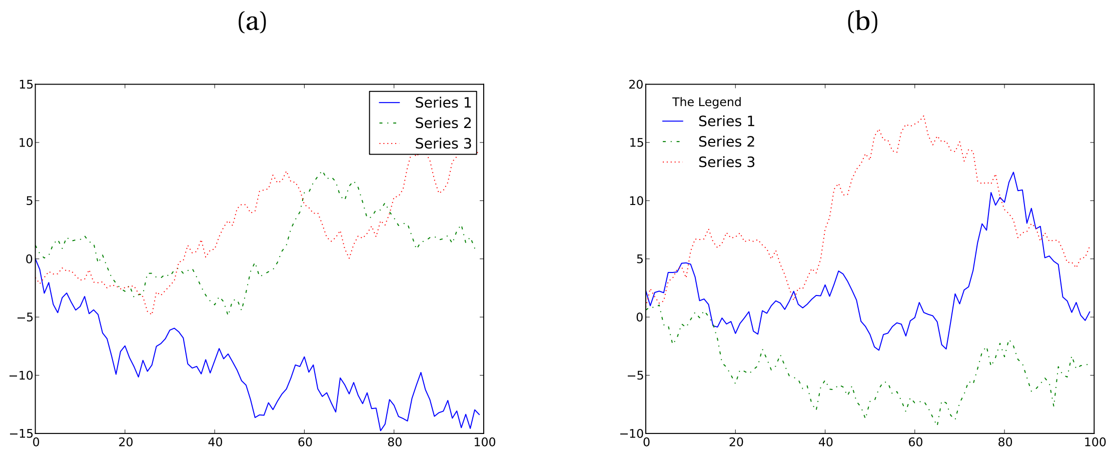


Figure 17.8: Figures with titles and legend produced using `title` and `legend`.

```
>>> plt.plot(x[:,1],'g-.',label = 'Series 2')
>>> plt.plot(x[:,2],'r:',label = 'Series 3')
>>> plt.legend()
>>> plt.title('Basic Legend')
```

`legend` takes keyword arguments which can be used to change its location (`loc` and an integer, see the docstring), remove the frame (`frameon`) and add a title to the legend box (`title`). The output of a simple example using these options is presented in panel (b).

```
>>> plt.plot(x[:,0],'b-',label = 'Series 1')
>>> plt.hold(True)
>>> plt.plot(x[:,1],'g-.',label = 'Series 2')
>>> plt.plot(x[:,2],'r:',label = 'Series 3')
>>> plt.legend(loc = 0, frameon = False, title = 'Data')
>>> plt.title('Improved Legend')
```

17.2.4 Dates on Plots

Plots with date x-values on the x-axis are important when using time series data. Producing basic plots with dates is as simple as `plot(x,y)` where x is a list or array of dates. This first block of code simulates a random walk and constructs 2000 datetime values beginning with March 1, 2012 in a list.

```
import numpy as np
import numpy.random as rnd
import matplotlib.pyplot as plt
import matplotlib.dates as mdates
import datetime as dt

# Simulate data
T = 2000
x = []
for i in xrange(T):
    x.append(dt.datetime(2012,3,1)+dt.timedelta(i,0,0))
y = np.cumsum(rnd.randn(T))
```

A basic plot with dates only requires calling `plot(x,y)` on the *x* and *y* data. The output of this code is in panel (a) of figure 17.9.

```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(x,y)
plt.draw()
```

Once the plot has been produced `autofmt_xdate()` is usually called to rotate and format the labels on the x-axis. The figure produced by running this command on the existing figure is in panel (b).

```
fig.autofmt_xdate()
plt.draw()
```

Sometime, depending on the length of the sample plotted, automatic labels will not be adequate. To show a case where this issue arises, a shorted sample with only 100 values is simulated.

```
T = 100
x = []
for i in xrange(T):
    x.append(dt.datetime(2012,3,1)+dt.timedelta(i,0,0))
y = np.cumsum(rnd.randn(T))
```

A basic plot is produced in the same manner, and is depicted in panel (c). Note the labels overlap and so this figure is not acceptable.

```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(x,y)
plt.draw()
```

A call to `autofmt_xdate()` can be used to address the issue of overlapping labels. This is shown in panel (d).

```
fig.autofmt_xdate()
plt.draw()
```

While the formatted x dates are are an improvement, they are still unsatisfactory in that the date labels have too much information (month, day and year) and are not at the start of the month. The next piece of code shows how markers can be placed at the start of the month using `MonthLocator` which is in the `matplotlib.dates` module. This idea is to construct a `MonthLocator` instance (which is a class), and then to pass this axes using `xaxis.set_major_locator` which determines the location of major tick marks (minor tick marks can be set using `xaxis.set_mijor_locator`). This will automatically place ticks on the 1st of every month. Other locators are available, including `YearLocator` and `WeekdayLocator`, which place ticks on the first day of the year and on week days, respectively. The second change is to format the labels on the x-axis to have the short month name and year. This is done using `DateFormatter` which takes a custom format string containing the desired text format. Options for formatting include:

- `%Y` - 4 digit numeric year
- `%m` - Numeric month
- `%d` - Numeric day
- `%b` - Short month name

- %H - Hour
- %M - Minute
- %D - Named day

These can be combined along with other characters to produce format strings. For example, `%b %d, %Y` would produce a string with the format Mar 1, 2012. The format is used by calling `DateFormatter`. Finally `autofmt_xdate` is used to rotate the labels. The result of running this code is in panel (e).

```
months = mdates.MonthLocator()
ax.xaxis.set_major_locator(months)
fmt = mdates.DateFormatter('%b %Y')
ax.xaxis.set_major_formatter(fmt)
fig.autofmt_xdate()
plt.draw()
```

Note that March 1 is not present in the figure in panel (e). This is because the plot doesn't actually include the date March 1 12:00:00 AM, but starts slightly later. To address this, simply change the axis limits using first calling `get_xlim` to get the 2-element tuple containing the limits, change the it to include March 1 12:00:00 AM using `set_xlim`. The line between these call is actually constructing the correctly formatted date. Internally, matplotlib uses serial dates which are simply the number of days past some initial date. For example March 1, 2012 12:00:00 AM is 734563.0, March 2, 2012 12:00:00 AM is 734564.0 and March 2, 2012 12:00:00 PM is 734563.5. The function `date2num` can be used to convert datetimes to serial dates. The output of running this final price of code on the existing figure is presented in panel (f)

```
xlim = list(ax.get_xlim())
xlim[0] = mdates.date2num(dt.datetime(2012,3,1))
ax.set_xlim(xlim)
plt.draw()
```

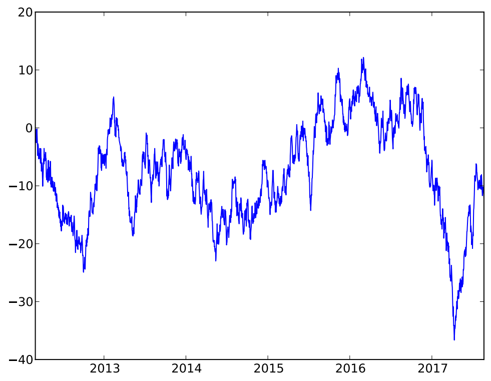
17.2.5 Shading Areas

For a simple demonstration of the range of matplotlib and Python graphics, consider the problem of producing a plot of Macroeconomic time series which has business cycle fluctuations. Capacity utilization data from FRED has been used to illustrate the steps needed to produce a plot with the time series, dates and shaded regions representing periods where the NBER has decided were recessions.

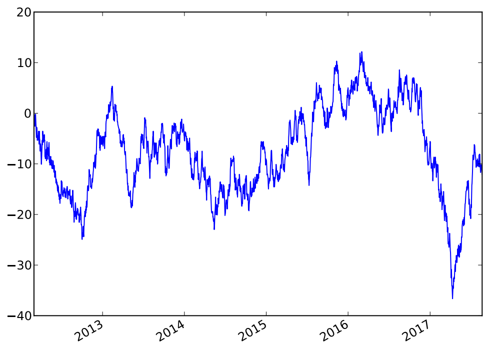
The code has been split into two parts. The first is the code needed to read the data, find the common dates, and finally format the data so that only the common sample is retained.

```
# Reading the data
import matplotlib.pyplot as plt
import matplotlib.mlab as mlab
# csv2rec for simplicity
recessionDates = mlab.csv2rec('USREC.csv',skiprows=0)
capacityUtilization = mlab.csv2rec('TCU.csv')
d1 = set(recessionDates['date'])
d2 = set(capacityUtilization['date'])
# Find the common dates
commonDates = d1.intersection(d2)
```

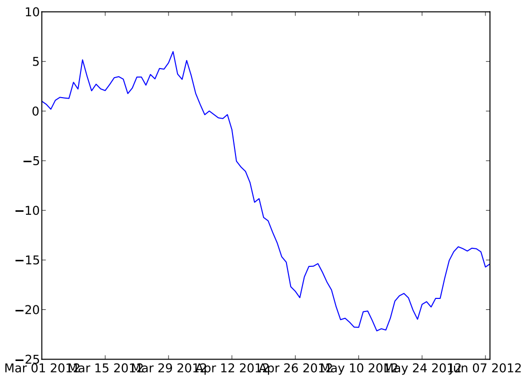
(a)



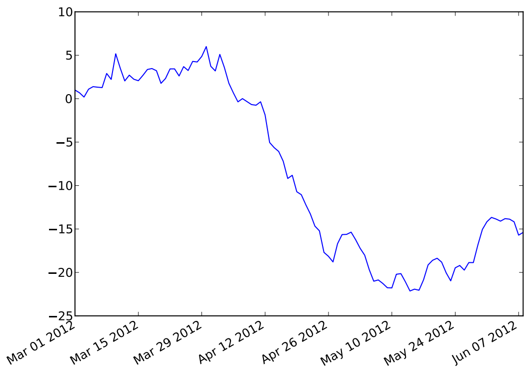
(b)



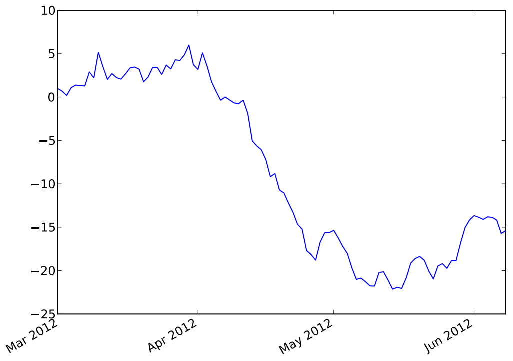
(c)



(d)



(e)



(f)

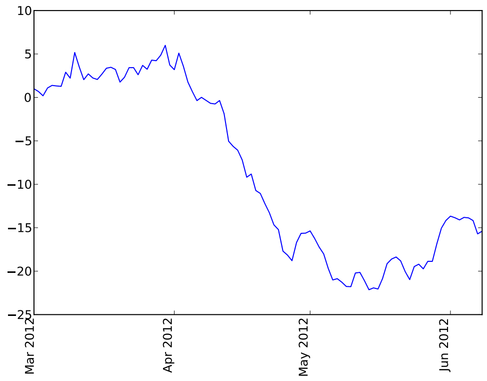


Figure 17.9: Figures with dates and additional formatting.

```
commonDates = list(commonDates)
commonDates.sort()
# And the first date
firstDate = min(commonDates)
# Find the data after the first date
plotData = capacityUtilization[capacityUtilization['date']>firstDate]
shadeData = recessionDates[recessionDates['date']>firstDate]
```

The second part of the code produces the plot. Most of the code is very simple. It begins by constructing a `figure`, then add_subplot to the figure, and the plotting the data using `plot`. `fill_between` is only one of many useful functions in matplotlib – it fills an area whenever a variable is 1, which is the structure of the recession indicator. The final part of the code adds a title with a custom font (set using a dictionary), and then changes the font and rotation of the axis labels. The output of this code is figure 17.10.

```
# The shaded plot
x = plotData['date']
y = plotData['value']
# z is the shading values, 1 or 0
z = shadeData['value']!=0
# Figure
fig = plt.figure()
ax = fig.add_subplot(111)
plt.plot_date(x,y,'r-')
limits = plt.axis()
font = { 'fontname':'Times New Roman', 'fontsize':14 }
ax.fill_between(x, limits[2], limits[3], where=z, edgecolor='#BBBBBB', \
    facecolor='#BBBBBB', alpha=0.5)
ax.set_title('Capacity Utilization',font)
xl = ax.get_xticklabels()
for label in xl:
    label.set_fontname('Times New Roman')
    label.set_fontsize(14)
    label.set_rotation(45)
yl = ax.get_yticklabels()
for label in yl:
    label.set_fontname('Times New Roman')
    label.set_fontsize(14)
plt.draw()
```

17.2.6 TEX in plots

Matplotlib supports using TEX in plots. The only steps needed are the first three lines in the code below, which configure some settings. the labels use raw mode to avoid needing to escape the \ in the TEX string. The final plot with TEX in the labels is presented in figure 17.11.

```
>>> from matplotlib import rc
>>> rc('text', usetex=True)
```

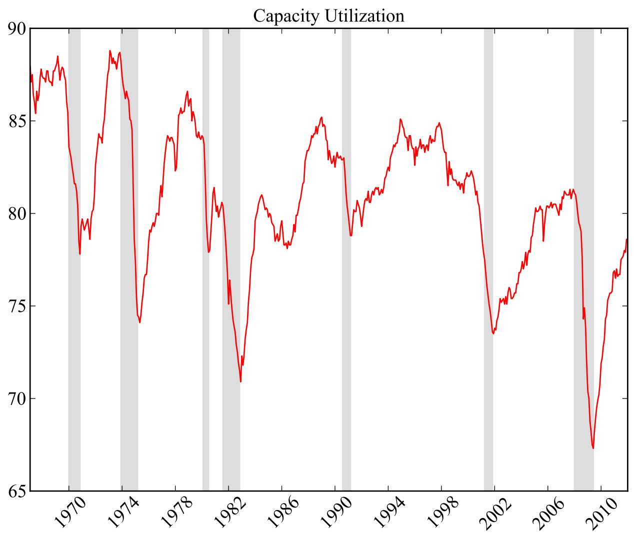


Figure 17.10: A plot of capacity utilization (US) with shaded regions indicating NBER recession dates.

```
>>> rc('font', family='serif')
>>> y = 50*np.exp(.0004 + np.cumsum(.01*np.random.randn(100)))
>>> plt.plot(y)
>>> plt.xlabel(r'\textbf{time ($\tau$)}')
>>> plt.ylabel(r'\textit{Price}',fontsize=16)
>>> plt.title(r'Geometric Random Walk: $d\ln p_t = \mu dt + \sigma dW_t$',fontsize=16)
>>> rc('text', usetex=False)
```

17.3 3D Plotting

The 3D plotting capabilities of matplotlib are decidedly weaker than the 2D plotting facilities. Despite this warning, the 3D capabilities are still more than adequate for most application – especially since 3D graphics are rarely necessary, and often not even useful when used.

17.3.1 Line Plots

Line plot in 3D are virtually identical to plotting in 2D, except that 3 1-dimensional vectors are needed, x, y and z (height). The simple example demonstrates how how `plot` can be used with the keyword argument zs to construct a 3D line plot. The line that sets up the axis using `Axed3D(fig)` is essential when producing 3D graphics. The other new command, `view_init`, is used to rotate the view using code (the view can be interactive rotated in the figure window). The result of running the code below is presented in figure 17.12.

```
>>> from mpl_toolkits.mplot3d import Axes3D
>>> x = np.linspace(0,6*np.pi,600)
>>> z = x.copy()
>>> y = np.sin(x)
>>> x=  np.cos(x)
>>> fig = plt.figure()
>>> ax = Axes3D(fig) # Different usage
```

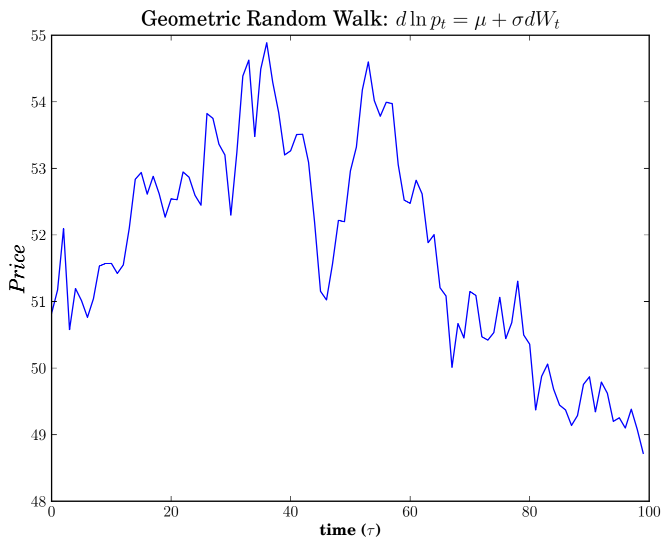


Figure 17.11: A plot that uses TEXin the labels.

```
>>> ax.plot(x, y, zs=z, label='Spiral')
>>> ax.view_init(15,45)
>>> plt.draw()
```

17.3.2 Surface and Mesh (Wireframe) Plots

Surface and mesh or wireframe plots are occasionally useful for visualizing functions with 2 inputs, such as a bivariate distribution. This example produces both for the bivariate normal PDF with mean 0, unit variances and correlation of 50%. The first block of code generates the points to use in the plot with `meshgrid` and evaluates the PDF for all combinations of x and y.

```
x = np.linspace(-3,3,100)
y = np.linspace(-3,3,100)
x,y = np.meshgrid(x,y)
z = np.mat(np.zeros(2))
p = np.zeros(np.shape(x))
R = np.matrix([[1,.5],[.5,1]])
Rinv = np.linalg.inv(R)
for i in xrange(len(x)):
    for j in xrange(len(y)):
        z[0,0] = x[i,j]
        z[0,1] = y[i,j]
        p[i,j] = 1.0/(2*np.pi)*np.sqrt(np.linalg.det(R))*np.exp(-(z*Rinv*z.T)/2)
```

The next code segment produces a mesh (wireframe) plot using `plot_wireframe`. The setup of the case is identical to that of the 3D line, and the call to `add_subplot(111, projection='3d')` is again essential. The figure is drawn using the 2-dimensional arrays x, y and p. The output of this code is presented in panel (a) of 17.13.

```
>>> fig = plt.figure()
```

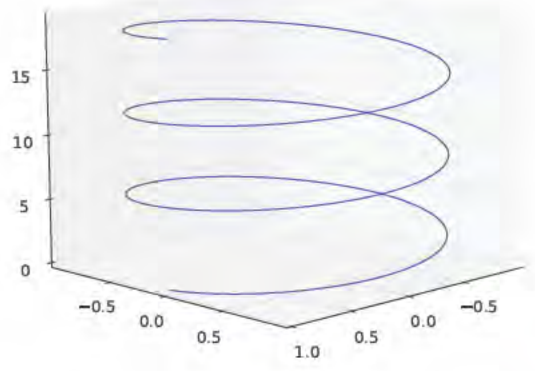
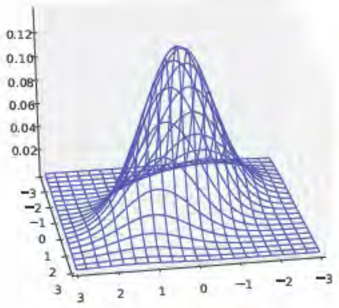


Figure 17.12: A 3D line plot constructed using plot.

(a) (b)



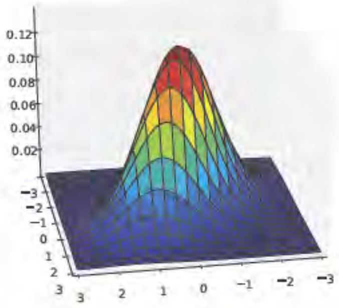


Figure 17.13: 3D figures produced using plot_wireframe and plot_surface.

```
>>> ax = fig.add_subplot(111, projection='3d')
>>> ax.plot_wireframe(x, y, p, rstride=5, cstride=5)
>>> ax.view_init(29,80)
>>> plt.draw()
```

Producing a surface plot is identical, only that a color map is needed from the module matplotlib.cm to provide different colors to highs versus lows. The output of this code is presented in panel (b).

```
>>> import matplotlib.cm as cm
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111, projection='3d')
>>> ax.plot_surface(x, y, p, rstride=5, cstride=5, cmap=cm.jet)
>>> ax.view_init(29,80)
>>> plt.draw()
```

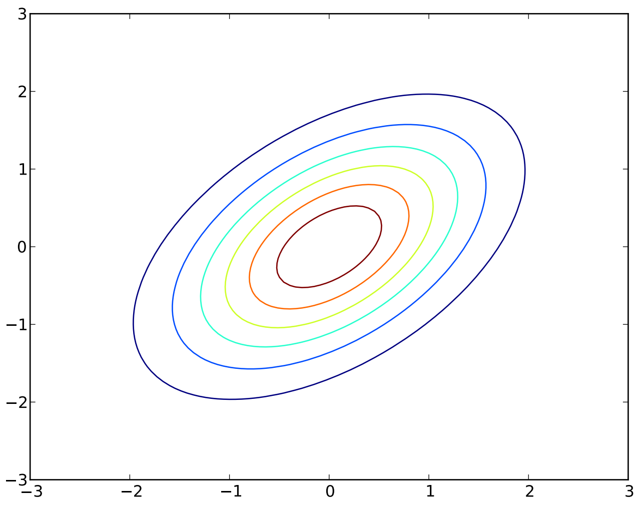


Figure 17.14: Contour plot produced using `contour`.

17.3.3 Contour Plots

Contour plots are not technically 3D, although they are used as a 2D representation of 3D data. Since they are ultimately 2D, little setup is needed, aside form a call to `contour` using the same inputs as `plot_surface` and `plot_wireframe`. The output of the code below is in figure 17.14.

```
>>> fig = plt.figure()
>>> ax = fig.gca()
>>> ax.contour(x,y,p)
>>> plt.draw()
```

17.4 General Plotting Functions

figure

`figure` is used to open a figure window, and can be used to generate axes. `fig = figure(n)` produces a figure object with id n, and assigns the object to `fig`.

add_subplot

`add_subplot` is used to add axes to a figure. `ax = fig.add_subplot(111)` can be used to add a basic axes to a figure. `ax = fig.add_subplot(m,n,i)` can be used to add an axes to a non-trivial figure with a m by n grid of plots.

close

`close` closes figures. `close(n)` closes the figure with id n, and `close('all')` closes all figure windows.

show

show is used to force an update to a figure, and to pause execution if not used in an interactive console. show should not be used in stand along Python programs. Instead draw should be used.

draw

draw forces an update to a figure.

17.5 Exporting Plots

Exporting plots is simple using `savefig('`*filename.ext*`')` where *ext* determine that type of exported file to produce. *ext* can be one of png, pdf, ps, eps and svg.

```
>>> plt.plot(randn(10,2))
>>> savefig('figure.pdf') # PDF export
>>> savefig('figure.png') # PNG export
>>> savefig('figure.svg') # Scalable Vector Graphics export
```

`savefig` has a number of useful keyword arguments. In particular, `dpi` is useful when exporting png files. The default `dpi` is 100.

```
>>> plt.plot(randn(10,2))
>>> savefig('figure.png', dpi = 600) # High resolution PNG export
```

17.6 Exercises

1. Download data for the past 20 years for the S&P 500 from Yahoo!. Plot the price against dates, and ensure the date display is reasonable.
2. Compute Friday-to-Friday returns using the log difference of closing prices and produce a histogram. Experiment with the number of bins.
3. Compute the percentage of weekly returns and produce a pie chart containing the percentage of weekly returns in each of:
   (a) $r \leq -2\%$
   (b) $-2\% < r \leq 0\%$
   (c) $0 < r \leq 2\%$
   (d) $r > 2\%$
4. Download 20 years of FTSE data, and compute Friday-to-Friday returns. Produce a scatter plot of the FTSE returns against the S&P 500 returns. Be sure to label the axes and provide a title.
5. Repeat exercise 4, but add in the fit line showing is the OLS fit of regressing FTSE on the S&P plus a constant.
6. Compute EWMA variance for both the S&P 500 and FTSE and plot against dates. An EWMA variance has $\sigma^2_{t'} = (1-\lambda)\,r^2_{t-1} + \sigma^2_{t-1}$ where $r^2_0 = \sigma^2_0$ is the full sample variance and $\lambda = 0.97$.

7. Explore the chart gallery on the matplotlib website.

Chapter 18

String Manipulation

Strings are usually less interesting than numerical values in econometrics and statistics. There are, however, some important uses for strings:

- Reading complex data formats
- Outputting formatted results to screen or file

Recall that strings are sliceable, but unlike arrays, are immutable, and so it is not possible to replace part of a string.

18.1 String Building

18.1.1 Adding Strings (+)

Strings can be concatenated using +.

```
>>> a = 'Python is'
>>> b = 'a rewarding language.'
>>> a + ' ' + b
'Python is a rewarding language.'
```

While + is a simple method to joint strings, the modern method is to use `join`. `join` is a string method which joins a list of strings (the input) using the object calling the string as the separator.

```
>>> a = 'Python is'
>>> b = 'a rewarding language.'
>>> ' '.join([a,b])
'Python is a rewarding language.'
```

Alternatively, the same output can be constructed using an empty string `''`.

```
>>> a = 'Python is'
>>> b = 'a rewarding language.'
>>> ''.join([a,' ',b])
'Python is a rewarding language.'
```

18.1.2 Multiplying Strings (*)

Strings can be repeated using *.

```
>>> a = 'Python is'
>>> 2*a
'Python isPython is'
```

18.2 String Functions

18.2.1 split and rsplit

split splits a string into a list based on a character, for example a comma. Takes an optional third argument maxsplit which limits the number of outs in the list. rsplit works identically to split, only scanning from the end of the string – split and rsplit only differ when maxsplit is used.

```
>>> s = 'Python is a rewarding language.'
>>> s.split(' ')
['Python', 'is', 'a', 'rewarding', 'language.']

>>> s.split(' ',3)
['Python', 'is', 'a', 'rewarding language.']

>>> s.rsplit(' ',3)
['Python is', 'a', 'rewarding', 'language.']
```

18.2.2 join

join concatenates a list or tuple of strings, using an optional argument sep which specified a separator (default is space).

```
>>> import string
>>> a = 'Python is'
>>> b = 'a rewarding language.'
>>> string.join((a,b))
'Python is a rewarding language.'

>>> string.join((a,b),':')
'Python is:a rewarding language.'
```

18.2.3 strip, lstrip, and rstrip

strip removes leading and trailing whitespace from a string. An optional input char removes leading and trailing occurrences of the input value (instead of space). lstrip and rstrip work identically, only stripping from the left and right, respectively.

```
>>> s = '    Python is a rewarding language.    '
>>> s=s.strip()
'Python is a rewarding language.'
```

```
>>> s.strip('P')
'ython is a rewarding language.'
```

18.2.4 find and rfind

find locates the lowest index of a substring, and returns -1 if not found. Optional arguments limit the range of the search, and s.find('i',10,20) is identical to s[10:20].find('i'). rfind works identically, only returning the highest index of the substring.

```
>>> s = 'Python is a rewarding language.'
>>> s.find('i')
7

>>> s.find('i',10,20)
18

>>> s.rfind('i')
18
```

18.2.5 index and rindex

index returns the lowest index of a substring, and is identical to find except that an error is raised if the substring does not exist. As a result, index is only safe to use in a try . . . except block.

```
>>> s = 'Python is a rewarding language.'
>>> s.index('i')
7

>>> s.index('q') # Error
ValueError: substring not found
```

18.2.6 count

count counts the number of occurrences of a substring. It takes optional arguments which limit the search range.

```
>>> s = 'Python is a rewarding language.'
>>> s.count('i')
2

>>> s.count('i', 10, 20)
1
```

18.2.7 lower and upper

lower and upper convert strings to lower and upper case, respectively. They are useful to remove case ambiguity when comparing string to known constants.

```
>>> s = 'Python is a rewarding language.'
>>> s.upper()
'PYTHON IS A REWARDING LANGUAGE.'

>>> s.lower()
'python is a rewarding language.'
```

18.2.8 `ljust`, `rjust` and `center`

`ljust`, `rjust` and `center` left justify, right justify and center, respectively, a string while expanding its size to a given length. If the desired length is smaller than the string, the unchanged string is returned.

```
>>> s = 'Python is a rewarding language.'
>>> s.ljust(40)
'Python is a rewarding language.         '

>>> s.rjust(40)
'         Python is a rewarding language.'

>>> s.center(40)
'    Python is a rewarding language.     '
```

18.2.9 `replace`

`replace` replaces a substring with an alternative string, which can have different size. An optional argument limits the number of replacement.

```
>>> s = 'Python is a rewarding language.'
>>> s.replace('g','Q')
'Python is a rewardinQ lanQuaQe.'

>>> s.replace('is','Q')
'Python Q a rewarding language.'

>>> s.replace('g','Q',2)
'Python is a rewardinQ lanQuage.'
```

18.2.10 `textwrap.wrap`

The module `textwrap` contains a function `wrap` which reformats a long string into a fixed width paragraph, stored line-by-line in a list. An optional argument changes the width of the output paragraph form the default of 70 characters.

```
>>> import textwrap
>>> s = 'Python is a rewarding language. '
>>> s = 10*s
>>> textwrap.wrap(s)
['Python is a rewarding language. Python is a rewarding language. Python',
'is a rewarding language. Python is a rewarding language. Python is a',
'rewarding language. Python is a rewarding language. Python is a',
'rewarding language. Python is a rewarding language. Python is a',
'rewarding language. Python is a rewarding language.']

>>> textwrap.wrap(s,50)
['Python is a rewarding language. Python is a',
'rewarding language. Python is a rewarding',
'language. Python is a rewarding language. Python',
'is a rewarding language. Python is a rewarding',
'language. Python is a rewarding language. Python',
'is a rewarding language. Python is a rewarding',
'language. Python is a rewarding language.']
```

18.3 Formatting Numbers

Formatting numbers when converting to string allow for automatic generation of tables and well formatted print statements. Numbers are formatted using the `format` function, which is used in conjunction with a format specifier. For example, consider these examples which format π.

```
>>> pi
3.141592653589793

>>> '{:12.5f}'.format(pi)
'     3.14159'

>>> '{:12.5g}'.format(pi)
'      3.1416'

>>> '{:12.5e}'.format(pi)
' 3.14159e+00'
```

These all provide alternative formats and the difference is determined by the letter in the format string. The generic form of a format string is $\{n:faswm,.pt\}$. To understand the the various choices, consider the output produced by the basic output string `'{0:}'`

```
>>> '{0:}'.format(pi)
'3.14159265359'
```

- n is a number 0,1,... indicating which value to take from the format function

  ```
  >>> '{0:}, {1:} and {2:} are all related to pi'.format(pi,pi+1,2*pi)
  '3.14159265359'

  >>> '{2:}, {0:} and {1:} reorder the output.'.format(pi,pi+1,2*pi)
  '6.28318530718, 3.14159265359 and 4.14159265359 reorder the output.
  ```

- fa are fill and alignment characters, typically a 2 character string. Fill can be any character except }. Alignment can < (left) ,> (right), ^ (top), = (pad to the right of the sign). Simple left 0-fills can omit the alignment character so that $fa = 0$.

```
>>> '{0:0<20}'.format(pi) # Left, 0 padding, precion 20
'3.141592653590000000'

>>> '{0:0>20}'.format(pi) # Right, 0 padding, precion 20
'00000003.14159265359'

>>> '{0: >20}'.format(pi) # Right, space padding, precion 20
'       3.14159265359'

>>> '{0:$^20}'.format(pi) # Center, dollar sign padding, precion 20
'$$$3.14159265359$$$$'
```

- *s* indicates whether a sign should be included. + indicates always include sign, - indicates only include if needed, and a blank space indicates to use a blank space for positive numbers, and a − sign for negative numbers – this format is useful for producing aligned tables.

```
>>> '{0:+}'.format(pi)
'+3.14159265359'

>>> '{0:-}'.format(pi)
'3.14159265359'

>>> '{0: }'.format(pi)
' 3.14159265359'

>>> '{0: }'.format(-1 * pi)
'-3.14159265359'
```

- *m* is the minimum total size of the formatted string. If the formatted string is shorter than *m*, character *w* is prepended.

```
>>> '{0:10}'.format(pi)
'3.14159265359'
>>> '{0:20}'.format(pi)
'       3.14159265359'
>>> '{0:30}'.format(pi)
'                 3.14159265359'
```

- *c* can be , or omitted. , produces numbers with 1000s separated using a ,. In order to use *c* it is necessary to include the . before the precision.

```
>>> '{0:.10}'.format(1000000 * pi)
'3141592.654'

>>> '{0:,.10}'.format(1000000 * pi)
'3,141,592.654'
```

- p is the precision. The interpretation of precision depends on t. In order to use p, it is necessary to include a `.` (dot). If not included, p will be interpreted as m.

  ```
  >>> '{0:.1}'.format(pi)
  '3e+00'

  >>> '{0:.2}'.format(pi)
  '3.1'

  >>> '{0:.5}'.format(pi)
  '3.1416'
  ```

- t is the type. Options include:

Type	Description
e, E	Exponent notation, e produces e+ and E produces E+ notation
f, F	Display number using a fixed number of digits
g, G	General format, which uses f for smaller numbers, and e for larger. G is equivalent to switching between F and E. g is the default format if no presentation format is given
n	Similar to g, except that it uses locale specific information.
%	Multiplies numbers by 100, and inserts a % sign

```
>>> '{0:.5e}'.format(pi)
'3.14159e+00'

>>> '{0:.5g}'.format(pi)
'3.1416'

>>> '{0:.5f}'.format(pi)
'3.14159'

>>> '{0:.5%}'.format(pi)
'314.15927%'

>>> '{0:.5e}'.format(100000 * pi)
'3.14159e+05'

>>> '{0:.5g}'.format(100000 * pi)
'3.1416e+05'

>>> '{0:.5f}'.format(100000 * pi)
'314159.26536'
```

All of these features can be combined in a single format string to produce complexly presented data.

```
>>> '{0: > 20.4f}, {1: > 20.4f}'.format(pi,-pi)
'              3.1416,              -3.1416'

>>> '{0: > 20,.2f}, {1: > 20,.2f}'.format(100000 * pi,-100000 * pi)
```

```
'         314,159.2654,        -314,159.2654'
```

In the first example, reading from left to right after the colon, the format string consists of:

1. Space fill (the blank space after the colon)
2. Right align (>)
3. Use no sign for positive numbers, − sign for negative numbers (the blank space after >)
4. Minimum 20 digits
5. Precision of 4 fixed digits

The second is virtually identical to the first, except that it includes a `,` to show the 1000s separator.

18.3.1 Formatting Strings

`format` can be used to output formatted strings using a similar syntax to number formatting, although some options, precision, sign, comma and type are not relevant.

```
>>> s = 'Python'
>>> '{0:}'.format(s)
'Python'

>>> '{0: >20}'.format(s)
'              Python'

>>> '{0:!>20}'.format(s)
'!!!!!!!!!!!!!!Python'

>>> 'The formatted string is: {0:!<20}'.format(s)
'The formatted string is: Python!!!!!!!!!!!!!!'
```

18.3.2 Formatting Multiple Objects

`format` can be used to format multiple objects in the same string output. There are three methods to do this:

- No position arguments, in which case the objects are matched to format strings in order
- Numeric positional arguments, in which case the first object is mapped to `'{0:}'`, the second to `'{1:}'`, and so on.
- Named arguments such as `'{price:}'` and volume `'{volume:}'`, which match keyword arguments inside format.

```
>>> price = 100.32
>>> volume = 132000
>>> 'The price yesterday was {:} with volume {:}'.format(price,volume)
'The price yesterday was 100.32 with volume  132000'

>>> 'The price yesterday was {0:} and the volume was {1:}'.format(price,volume)
```

```
'The price yesterday was 100.32 with volume 132000'

>>> 'The price yesterday was {1:} and the volume was {0:}'.format(volume,price)
'The price yesterday was 100.32 with volume 132000'

>>> 'The price yesterday was {price:} and the volume was {volume:}'.format(price=price,volume=volume)
'The price yesterday was 100.32 with volume 132000'
```

18.3.3 Old style format strings

Some Python code still uses an older style format string. Old style format strings have %$(map)flm.pt$, where:

- (map) is a mapping string containing a name, for example `(price)`
- fl is a flag which can be one or more of:
  - 0: Zero pad
  - (blank space)
  - – Left adjust output
  - + Include sign character
- m, p and t are identical to those of the new format strings.

In general, the old format strings should only be used when required by other code (e.g. matplotlib). Below are some examples of their use in strings.

```
>>> price = 100.32
>>> volume = 132000
>>> 'The price yesterday was %0.2f with volume %d' % (price, volume)
'The price yesterday was 100.32 with volume  132000'

>>> 'The price yesterday was %(price)0.2f with volume %(volume)d' \
...     % {'price': price, 'volume': volume}
'The price yesterday was 100.32 with volume 132000'

>>> 'The price yesterday was %+0.3f and the volume was %010d' % (price, volume)
'The price yesterday was +100.320 and the volume was 0000132000'
```

18.4 Regular Expressions

Regular expressions are powerful tools for matching patterns in strings. While teaching regular expressions is beyond the scope of these notes – there are 500 page books dedicated to regular expressions – they are sufficiently useful to warrant coverage. Fortunately there are a large number of online regular expression generators which can assist in finding the pattern to use, and so they are useful to anyone working with unformatted text.

Using regular expression requires the re module. The most useful function for regular expression matching are findall, finditer and sub. findall and finditer work in similar manners, except that findall returns a list while finditer returns an iterable. finditer is preferred if a large number of matches is possible. Both search through a string and find all non-overlapping matches of a regular expression.

```
>>> import re
>>> s = 'Find all numbers in this string: 32.43, 1234.98, and 123.8.'
>>> re.findall('[\s][0-9]+\.\d*',s)
[' 32.43', ' 1234.98', ' 123.8']

>>> matches = re.finditer('[\s][0-9]+\.\d*',s)
>>> for m in matches:
...     print(s[m.span()[0]:m.span()[1]])
 32.43
 1234.98
 123.8
```

finditer returns MatchObjects which contain the method span. span returns a 2 element tuple which contains the start and end position of the match.

sub replaces all matched text with another text string (or a function which takes a MatchObject).

```
>>> s = 'Find all numbers in this string: 32.43, 1234.98, and 123.8.'
>>> re.sub('[\s][0-9]+\.\d*',' NUMBER',s)
'Find all numbers in this string: NUMBER, NUMBER, and NUMBER.'

>>> def reverse(m):
...     """Reverse the string in the MatchObject group"""
...     s = m.group()
...     s = s.rstrip()
...     return ' ' + s[::-1]

>>> re.sub('[\s][0-9]+\.\d*',reverse,s)
'Find all numbers in this string: 34.23, 89.4321, and 8.321.'
```

18.4.1 Compiling Regular Expressions

When repeatedly using a regular expression, for example running it on all lines in a file, it is better to compile the regular expression, and then to use the resulting RegexObject.

```
>>> import re
>>> s = 'Find all numbers in this string: 32.43, 1234.98, and 123.8.'
>>> numbers = re.compile('[\s][0-9]+\.\d*')
>>> numbers.findall(s)
[' 32.43', ' 1234.98', ' 123.8']
```

Parsing the regular expression text is relatively expensive, and compiling the expression avoids this cost.

18.5 Conversion of Strings

When reading data into Python using a mixed format, blindly converting text to integers or floats is dangerous. For example, float('a') returns a ValueError since Python doesn't know how to convert 'a' to a string. The simplest method to safely convert potentially non-numeric data is to use a `try` . . . `except` block.

```python
from __future__ import print_function
from __future__ import division

S = ['1234','1234.567','a','1234.a34','1.0','a123']
for s in S:
    try:
        int(s)
        print(s, 'is an integer.')
    except:
        try:
            float(s)
            print(s, 'is a float.')
        except:
            print('Unable to convert', s)
```

Chapter 19

File System and Navigation

Manipulating the file system is surprising useful when working with data. The most important file system commands are located in the modules `os` and `shutil`. This chapter assumes that

```
import os
import shutil
```

have been included.

19.1 Changing the Working Directory

The working directory is where files can be created and accessed without any path information. `os.getcwd()` can be used to determine the current working directory, and `os.chdir(`*path*`)` can be used to change the working directory, where *path* is a directory, such as /temp or c:\\temp.[1] Alternatively, *path* can can be .. to more up the directory tree.

```
pwd = os.getcwd()
os.chdir('c:\\temp')
os.chdir('c:/temp')  # Identical
os.chdir('..')
os.getcwd()          # Now in 'c:\\'
```

19.2 Creating and Deleting Directories

Directories can be created using `os.mkdir(`*dirname*`)`, although it must be the case that the higher level directories exist (e.g. to create /home/username/ Python/temp, it must be the case that /home/username/Python already exists). `os.makedirs(`*dirname*`)` works similar to `os.mkdir(`*dirname*`)`, except that is will create any higher level directories needed to create the target directory.

Empty directories can be deleted using `os.rmdir(`*dirname*`)` – if the directory is not empty, an error occurs. `shutil.rmtree(`*dirname*`)` works similarly to `os.rmdir(`*dirname*`)`, except that it will delete the directory, and any files or other directories contained in the directory.

[1]On Windows, directories use the backslash, which is used to escape characters in Python, and so an escaped backslash – \\ – is needed when writing Windows' paths. Alternatively, the forward slash can be substituted, so that c:\\temp and c:/temp are equivalent.

```
os.mkdir('c:\\temp\\test')
os.makedirs('c:/temp/test/level2/level3')  # mkdir will fail
os.rmdir('c:\\temp\\test\\level2\\level3')
shutil.rmtree('c:\\temp\\test')  # rmdir fails, since not empty
```

19.3 Listing the Contents of a Directory

The contents of a directory can be retrieved in a list using `os.listdir(`*dirname*`)`, or simply `os.listdir('.')` to list the current working directory. The list returned will contain all all files and directories. `os.path.isdir(` *name* `)` can be used to determine whether a value in the list is a directory, and `os.path.isfile(`*name*`)` can be used to determine if it is a file. `os.path` contains other useful functions for working with directory listings and file attributes.

```
os.chdir('c:\\temp')
files = os.listdir('.')
for f in files:
    if os.path.isdir(f):
        print(f, ' is a directory.')
    elif os.path.isfile(f):
        print(f, ' is a file.')
    else:
        print(f, ' is a something else.')
```

A more sophisticated listing which accepts wildcards and is similar to `dir` (Windows) and `ls` (Linux) can be constructed using the glob module.

```
import glob
files = glob.glob('c:\\temp\\*.txt')

for file in files:
    print(file)
```

19.4 Copying, Moving and Deleting Files

File contents can be copied using `shutil.copy(` *src* , *dest* `)`, `shutil.copy2(` *src* , *dest* `)` or `shutil.copyfile(` *src* , *dest* `)`. These functions are all similar, and the differences are:

- `shutil.copy` will accept either a filename or a directory as *dest*. If a directory is given, the a file is created in the directory with the same name as the original file
- `shutil.copyfile` requires a filename for *dest*.
- `shutil.copy2` is identical to `shutil.copy`, except that metadata such as access times, is also copied.

Finallly, `shutil.copytree(` *src* , *dest* `)` will copy an entire directory tree, starting from the directory *src* to the directory *dest*, which must not exist. `shutil.move(` *src*, *dest*`)` is similar to `shutil.copytree`, except that it moves a file or directory tree to a new location. If preserving file metadata (such as permissions or file streams) is important, it is better use system commands (copy or move on Windows, cp or mv on Linux) as an external program.

```
os.chdir('c:\\temp\\python')
# Make an empty file
f = file('file.ext','w')
f.close()
# Copies file.ext to 'c:\temp\'
shutil.copy('file.ext','c:\\temp\\')
# Copies file.ext to 'c:\temp\\python\file2.ext'
shutil.copy('file.ext','file2.ext')
# Copies file.ext to 'c:\\temp\\file3.ext', plus metadata
shutil.copy2('file.ext','file3.ext')
shutil.copytree('c:\\temp\\python\\','c:\\temp\\newdir\\')
shutil.move('c:\\temp\\newdir\\','c:\\temp\\newdir2\\')
```

19.5 Executing Other Programs

Occasionally it is necessary to call other programs, for example to decompress a file compressed in an unusual format or to call system copy commands to preserve metadata and file ownership. Both `os.system` and `subprocess.call` (which requires `import subprocess`) can be used to execute commands as if they were executed directly in the shell.

```
import subprocess

# Copy using xcopy
os.system('xcopy /S /I c:\\temp c:\\temp4')
subprocess.call('xcopy /S /I c:\\temp c:\\temp5',shell=True)
# Extract using 7-zip
subprocess.call('"C:\\Program Files\\7-Zip\\7z.exe" e -y c:\\temp\\zip.7z')
```

19.6 Creating and Opening Archives

Creating and extracting files from archives often allows for further automation in data processing. Python has native support for zip, tar, gzip and bz2 file formats using `shutil.make_archive(` *archivename* , *format*, *root*) where *archivename* is the name of the archive to create, without the extension, *format* is one of the supported formats (e..g `'zip'` for a zip archive or `'gztar'`, for a gzipped tar file) and *root* is the root directory which can be `'.'` for the current working directory.

```
# Creates files.zip
shutil.make_archive('files','zip','c:\\temp\\folder_to_archive')
# Creates files.tar.gz
shutil.make_archive('files','gztar','c:\\temp\\folder_to_archive')
```

Creating a standard gzip from an existing file is slightly more complicated, and requires using the `gzip` module.[2]

```
import gzip

# Create file.csv.gz from file.csv
```

[2]A gzip can only contain 1 file, and is usually used with a tar file to compress a directory or set of files.

```
csvin = file('file.csv','rb')
gz = gzip.GzipFile('file.csv.gz','wb')
gz.writelines(csvin.read())
gz.close()
csvin.close()
```

Zip files can be extracted using the module `zipfile`, and gzip files can be extracted using `gzip`, and gzipped tar files can be extracted using `tarfile`.

```
import zipfile
import gzip
import tarfile

# Extract zip
zip = zipfile.ZipFile('files.zip')
zip.extractall('c:\\temp\\zip\\')
zip.close()

# Extract gzip tar 'r:gz' indicates read gzipped
gztar = tarfile.open('file.tar.gz', 'r:gz')
gztar.extractall('c:\\temp\\gztar\\')
gztar.close()

# Extract csv from gzipped csv
gz = gzip.GzipFile('file.csv.gz','rb')
csvout = file('file.csv','wb')
csvout.writelines(gz.read())
csvout.close()
gz.close()
```

19.7 Reading and Writing Files

Occasionally it may be necessary to read or write a file, for example to output a formatted LaTeX table. Python contains low level file access tools which can be used to to generate files with any structure. Custom files all begin by using `file` to create a new or open an existing file. Files can be opened in different modes, 'r' for reading, 'w' for writing, and 'a' for appending ('w' will overwrite an existing file). An additional modifier 'b' can be be used if the file is binary (not text), so that 'rb', 'wb' and 'ab' allow reading, writing and appending binary files.

Reading text files is usually implemented using `readline()` to read a single line, `readlines(` n`)` to read at most n lines or `readlines()` to read all lines in a file. `readline()` and `readlines(` n`)` are usually used inside a while loop which terminates if teh value retured is an empty string (`''`).

```
# Read all lines using readlines()
f = file('file.csv','r')
lines = f.readlines()
for line in lines:
    print(line)
f.close()
```

```
# Using readline(n)
f = file('file.csv','r')
line = f.readline()
while line != '':
    print(line)
    line = f.readline()

f.close()

# Using readlines(n)
f = file('file.csv','r')
lines = f.readlines(2)
while lines != '':
    for line in lines:
        print(line)
    lines = f.readline(2)

f.close()
```

In practice, the information from the file is usually transformed in a more meaningful way than using `print`.

Writing text files is similar, and begins by using `file` to create a file, and then `fwrite` to output information. `frwite` is conceptually similar to using print, except that the output will be written to a file rather than printed on screen. The next example show how to create a LaTeXtable from an array.

```
import numpy as np
import scipy.stats as stats

x = np.random.randn(100,4)
mu = np.mean(x,0)
sig = np.std(x,0)
sk = stats.skew(x,0)
ku = stats.kurtosis(x,0)

summaryStats = np.vstack((mu,sig,sk,ku))
rowHeadings = ['Var 1','Var 2','Var 3','Var 4']
colHeadings = ['Mean','Std Dev','Skewness','Kurtosis']

# Build table, then print
latex = []
latex.append('\\begin{tabular}{r|rrrr}')
line = ' '
for i in xrange(len(colHeadings)):
    line += ' & ' + rowHeadings[i]

line += ' \\ \hline'
latex.append(line)

for i in xrange(size(summaryStats,0)):
    line = rowHeadings[i]
    for j in xrange(size(summaryStats,1)):
        line += ' & ' + str(summaryStats[i,j])

    latex.append(line)

latex.append('\\end{tabular}')

# Output using write()
f = file('latex_table.tex','w')
for line in latex:
    f.write(line + '\n')

f.close()
```

19.8 Exercises

1. Create a new directory, *chapter19*.
2. Change into this directory.
3. Create a new file names *tobedeleted.py* a text editor in this new directory (It can be empty).
4. Create a zip file *tobedeleted.zip* containing *tobedeleted.py*.
5. Get and print the directory listing.
6. Delete the newly created file, and then delete this directory.

Chapter 20

Structured Arrays

The arrays and matrices used in most of these notes are highly optimized data structures where all elements have the same data type (e.g. float), and elements can be accessed using slicing. They are essential for high-performance numerical computing, such as computing inverses of large matrices. Unfortunately, actual data often have meaningful names – not just “column 0” – or may have different types – dates, strings, integers and floats – that cannot be combined in a uniform NumPy array. NumPy supports mixed arrays which solve both of these issues and so are a useful data structures for managing data prior to statistical analysis. Conceptually, a mixed array with named columns is similar to a spreadsheet where each column can have its own name and data type.

20.1 Mixed Arrays with Column Names

A mixed NumPy array can be initialized using `array` or `zeros`, among other functions. Mixed arrays are in many ways similar to standard NumPy arrays, except that the `dtype` input to the function is specified either using tuples of the form (*name*, *type*), or using a dictionary.

```
>>> x = zeros(4,[('date','int'),('ret','float')])
>>> x = zeros(4,{'names': ('date','ret'), 'formats': ('int', 'float')})
>>> x
array([(0, 0.0), (0, 0.0), (0, 0.0), (0, 0.0)],
      dtype=[('date', '<i4'), ('ret', '<f8')])
```

These two command are identical, and illustrate the two methods to create an array which contain a named column “date”, for integer data, and a named column “ret” for floats. These columns can be accessed by name.

```
>>> x['date']
array([0, 0, 0, 0])

>>> x['ret']
array([0.0, 0.0, 0.0, 0.0])
```

20.1.1 Data Types

A large number of primitive data types are available in NumPy.

Type	Syntax	Description
Boolean	b	True/False
Integers	i1,i2,i4,i8	1 to 8 byte signed integers ($-2^{B-1}, \ldots 2^{B-1}-1$)
Unsigned Integers	u1,u2,u4,u8	1 to 8 byte signed integers ($0, \ldots 2^B$)
Floating Point	f4,f8	Single (4) and double (8) precision float
Complex	c8,c16	Single (8) and double (16) precision complex
Object	On	Generic n-byte object
String	Sn, an	n-letter string
Unicode String	Un	n-letter unicode string

The majority of data types are for numeric data, and are simple to understand. The n in the string data type indicates the maximum length of a string. Attempting to insert a stringer with more than n characters will truncate the string. The object data type is somewhat abstract, but allows for storing Python objects such as `datetime`s.

Custom data types can be built using `dtype`. The constructed data type can then be used in the construction of a mixed array.

```
t = dtype([('var1','f8'), ('var2','i8'), ('var3','u8')])
```

Data types can even be nested to create a structured environment where one of the "variables" has multiple values. Consider this example which uses a nested data type to contain the bid and ask price or a stock, along with the time of the transaction.

```
ba = dtype([('bid','f8'), ('ask','f8')])
t = dtype([('date', 'O8'), ('prices', ba)])
data = zeros(2,t)
```

In this example, data is an array where each item has 2 elements, the date and the price. Price is also an array with 2 elements. Names can also be used to access values in nested arrays (e.g. `data['prices']['bid']` returns an array containing all bid prices). In practice nested arrays can almost always be expressed as a non-nested array without loss of fidelity.

Determining the size of object NumPy arrays can store objects which are anything which fall outside of the usual data types. One example of a useful, but abstract, data type is `datetime`. One method to determine the size of an object is to create a plain array containing the object – which will automatically determine the data type – and then to query the size from the array.

```
import datetime as dt
x = array([dt.datetime.now()])
# The size in bytes
print(x.dtype.itemsize)
# The name and description
print(x.dtype.descr)
```

20.1.2 Example: TAQ Data

TAQ is the NYSE Trade and Quote database which contains all trades and quotes of US listed equities which trade on major US markets (not just the NYSE). A record from a trade contains a number of fields:

- Date - The Date in YYYYMMDD format stored as a 4-byte unsigned integer
- Time - Time in HHMMSS format, stored as a 4-byte unsigned integer
- Size - Number of shares trades, stores as a 4 byte unsigned integer
- G127 rule indicator - Numeric value, stored as a 2 byte unsigned integer
- Correction - Numeric indicator of a correction, stored as a 2 byte unsigned integer
- Condition - Market condition, a 2 character string
- Exchange - The exchange where the trade occurred, a 1-character string

First consider a data type which stores the data in an identical format.

```
t = dtype([('date', 'u4'), ('time', 'u4'),
           ('size', 'u4'), ('price', 'f8'),
           ('g127', 'u2'), ('corr', 'u2'),
           ('cond', 'S2'), ('ex', 'S2')])
taqData = zeros(10, dtype=t)
taqData[0] = (20120201,120139,1,53.21,0,0,'','N')
```

An alternative is to store the date and time as a `datetime`, which is an 8-byte object.

```
import datetime as dt

t = dtype([('datetime', 'O8'), ('size', 'u4'), ('price', 'f8'),
           ('g127', 'u2'), ('corr', 'u2'), ('cond', 'S2'), ('ex', 'S2')])
taqData = zeros(10, dtype=t)
taqData[0] = (dt.datetime(2012,2,1,12,01,39),1,53.21,0,0,'','N')
```

20.2 Record Arrays

Record arrays are closely related to mixed arrays with names. The primary difference is that elements record arrays can be accessed using *variable.name* format.

```
>>> x = zeros((4,1),[('date','int'),('ret','float')])
>>> y = rec.array(x)
>>> y.date
array([[0],
       [0],
       [0],
       [0]])

>>> y.date[0]
array([0])
```

In practice record arrays may be slower than standard arrays, and unless the *variable.name* is really important, record arrays are not compelling.

Chapter 21

Performance and Code Optimization

We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil.

Donald Knuth

21.1 Getting Started

Occasionally a direct implementation of a statistical algorithm will not be fast enough to be applied to interesting data sets. When this happens, there are a number of alternatives. Before starting to optimize code, it is *essential* that a clean, working implementation has been coded. This allows for both benchmarking improvements and to ensure that optimizations have not introduced bugs to code. The famous quote of Donald Knuth should also be heeded. Code optimization is only needed for a very small amount of code which is repeatedly used. Most code, on modern computers, is sufficiently fast and/or infrequently used that optimization is a wasted effort.

21.2 Timing Code

Timing code is an important step in performance tuning. IPython contains the magic keywords `%timeit` and `%time` which can be used to determine the performance of code. `%time` simply runs the code and reports the time needed. `%timeit` is smarter in that it will loop very fast code which will increase the accuracy of the timing measurements. Both are used in the same manner, `%timeit` *code to time*.

```
>>> x = randn(1000,1000)
>>> %timeit inv(dot(x.T,x))
1 loops, best of 3: 387 ms per loop

>>> %time inv(dot(x.T,x))
CPU times: user 0.52 s, sys: 0.00 s, total: 0.52 s
Wall time: 0.52 s

>>> x = randn(100,100)
>>> %timeit inv(dot(x.T,x))
1000 loops, best of 3: 797 us per loop
```

21.3 Vectorize to Avoid Unnecessary Loops

Vectorization is the key to getting top performance out of NumPy code. Code that is vectorized run inside NumPy and so executes as quickly as possible (with some small technical caveats, see NumExpr). Consider the difference between manually multiplying two matrices and using `dot`.

```
def pydot(a, b):
    M,N = shape(a)
    P,Q = shape(b)
    assert N==P
    c = zeros((M,Q))
    for i in xrange(M):
        for j in xrange(Q):
            for k in xrange(N):
                c[i,j] = c[i,j] + a[i,k] * b[k,j]
    return c
```

Timing the difference shows that in this extreme case, NumPy is about 10000x faster than looping Python.

```
>>> a = randn(100,100)
>>> b = randn(100,100)
>>> %timeit c = pydot(a,b)
1 loops, best of 3: 1.27 s per loop

>>> %timeit d = dot(a,b)
10000 loops, best of 3: 165 us per loop

>>> absolute(c-d).max()
0.0

>>> 1.72/0.000165
10424.242424
```

A less absurd example is to consider computing a weighted moving average across m consecutive values of a vector.

```
def naive_weighted_avg(x, w):
    T = x.shape[0]
    m = len(w)
    m12 = int(ceil(m/2))
    y = zeros(T)
    for i in xrange(len(x)-m+1):
        y[i+m12] = dot(x[i:i+m].T,w)

    return y
```

```
>>> w = r_[1:11,9:0:-1]
>>> w = w/sum(w)
>>> x = randn(10000,1)
>>> %timeit naive_weighted_avg(x,w)
100 loops, best of 3: 22.6 ms per loop
```

```
def clever_weighted_avg(x,w):
    T = x.shape[0]
    m = len(w)
    wc = copy(w)
    wc.shape = m,1
    T = x.size
    xc = copy(x)
    xc.shape=T,1
    y = vstack((xc,zeros((m,1))))
    y = tile(y,(m,1))

    y = reshape(y[:len(y)-m],(m,T+m-1))
    y = y.T
    y = y[m-1:T,:]

    return dot(y,flipud(wc))
```

```
>>> %timeit clever_weighted_avg(x,w)
1000 loops, best of 3: 1.31 ms per loop
```

The loop-free method which uses copying and slicing is about 20 times faster than the simple looping specification.

21.4 Alter the loop dimensions

In many applications, it may be natural to loop over the long dimension in a time series. This is especially common if the mathematical formula underlying the program has a sum of $t = 1$ to T. In some cases, it is possible to replace a loop over time, which is assumed to be the larger dimension, with an alternative loop across another iterable. For example, in the moving average, it is possible to loop over the weights rather than the data, and if the moving windows length is much smaller than the length of the data, the code should run much faster.

```
def sideways_weighted_avg(x, w):
    T = x.shape[0]
    m = len(w)
    y = zeros(T)
    m12 = int(ceil(m/2))
    y = zeros(x.shape)
    for i in xrange(m):
        y[m12:T-m+m12] = x[i:T+i-m] * w[i]

    return y
```

```
>>> %timeit sideways_weighted_avg(x,w)
1000 loops, best of 3: 574 us per loop
```

In this example, the "sideways" loop is much faster than fully vectorized version since it avoids allocating a large amount of memory, and the loop dimension is very small.

21.5 Utilize Broadcasting

NumPy uses broadcasting for virtually all primitive mathematical operations (and for some more complicated functions). Broadcasting avoids unnecessary multiplication of matrix repetition, and so improves performance.

```
>>> x = randn(1000,1)
>>> y = randn(1,1000)
>>> %timeit x*y
100 loops, best of 3: 8.77 ms per loop

>>> %timeit dot(x,ones((1,1000))) * dot(ones((1000,1)),y)
10 loops, best of 3: 36.7 ms per loop
```

Broadcasting is about 4 times as fast as manually expanding the arrays.

21.6 Avoid Allocating Memory

Memory allocation is expensive, especially if it occurs inside a for loop. It is often better to pre-allocate storage space for computed values, and also to reuse existing space.

21.7 Inline Frequent Function Calls

Function calls are fast but not completely free. Simple functions, especially inside loops, should be in-lined to avoid the cost of calling functions.

21.8 Think about data storage

Arrays are stored using row major format, which means that data is stored across a row first, and then down columns second. This means that in an m by n array, element i,j is stored next to elements $i,j+1$ and $i,j-1$ (except when j is the first (previous is $i-1,n$) or last element in a row (next is $i+1,1$)). Spatial location matters for performance, and it is much faster to access data which is physically adjacent than it is to access data adjacent in the array $(i+1,j)$, but not as the array is stored in the computer's memory. The simplest method to understand array storage is to use:

```
>>> x = arange(16.0)
>>> x.shape = 4,4
>>> x
array([[  0.,   1.,   2.,   3.],
       [  4.,   5.,   6.,   7.],
       [  8.,   9.,  10.,  11.],
       [ 12.,  13.,  14.,  15.]])
```

21.9 Cython

Cython is a powerful, but somewhat complex, solution for situations where pure NumPy cannot achieve performance targets. Unless you are very familiar with C, Cython should be considered a last resort. Cython translates Python code into C code, which can then be compiled into a Python extension. Using Cython on Linux is relatively painless, and only requires that the system compiler is installed in addition to Cython. To use Cython in Python x64, it is necessary to have the x64 version of Cython installed along with the Windows 7 and .NET 3.5 SDK – it must be this SDK and not a newer SDK – which ships with the Microsoft Optimizing Compiler version 15, which is what Python 2.7.2 x64 is compiled with.

The main idea behind Cython is to write standard Python, and then to add some special syntax and hints about the type of data used. The first example is a real world case which uses a generic recursion from a GARCH(P,Q) model to compute the conditional variance given parameters, data, and a backcast value. To begin, start with writing a pure Python function which is known to work.

```
def garch_recursion(parameters, data, sigma2, p, q, backcast):
    T = size(data,0)
    for i in xrange(T):
        sigma2[i] = parameters[0]
        for j in xrange(p):
            if (i-j)<0:
                sigma2[i] = parameters[1+j] * backcast
            else:
                sigma2[i] = parameters[1+j] * (data[i-j]*data[i-j])
        for j in xrange(q):
            if (i-j)<0:
                sigma2[i] = parameters[1+p+j] * backcast
            else:
                sigma2[i] = parameters[1+p+j] * sigma2[i-j]

    return sigma2
```

This example is fairly straight forward and only involves the (slow) recursive calculation of the conditional variance, not the other portions of the log-likelihood (which can be vectorized using NumPy). Applying Cython to an existing Python function requires a number of steps (for standard numeric code):

- Save the file with the extension pyx – for Python Extension.
- Use `cimport`, which is a special version of `import` for Cython, to import both `cython` and `numpy as np`.

- Declare types for *every* variable:
  - Scalars have standard C-types, and in almost all cases should be `double` (same as `float64` in NumPy, and `float` in Python), `int` (signed integer) or `uint` (unsigned integer).
  - NumPy arrays `np.ndarray[` *type* `,ndim=` *numdims* `]` where *type* is a Cython NumPy type, and should almost always be `np.float64_t` for numeric data, and *numdims* is the number of dimensions of the NumPy array, likely to be 1 or 2.
- Declare all arrays as `not None`.
- Ensure that all array access uses only single item access and not more complex slicing. For example is `x` is a 2-dimensional array, `x[i,j]` must be used and not `x[i,:]` or `x[:,j]`.

The "Cythonized" version of the variance recursion is presented below. All arrays are declared using `np.ndarray[np.floa` and so the inputs must all have 1 dimension (and 1 dimension only). The inputs `p` and `q` are declared to be integers, and `backcast` is declared to be a `double`. The three local variables `T`, `i` and `j` are all declared to be `int`s. Note that is is crucial that the variables used as iterators are declared as `int` (or other integer type, such as `uint` or `long`). The remainder of the function is *unchanged*.

```
import numpy as np
cimport numpy as np
cimport cython

@cython.boundscheck(False)
@cython.wraparound(False)
def garch_recursion(np.ndarray[np.float64_t, ndim=1] parameters not None,
                      np.ndarray[np.float64_t, ndim=1] data not None,
                      np.ndarray[np.float64_t, ndim=1] sigma2 not None,
                      int p,
                      int q,
                      double backcast):

    cdef int T = np.size(data,0)
    cdef int i, j

    for i in xrange(T):
        sigma2[i] = parameters[0]
        for j in xrange(p):
            if (i-j)<0:
                sigma2[i] = parameters[1+j] * backcast
            else:
                sigma2[i] = parameters[1+j] * (data[i-j]*data[i-j])
        for j in xrange(q):
            if (i-j)<0:
                sigma2[i] = parameters[1+p+j] * backcast
            else:
                sigma2[i] = parameters[1+p+j] * sigma2[i-j]

    return sigma2
```

Two additional lines were included in the Cython version of the function, `@cython.boundscheck(False)` and `@cython.wraparound(False)`. The first disables bounds checking which speeds up the final code, but is dangerous if the loop has the wrong ending point. The second rules out the use of negative indices, which is simple to verify and enforce. The `@cython` syntax means that these are only set for the next function and wouldn't affect any other code in the extension.

The next step is to code up a `setup.py` file which is used to build the actual extension. The code is located in a file named `garch_ext.pyx`, which will be the name of the extension. The setup code is standard, and is unlikely to require altering (aside from the extension and file name).

```
from distutils.core import setup
from distutils.extension import Extension
from Cython.Distutils import build_ext
import numpy

setup(
    cmdclass = {'build_ext': build_ext},
    ext_modules = [Extension("garch_ext", ["garch_ext.pyx"])],
    include_dirs = [numpy.get_include()]
)
```

The final step is to build the extension by running `python setup.py build_ext --inplace` from the terminal. This will produce `garch_ext.pyd` which contains the compiled code.

```
>>> parameters = array([.1,.1,.8])
>>> data = randn(10000)
>>> sigma2 = zeros(shape(data))
>>> p,q = 1,1
>>> backcast = 1.0
>>> %timeit -n 100 garch_recursion(parameters, data, sigma2, p, q, backcast)
100 loops, best of 3: 27.7 ms per loop

>>> import garch_ext
>>> %timeit garch_ext.garch_recursion(parameters, data, sigma2, p, q, backcast)
10000 loops, best of 3: 134 us per loop

>>> .0277/.000134
```

The Cythonized version is about 200 times faster than the standard Python version, and only required about 3 minutes to write (after the main Python function has been written).

The function `pydot` was similarly Cythonized. This Cython program demonstrates how arrays should be allocated within the function. Not that the Cython type for an array is `np.float64_t` which corresponds to the usual NumPy data type of `np.float64` (other `_t` types are available for different NumPy data types). Again, it only required a couple of minutes to Cythonize the original Python function.

```
import numpy as np
cimport numpy as np
cimport cython

@cython.boundscheck(False)
@cython.wraparound(False)
def pydot(np.ndarray[np.float64_t, ndim=2] a not None,
           np.ndarray[np.float64_t, ndim=2] b not None):
    cdef int M, N, P, Q
    M,N = np.shape(a)
    P,Q = np.shape(b)
    assert N==P
    cdef np.ndarray[np.float64_t, ndim=2] c = np.zeros((M,N), dtype=np.float64)
    for i in xrange(M):
        for j in xrange(Q):
            for k in xrange(N):
                c[i,j] = c[i,j] + a[i,k] * b[k,j]
    return c
```

The Cythonized function is about 340 times faster than straight Python, although it is still much slower than the native NumPy routing dot.

```
>>> x = randn(100,100)
>>> y = randn(100,100)
>>> %timeit pydot(x,y)
1 loops, best of 3: 1.29 s per loop

>>> import pydot as p
>>> %timeit p.pydot(x,y)
100 loops, best of 3: 3.76 ms per loop

>>> 1.29/.00379
340.3693931398417

>>> %timeit dot(x,y)
10000 loops, best of 3: 181 us per loop

>>> .00379/0.000181
20.939226519337016
```

The final example will produce a Cython version of the weighted average. Since the original Python code used slicing, which is removed and replaced with a second loop.

```
def super_slow_weighted_avg(x, w):
    T = x.shape[0]
    m = len(w)
    m12 = int(ceil(m/2))
    y = zeros(T)
    for i in xrange(len(x)-m+1):
        for j in xrange(m):
            y[i+m12] += x[i+j] * w[j]

    return y
```

This makes writing the Cython version simple.

```
import numpy as np
cimport numpy as np
cimport cython
```

```
@cython.boundscheck(False)
@cython.wraparound(False)
def cython_weighted_avg(np.ndarray[np.float64_t, ndim=1] x,
                        np.ndarray[np.float64_t, ndim=1] w):
    cdef int T, m, m12, i, j
    T = x.shape[0]
    m = len(w)
    m12 = int(np.ceil(float(m)/2))
    cdef np.ndarray[np.float64_t, ndim=1] y = np.zeros(T, dtype=np.float64)
    for i in xrange(T-m+1):
        for j in xrange(m):
            y[i+m12] += x[i+j] * w[j]

    return y
```

The Cython version can be compiled using a setup function in the same way that the GARCH recursion was compiled.

```
>>> %timeit super_slow_weighted_avg(x,w)
1 loops, best of 3: 195 ms per loop

>>> import cython_weighted_avg as c
>>> %timeit c.cython_weighted_avg(x,w)
1000 loops, best of 3: 688 us per loop
```

The gains are unsurprisingly large (about 300$\times$) – however, the Cython code is no faster than the sideways version. This shows that Cython is not a magic bullet and that good vectorized code, even with a small amount of looping, can be very fast.

21.10 Exercises

1. Write a Python function which will accept a $p+q+1$ vector of parameters, a T vector of data, and p and q (integers, AR and MA order, respectively) and recursively computes the the ARMA error beginning with observation $p+1$. If an MA index is negative it should be backcast to 0.

2. [Only for the brave] Convert the function written in the previous function to Cython, compile it, and compare the pure Python implementation to Cython.

Chapter 22

Examples

These examples are all actual econometric problems chosen to demonstrate the use of Python in an end-to-end manner, form importing data to presenting estimates. A reasonable familiarity with the underlying econometric models and methods is assumed so that the focus can be on the translation of mathematics to Python.

22.1 Estimating the Parameters of a GARCH Model

This example will highlight the steps needed to estimate the parameters of a GJR-GARCH(1,1,1) model with a constant mean. The volatility dynamics in a GJR-GARCH model are given by

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{j=1}^{o} \gamma_j r_{t-j}^2 I_{[r_{t-j}<0]} + \sum_{k=1}^{q} \beta_k \sigma_{t-k}^2.$$

Returns are assumed to be conditionally normal, $r_t|\mathcal{F}_{t-1} \sim N\left(\mu, \sigma_t^2\right)$, and parameters are estimated by maximum likelihood. To estimate the parameters, it is necessary to:

1. Produce some starting values
2. Estimate the parameters using (quasi) maximum likelihood
3. Compute standard errors using a "sandwich" covariance estimator (also known as the Bollerslev & Wooldridge (1992) covariance estimator)

The first task is to write the log-likelihood which can be used in an optimizer. The log-likelihood function will compute the recursion and the log-likelihood. It will also, optionally, return the T by 1 vector of individual log-likelihoods which are useful for numerically computing the scores.

First, I will discuss the imports needed for this. I prefer to use the import form `from` *module* `import` *func1*, *func2* for commonly used functions. This both saves typing and increases code readability. For functions which are used less, I use `import` *module* `as` *shortname*. This is a personal choice, and any combination is acceptable, although `from` *module* `import *` should be avoided.

```
import numpy as np
import matplotlib.pyplot as plt
from numpy import size, log, pi, sum, diff, array, zeros, diag, dot, mat, asarray, sqrt
```

```python
from numpy.linalg import inv
from scipy.optimize import fmin_slsqp
from matplotlib.mlab import csv2rec
```

The log-likelihood can be defined using the normal log-likelihood

$$\ln f\left(r_t|\mu,\sigma_t^2\right) = -\frac{1}{2}\left(\ln 2\pi + \ln\sigma_t^2 + \frac{\left(r_t-\mu\right)^2}{\sigma_t^2}\right),$$

which is negated in the code since the optimizers all minimize.

```python
def gjr_garch_likelihood(parameters, data, sigma2, out=None):
    ''' Returns likelihood for GJR-GARCH(1,1,1) model.'''
    mu = parameters[0]
    omega = parameters[1]
    alpha = parameters[2]
    gamma = parameters[3]
    beta = parameters[4]

    T = size(data,0)
    eps = data - mu
    # Data and sigma2 are T by 1 vectors
    for t in xrange(1,T):
        sigma2[t] = (omega + alpha * eps[t-1]**2
                     + gamma * eps[t-1]**2 * (eps[t-1]<0) + beta * sigma2[t-1])

    logliks = 0.5*(log(2*pi) + log(sigma2) + eps**2/sigma2)
    loglik = sum(logliks)

    if out is None:
        return loglik
    else:
        return loglik, logliks, copy(sigma2)
```

The keyword argument out has a default value of None, and is used to determine whether to return 1 output or 3. This is common practice since the optimizer requires a single output – the log-likelihood function value, but it is also useful to be able to output other useful quantities, such as $\left\{\sigma_t^2\right\}$.

The optimization is constrained so that $\alpha+\gamma/2+\beta\leq 1$, which constraint function.

```python
def gjr_constraint(parameters, data, sigma2, out=None):
    ''' Constraint that alpha+gamma/2+beta<=1'''

    alpha = parameters[2]
    gamma = parameters[3]
    beta = parameters[4]

    return array([1-alpha-gamma/2-beta])
```

The constraint function takes the same inputs as the negative of the log-likelihood function.

It is necessary to discuss one other function before proceeding with the main block of code. The asymptotic variance takes the "sandwich" form, which is commonly expressed as

$$\mathcal{J}^{-1}\mathcal{I}\mathcal{J}^{-1}$$

where J is the expected Hessian and $\mathcal{I}$ is the covariance of the scores. Both are numerically computed. The strategy for computing the Hessian is to use the definition that

$$\mathcal{J}_{ij} \approx \frac{f\left(\theta + e_i h_i + e_j h_j\right) - f\left(\theta + e_i h_i\right) - f\left(\theta + e_j h_j\right) + f\left(\theta\right)}{h_i h_j}$$

where h_i is a scalar "step size" and e_i is a vector of 0s except for element i, which is 1. A 2-sided version of this approximation, which takes both forward and backward steps and then averages, is below. For more on numerical derivatives, see Flannery et al. (1992).

```
def hessian_2sided(fun, theta, args):
    f = fun(theta, *args)
    h = 1e-5*np.abs(theta)
    thetah = theta + h
    h = thetah - theta
    K = size(theta,0)
    h = np.diag(h)

    fp = zeros(K)
    fm = zeros(K)
    for i in xrange(K):
        fp[i] = fun(theta+h[i], *args)
        fm[i] = fun(theta-h[i], *args)

    fpp = zeros((K,K))
    fmm = zeros((K,K))
    for i in xrange(K):
        for j in xrange(i,K):
            fpp[i,j] = fun(theta + h[i] + h[j],  *args)
            fpp[j,i] = fpp[i,j]
            fmm[i,j] = fun(theta - h[i] - h[j],  *args)
            fmm[j,i] = fmm[i,j]

    hh = (diag(h))
    hh = hh.reshape((K,1))
    hh = dot(hh,hh.T)

    H = zeros((K,K))
    for i in xrange(K):
        for j in xrange(i,K):
            H[i,j] = (fpp[i,j] - fp[i] - fp[j] + f
                       + f - fm[i] - fm[j] + fmm[i,j])/hh[i,j]/2
            H[j,i] = H[i,j]

    return H
```

Finally, the code that does the actual work can be written. The first block imports the data, flips it using a slicing operator, and computes 100 times returns. Scaling data can be useful to improve optimizer performance, since ideally estimated parameters should have similar magnitude (i.e. $\omega \approx .01$ and $\alpha \approx .05$)

```
# Import data
FTSEdata = csv2rec('FTSE_1984_2012.csv')
# Flip upside down
FTSEdata = FTSEdata[::-1]
# Compute returns
FTSEprice = FTSEdata['adj_close']
FTSEreturn = 100*diff(log(FTSEprice))
```

Good starting values are important. These are a good guess based on more than a decade of fitting models. An alternative is to attempt a crude grid search and use the best (smallest) value from the grid search.

```
# Starting values
startingVals = array([FTSEreturn.mean(),
                      FTSEreturn.var() * .01,
                      .03, .09, .90])
```

Bounds are used in estimation to ensure that all parameters are ≥ 0, and to set sensible upper bounds in the parameters. The vector `sigma2` is then initialized, and the arguments are placed in a tuple.

```
# Estimate parameters
finfo = np.finfo(np.float64)
bounds = [(-10*FTSEreturn.mean(), 10*FTSEreturn.mean()),
          (finfo.eps, 2*FTSEreturn.var() ),
          (0.0,1.0), (0.0,1.0), (0.0,1.0)]

T = size(FTSEreturn,0)
sigma2 = np.repeat(FTSEreturn.var(),T)
args = (FTSEreturn, sigma2)
estimates = fmin_slsqp(gjr_garch_likelihood, startingVals, \
           f_ieqcons=gjr_constraint, bounds = bounds, \
           args = args)
```

The optimized log-likelihood and the time series of variances are computed by calling the objective using the keyword argument `out=True` (anything that is not `None` would work).

```
loglik, logliks, sigma2final = gjr_garch_likelihood(estimates, \
                               FTSEreturn, sigma2, out=True)
```

Next, the numerical scores and the covariance of the scores are computed. These exploit the definition of a derivative, so that for a scalar function,

$$\frac{\partial f(\theta)}{\partial \theta_i} \approx \frac{f(\theta + e_i h_i) - f(\theta)}{h_i}.$$

The covariance is computed as the outer product of the scores since the scores should have mean 0 when evaluated at the solution to the optimization problem.

```
step = 1e-5 * estimates
scores = np.zeros((T,5))
for i in xrange(5):
    h = step[i]
    delta = np.zeros(5)
    delta[i] = h

    loglik, logliksplus, sigma2 = gjr_garch_likelihood(estimates + delta, \
                                 FTSEreturn, sigma2, out=True)
    loglik, logliksminus, sigma2 = gjr_garch_likelihood(estimates - delta, \
                                 FTSEreturn, sigma2, out=True)
    scores[:,i] = (logliksplus - logliksminus)/(2*h)

B = np.dot(scores.T,scores)/T
```

The final block of the numerical code calls `hessian_2sided` to estimate the Hessian, and finally computes the asymptotic covariance.

```
A = hessian_2sided(gjr_garch_likelihood, estimates, args)
A = A/T
Ainv = mat(inv(A))
vcv = Ainv*mat(B)*Ainv/T
vcv = asarray(vcv)
```

The final steps are to pretty print the results and to produce a plot of the conditional variances,

```
output = np.vstack((estimates,sqrt(diag(vcv)),estimates/sqrt(diag(vcv)))).T
print('Parameter   Estimate       Std. Err.      T-stat')
param = ['mu','omega','alpha','gamma','beta']
for i in xrange(len(param)):
    print('{0:<11} {1:>0.6f}        {2:0.6f}    {3: 0.5f}'.format(param[i],output[i,0],output[i,1],outpu
```

This final code block produce a plot of the annualized conditional standard deviations.

```
# Produce a plot
dates = FTSEdata['date'][1:]
fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(dates,np.sqrt(252*sigma2))
fig.autofmt_xdate()
ax.set_ylabel('Volatility')
ax.set_title('FTSE Volatility (GJR GARCH(1,1,1))')
plt.show()
```

22.2 Estimating the Risk Premia using Fama-MacBeth Regressions

This example highlights how to implement a Fama-MacBeth 2-stage regression to estimate factor risk premia, make inference on the risk premia, and test whether a linear factor model can explain a cross-section of portfolio returns. This example closely follows Cochrane (2001) (See also Jagannathan et al. (2010)). As in the previous example, the first segment contains the imports.

```
from __future__ import print_function
from numpy import mat, dot, cov, mean, hstack, multiply,sqrt,diag, genfromtxt, \
    squeeze, ones, array
from numpy.linalg import lstsq, inv
from scipy.stats import chi2
```

Next, the data are imported. I formatted the data downloaded from Ken French's website into an easy-to-import CSV which can be read by `genfromtxt`. The data is split using slicing, and the dimensions are determined using `shape`. Finally the risk free rate is forces to have 2 dimensions so that it will be broadcastable with the portfolio returns in order to construct the excess returns to the Size and Value-weighted portfolios. Fama-MacBeth makes heavy use of matrix algebra, and so `mat` is used in the import statement to ensure the imported data is a matrix. Since all other series are slices of a matrix, they are also matrices.

```
# Import data
data = mat(genfromtxt('famafrench.csv',delimiter=','))
# Split using slices
data = data[1:]
dates = data[:,0]
factors = data[:,1:4]
riskfree = data[:,4]
portfolios = data[:,5:]
# Shape information
T,K = factors.shape
T,N = portfolios.shape
# Reshape rf and compute excess returns
riskfree.shape = T,1
excessReturns = portfolios - riskfree
```

The next block does 2 things:

1. Compute the time-series βs. This is done be regressing the full array of excess returns on the factors (augmented with a constant) using `lstsq`.

2. Compute the risk premia using a cross-sectional regression of average excess returns on the estimates βs. This is a standard regression where the step 1 β estimates are used as regressors, and the dependent variable is the average excess return.

```
# Time series regressions
X = hstack((ones((T,1)),factors))
out = lstsq(X,excessReturns)
alpha = out[0][0]
beta = out[0][1:]
avgExcessReturns = mean(excessReturns,0)
# Cross-section regression
out = lstsq(beta.T,avgExcessReturns.T)
lam = out[0]
```

The asymptotic variance requires computing the covariance of the demeaned returns and the weighted pricing errors. The problem is formulated as a 2-step GMM estimation where the moment conditions are

$$g_t(\theta) = \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{1t} f_t \\ \epsilon_{2t} \\ \epsilon_{2t} f_t \\ \vdots \\ \epsilon_{Nt} \\ \epsilon_{Nt} f_t \\ \beta u_t \end{bmatrix}$$

where $\epsilon_{it} = r^e_{it} - \alpha_i - \beta_i' f_t$, β_i is a K by 1 vector of factor loadings, f_t is a K by 1 set of factors, $\beta = \left[\beta_1\, \beta_2 \ldots \beta_N\right]$ is a K by N matrix of all factor loadings, $u_t = r^e_t - \beta'\lambda$ are the N by 1 vector of pricing errors and λ is a K by 1 vector of risk premia. $\theta = \left[\alpha_1\, \beta_1'\, \alpha_2\, \beta_2'\, \ldots\, \alpha_N\, \beta_N'\, \lambda'\right]'$To make inference on this problem, the derivative of the moments with respect to the parameters, $\partial g_t(\theta)/\partial\theta'$ is needed. With some work, the estimator of this matrix can be seen to be

$$G = E\left[\frac{\partial g_t(\theta)}{\partial \theta'}\right] = \begin{bmatrix} -I_n \otimes \Sigma_X & 0 \\ G_{21} & -\beta\beta' \end{bmatrix}.$$

where $X_t = \left[1\; f_t'\right]'$ and $\Sigma_X = E\left[X_t X_t'\right]$. G_{21} is a matrix with the structure

$$G_{21} = \left[G_{21,1}\, G_{21,2}\, \ldots G_{21,N}\right]$$

where

$$G_{21,i} = \begin{bmatrix} 0_{K,1} & \operatorname{diag}\left(E\left[u_i\right] - \beta_i \odot \lambda\right) \end{bmatrix}$$

and where $E\left[u_i\right]$ is the expected pricing error. In estimation, all expectations are replaced with their sample analogues.

```python
# Moment conditions
X = hstack((ones((T,1)),factors))
p = vstack((alpha,beta))
epsilon = excessReturns - X*p
moments1 = kron(epsilon,ones((1,K+1)))
moments1 = multiply(moments1, kron(ones((1,N)),X))
u = excessReturns - lam.T*beta
moments2 = u*beta.T
# Score covariance
S = mat(cov(hstack((moments1,moments2)).T))
# Jacobian
G = mat(zeros((N*K+N+K,N*K+N+K)))
SigmaX = X.T*X/T
G[:N*K+N,:N*K+N] = kron(eye(N),SigmaX)
G[N*K+N:,N*K+N:] = -beta*beta.T
for i in xrange(N):
    temp = zeros((K,K+1))
    values = mean(u[:,i])-multiply(beta[:,i],lam)
    temp[:,1:] = diag(values.A1)
    G[N*K+N:,i*(K+1):(i+1)*(K+1)] = temp

vcv = inv(G.T)*S*inv(G)/T
```

The J test examines whether the average pricing errors, $\hat{\alpha}$, are zero. The J statistic has an asymptotic χ^2_N distribution, and the model is badly rejected.

```
vcvAlpha = vcv[0:N*K+N:4,0:N*K+N:4]
J = alpha*inv(vcvAlpha)*alpha.T
J = J[0,0]
Jpval = 1 - chi2(25).cdf(J)
```

The final block using formatted output to present all of the results in a readable manner.

```
riskPremia = lam
vcvLam = vcv[N*K+N:,N*K+N:]
annualizedRP = 12*riskPremia
arp = list(squeeze(annualizedRP.A))
arpSE = list(sqrt(12*diag(vcvLam)))
print('        Annualized Risk Premia')
print('           Market       SMB        HML')
print('--------------------------------------')
print('Premia     {0:0.4f}    {1:0.4f}     {2:0.4f}'.format(arp[0],arp[1],arp[2]))
print('Std. Err.  {0:0.4f}    {1:0.4f}     {2:0.4f}'.format(arpSE[0],arpSE[1],arpSE[2]))
print('\n\n')

print('J-test:   {:0.4f}'.format(J))
print('P-value:   {:0.4f}'.format(Jpval))

i=0
betaSE = []
for j in xrange(5):
    for k in xrange(5):
        a = alpha[0,i]
        b = beta[:,i].A1
        variances = diag(vcv[(K+1)*i:(K+1)*(i+1),(K+1)*i:(K+1)*(i+1)])
        betaSE.append(sqrt(variances))
        s = sqrt(variances)
        c = hstack((a,b))
        t = c/s
        print('Size: {:}, Value:{:}   Alpha   Beta(VWM)   Beta(SMB)   Beta(HML)'.format(j+1,k+1))
        print('Coefficients: {:>10,.4f}  {:>10,.4f}  {:>10,.4f}  {:>10,.4f}'.format(a,b[0],b[1],b[2]))
        print('Std Err.      {:>10,.4f}  {:>10,.4f}  {:>10,.4f}  {:>10,.4f}'.format(s[0],s[1],s[2],s[3]
        print('T-stat        {:>10,.4f}  {:>10,.4f}  {:>10,.4f}  {:>10,.4f}'.format(t[0],t[1],t[2],t[3]
        i +=1
```

The final block converts the standard errors of β to be an array and saves the data.

```
betaSE = array(betaSE)
savez_compressed('Fama-MacBeth_results',alpha=alpha, \
beta=beta, betaSE=betaSE, arpSE=arpSE, arp=arp,J=J, Jpval=Jpval)
```

22.3 Estimating the Risk Premia using GMM

The final numeric example estimates examines the same problem, only using GMM rather than 2-stage regression. The first block imports relevant modules and functions. GMM requires non-linear optimization, and fmin_bfgs will be used.

```
from __future__ import print_function
from numpy import hstack, ones, array, mat, tile, dot, reshape, squeeze, eye
from numpy.linalg import lstsq, inv
from scipy.linalg import kron
from scipy.optimize import fmin_bfgs
import numpy as np
```

Before defining the problem, the next code block contains a *callback* function which is called after each iteration of the minimizer. This function will be used to display information about the progress of the optimizer: the current function value, the iteration and the number of function calls. Because the callback only gets 1 input, the current value of the parameters used by the optimizer, it is necessary to use global variables to pass information between functions. Three variables have been declared as global, iteration, lastValue and functionCount. iteration is updated only by the callback function since it should report the number of completed iterations of the optimizer, and the callback is called once per iteration. lastValue is updated each time the main GMM objective function is called, and functionCount is incremented by 1 each time the main GMM objective is called.

```
global iteration, lastValue, functionCount
iteration = 0
lastValue = 0
functionCount = 0

def iter_print(params):
    global iteration, lastValue, functionCount
    iteration += 1
    print('Func value: {0:}, Iteration: {1:}, Function Count: {2:}'.format(lastValue, iteration, functic
```

The GMM objective takes the parameters, portfolio returns, factor returns and the weighting matrix and computes the moments, average moments and the objective value. The moments used can be described as

$$\left(r_{it}^2-\beta_i f_t\right)f_t \quad \forall i=1,\ldots N$$

and

$$r_{it}-\beta_i\lambda \quad \forall i=1,\ldots N.$$

```
def gmm_objective(params, pRets, fRets, Winv, out=False):
    global lastValue, functionCount
    T,N = pRets.shape
    T,K = fRets.shape
    beta = squeeze(array(params[:(N*K)]))
    lam = squeeze(array(params[(N*K):]))
    beta = reshape(beta,(N,K))
    lam = reshape(lam,(K,1))
    betalam = dot(beta,lam)
    expectedRet = dot(factors,beta.T)
    e = pRets - expectedRet
    instr = tile(fRets,N)
    moments1  = kron(e,ones((1,K)))
    moments1 = moments1 * instr
    moments2 = pRets - betalam.T
    moments = hstack((moments1,moments2))

    avgMoment = moments.mean(axis=0)

    J = T * mat(avgMoment)*mat(Winv)*mat(avgMoment).T
    J = J[0,0]
    lastValue = J
    functionCount += 1
    if not out:
        return J
    else:
        return J, moments
```

The final function needed is the Jacobian of the moment conditions. Mathematically it is simply to express the Jacobian using $\otimes$(Kronecker product). This code is so literal that it is simple to reverse engineer the mathematical formulas used to implement this estimator.

$$\hat{G} = \begin{bmatrix} I_N \otimes \Sigma_F & 0 \\ I_N \otimes \lambda & -\beta \end{bmatrix}$$

```
def gmm_G(params, pRets, fRets):
    T,N = pRets.shape
    T,K = fRets.shape
    beta = squeeze(array(params[:(N*K)]))
    lam = squeeze(array(params[(N*K):]))
    beta = reshape(beta,(N,K))
    lam = reshape(lam,(K,1))
    G = np.zeros((N*K+K,N*K+N))
    ffp = dot(fRets.T,fRets)/T
    G[:(N*K),:(N*K)]=kron(eye(N),ffp)
    G[:(N*K),(N*K):] = kron(eye(N),-lam)
    G[(N*K):,(N*K):] = -beta.T

    return G
```

The data import step is virtually identical to that in the previous example – although it shows some alternative functions to accomplish the same tasks. Note that only every other portfolio is used to speed up the GMM optimization.

```
data = np.genfromtxt('famafrench.csv',delimiter=',')
data = data[1:]
dates = data[:,0]
factors = data[:,1:4]
riskfree = data[:,4]
```

```
portfolios = data[:,5:]
T,N = portfolios.shape
portfolios = portfolios[:,np.arange(0,N,2)]
T,N = portfolios.shape
excessRet = portfolios - np.reshape(riskfree,(T,1))
K = np.size(factors,1)
```

Starting values are important in any optimization problem. The GMM problem is closely related to Fama-MacBeth regression, and so it is sensible to use the output from a FMB regression.

```
betas = []
augFactors = hstack((ones((T,1)),factors))
for i in xrange(N):
    out = lstsq(augFactors,excessRet[:,i])
    betas.append(out[0][1:])

avgReturn = excessRet.mean(axis=0)
avgReturn.shape = N,1
betas = array(betas)
out = lstsq(betas,avgReturn)
riskPremia = out[0]
```

The GMM objective can be minimized using an identity matrix as covariance of the moment conditions along with the starting values computed using a Fama-MacBeth regression. The keyword argument `callback` is used to pass the callback function to the optimizer.

```
riskPremia.shape = 3
startingVals = np.concatenate((betas.flatten(),riskPremia))

Winv = np.eye(N*(K+1))
args = (excessRet, factors, Winv)
iteration = 0
functionCount = 0
step1opt = fmin_bfgs(gmm_objective, startingVals, args=args, callback=iter_print)
```

Once the initial estimates have been computed, these can be used to estimate the covariance of the moment conditions, which is then used to estimate the optimal weighting matrix. The keyword input `out` is used to return the moments in addition to the objective function value. Note that the variables `iteration` and `lastValue` which are used in the callback are reset to 0 so that the count will be accurate.

```
out = gmm_objective(step1opt, excessRet, factors, Winv, out=True)
S = np.cov(out[1].T)
Winv2 = inv(S)
args = (excessRet, factors, Winv2)

iteration = 0
functionCount = 0
step2opt = fmin_bfgs(gmm_objective, step1opt, args=args, callback=iter_print)
```

The final block computes estimates the asymptotic covariance of the parameters using the usual efficient GMM covariance estimator, assuming that the moments are a martingale.

```
out = gmm_objective(step2opt, excessRet, factors, Winv2, out=True)
```

```
G = gmm_G(step2opt, excessRet, factors)
S = mat(np.cov(out[1].T))
vcv = inv(G*inv(S)*G.T)/T
```

22.4 Outputting LaTeX

Automatically outputting results to LaTeX or another format can eliminate export errors and avoid tedious work. This example show how two of the tables in the previous Fama-MacBeth example can be exported to a LaTeX document, and how, if desired, the document can be compiled to a PDF. The first code block contains the imports needed and defines a flag which determines whether the output LaTeX should be compiled.

```
# imports
from __future__ import print_function
import numpy as np
import subprocess

# Flag to compile output tables
compileLatex = True
```

The next code block loads the npz file created using the output from the Fama-MacBeth example. It restores the variables with the same name in the main program.

```
# Load variables
f = np.load('Fama-MacBeth_results.npz')
data = f.items()
# Restore the data
for key in f.keys():
    exec(key + " = f['" + key + "']")
f.close()
```

The document will be help in a list. The first few lines contain the required header for a LaTeX document, including some packages used to improve table display and to select a custom font.

```
# List to hold table
latex = []
# Initializd LaTeX document
latex.append(r'\documentclass[a4paper]{article}')
latex.append(r'\usepackage{amsmath}')
latex.append(r'\usepackage{booktabs}')
latex.append(r'\usepackage[adobe-utopia]{mathdesign}')
latex.append(r'\usepackage[T1]{fontenc}')
latex.append(r'\begin{document}')
```

Table 1 will be stored in its own list, and then `extend` will be used to add it to the main list. Building this table is simple string manipulation and use of `format`.

```
# Table 1
table1 = []
table1.append(r'\begin{center}')
table1.append(r'\begin{tabular}{lrrr} \toprule')
# Header
```

```python
colNames = [r'VWM$^e$','SMB','HML']
header = ''
for cName in colNames:
    header += ' & ' + cName

header += r'\\ \cmidrule{2-4}'
table1.append(header)
# Main row
row = ''
for a,se in zip(arp,arpSE):
    row += r' & $\underset{{({0:0.3f})}}{{{1:0.3f}}}$'.format(se,a)
table1.append(row)
# Blank row
row = r'\\'
table1.append(row)
# J-stat row
row = r'J-stat: $\underset{{({0:0.3f})}}{{{1:0.1f}}}$ \\'.format(float(Jpval),float(J))
table1.append(row)
table1.append(r'\bottomrule \end{tabular}')
table1.append(r'\end{center}')
# Extend latex with table 1
latex.extend(table1)
latex.append(r'\newpage')
```

Table 2 is a bit more complex, and uses loops to iterate over the rows of the arrays containing the βs and their standard errors.

```python
# Format information for table 2
sizes = ['S','2','3','4','B']
values = ['L','2','3','4','H']
# Table 2 has the same header as table 1, copy with a slice
table2 = table1[:3]
m = 0
for i in xrange(len(sizes)):
    for j in xrange(len(values)):
        row = 'Size: {:}, Value: {:} '.format(sizes[i],values[j])
        b = beta[:,m]
        s = betaSE[m,1:]
        for k in xrange(len(b)):
            row += r' & $\underset{{({0:0.3f})}}{{{1: .3f}}}$'.format(s[k],b[k])
        row += r'\\ '
        table2.append(row)
        m += 1
    if i<(len(sizes)-1):
        table2.append(r'\cmidrule{2-4}')

table2.append(r'\bottomrule \end{tabular}')
table2.append(r'\end{center}')
# Extend with table 2
latex.extend(table2)
```

The penultimate block finished the document, and uses `write` to write the lines to the LaTeX file. `write` does not break lines, so the new line character is added to each (`\n`).

```
# Finish document
latex.append(r'\end{document}')
# Write to table
fid = file('latex.tex','w')
for line in latex:
    fid.write(line + '\n')
fid.close()
```

Finally, if the flag is set, `subprocess` is used to compile the LaTeX. This assumes that `pdflatex` is on the system path.

```
# Compile if needed
if compileLatex:
    exitStatus = subprocess.call(r'pdflatex latex.tex', shell=True)
```

Chapter 23

Quick Reference

23.1 Built-ins

`import`

`import` is used to import modules for use in a program.

`from`

`from` is used to import selected functions from a module in a program.

`def`

`def` is used to denote the beginning of a function.

`return`

`return` is used return a value from a function.

`xrange`

`xrange` is an iterator commonly used in `for` loops.

`tuple`

A `tuple` is an immutable collection of other types, and is initialized using parentheses, e.g. `(a,)` for a single element tuple or `(a,b)` for a 2-element tuple.

`list`

A `list` is a mutable collection of other types, and is initialized using square brackets, e.g. `[a]` or `[a,b]`.

`dict`

`dict` initialized a dictionary, which is a list of named values where the names are unique.

`set`, `frozenset`

`set` initializes a set which is a unique, mutable collection. `frozenset` is an immutable counterpart.

`for`

`for` being a for loop, and should have the structure `for` *var* `in` *iterable*: .

`while`

`while` begins a while loop, and should have the structure `while` *logical* :

`break`

`break` terminates a loop prematurely.

`continue`

`continue` continues a loop without processing any code below the continue statement.

`try`

`try` begin a block which can handle code which may not succeed. It must be followed by an `except` statement.

`except`

`except` catches errors generated in a `try` statement.

`if`

`if` begins an `if` . . . `elif` . . . `else`, and should have the structure `if` *logical* :

`elif`

`elif` stands for else if, and can be used after an `if` statement to refine the expression. It should have the structure `elif` *logical* :

`else`

`else` finalizes an `if` block and executes if no previous path was taken.

`print`

`print` outputs information to the console. If used with `from __future__ import print_function`, `print` behaves like a function.

`file`

`file` opens a file for low-level file reading and writing.

23.1.1 `file` Methods

File methods operate on a file object, which is initialized using `file`. For example, `f = file('text.txt','r')` opens the file text.txt for reading, and `close` is used as `f.close()`.

`close`

`close` closes an open file handle, and flushes any unwritten data to disk.

`flush`

`flush` flushes any unwritten data to disk without closing the file.

`read`

`read` reads data from an open file.

`readline`

`readline` reads a single line from an open file.

`readlines`

`readlines` reads one or more lines from an open file.

`write`

`write` writes a single line to a file without appending the new line character.

`writelines`

`writelines` writes the contents of an iterable (e.g. a list) to a file without appending the new line character to each line.

23.1.2 String (`str`) Methods

String methods operate on strings. For example, `strip` can be used on a string `x = ' abc '` as `x.strip()`, or can be directly used as `' abc '.strip()`.

`split`

`split` splits a string at every occurrence of another string, left to right.

`rsplit`

`rsplit` splits a string at every occurrence of another string, right to left.

`join`

`join` combines an iterable of strings using a given string to combine.

`strip`

`strip` removes leading and trailing whitespace from a string.

`lstrip`

`lstrip` removes leading whitespace from a string.

`rstrip`

`rstrip` removes trailing whitespace from a string.

`find`

`find` returns the index of the first occurrence of a substring. `-1` is returned if not found.

`rfind`

`rfind` returns the index of the first occurrence of a substring, scanning from the right. `-1` is returned if not found.

`index`

`index` behaves like `find`, but raises an error when not found.

`count`

`count` counts the number of occurrences of a substring.

`upper`

`upper` converts a string to upper case.

`lower`

`lower` coverts a string to lower case.

`ljust`

`ljust` right pads a string with whitespace or a specified character up to a given width.

`rjust`

`rjust` left pads a string with whitespace or a specified character up to a given width.

`center`

`center` left and right pads a string with whitespace or a specified character up to a given width.

`replace`

`replace` returns a copy of a string with all occurrences of a substring replaced with an alternative substring.

`format`

`format` formats and inserts formattable objects (e.g. numbers) into a string.

23.1.3 Operating System (os)

`os.system`

`system` executes (external) system commands.

`os.getcwd`

`getcwd` returns the current working directory.

`os.chdir`

`chdir` changes the current working directory.

`os.mkdir`

`mkdir` creates a new directory, but requires all higher level directories to exist.

`os.makedirs`

`makedirs` creates a directory at any level, and creates higher level directories if needed.

`os.rmdir`

`rmdir` removes an empty directory.

`os.listdir`

`listdir` returns the contents of a directory. See `glob.glob` for a more useful form.

`os.path.isfile`

`path.isfile` determines whether a string corresponds to a file.

`os.path.isdir`

`path.isdir` determines whether a string corresponds to a directory.

23.1.4 Shell Utilities (`shutil`)

`shutil.copy`

`copy` copies a files using either a file name to use for the copy or a directory to use as a destination, in which case the current file name is used.

`shutil.copyfile`

`copyfile` copies a file using a file name.

`shutil.copy2`

`copy2` is identical to `copy` only that (some) file meta-data is also copied.

`shutil.copytree`

`copytree` copies an entire directory tree to a destination that must not exist.

`shutil.move`

`move` moves a directory tree to a destination which must not exist.

`shutil.make_archive`

`make_archive` creates zip, gztar and bztar archives.

`shutil.rmtree`

`rmtree` recursively removes a directory tree.

23.1.5 Regular Expressions (`re`)

`re.findall`

`findall` returns all occurrences of a regular expression in a string as a list.

`re.split`

`split` splits a string on occurrences of a regular expression.

`re.sub`

`sub` substitutes a string of each occurrence of a regular expression.

`re.finditer`

`finditer` works similarly to `findall`, only returning an iterable object rather than a list.

`re.compile`

`compile` compiles a regular expression for repeated use.

23.1.6 Dates and Times (`datetime`)

`datetime.datetime`

`datetime` initializes a date-time object.

`datetime.date`

`date` initializes a date object.

`datetime.time`

`time` initializes a time object.

`datetime.timedelta`

`timedelta` represents the difference between two `datetime`s.

`datetime.datetime.replace`

`replace` replaces fields in a date-time object. `replace` is a method of a date-time, date or time object.

`datetime.datetime.combine`

`combine` combines a date object and a time object and returns a date-time object. `combine` is a method of a date-time, date or time object.

23.1.7 Other

These function all are contained in other modules, as listed to the left of the dot.

`glob.glob`

`glob.glob` returns a directory listing allowing for wildcards.

`subprocess.call`

`subprocess.call` can be used to run external commands.

`textwrap.wrap`

`textwrap.wrap` wraps a long block of text at a fixed width.

23.2 NumPy (`numpy`)

The functions listed in this section are all provided by NumPy. When a function is listed using only the function name, this function appears in the NumPy module, and so can be accessed as `numpy.`*function*, assuming that NumPy was imported using `import numpy`. When a function name contains a dot, for example `linalg.eig`, then the function `eig` is in the `linalg` module of `numpy`, and so the function is accessed as `numpy.linalg.eig`.

23.2.1 Core NumPy Types

`dtype`

`dtype` is used for constructing data types for use in arrays.

`array`

`array` is the primary method for constructing arrays from iterables (e.g. list or matrix). Variables created using `array` have type `numpy.ndarray`.

`matrix`

`matrix` constructs a matrix from an iterable (e.g. list or array) that directly supports matrix mathematics. Variables created using `matrix` have type `numpy.matrixlib.defmatrix.matrix`, which is a subclass of `numpy.ndarray`. Since `matrix` is a subclass of `numpy.ndarray`, it inherits the methods of `numpy.ndarray`.

23.2.2 `ndarray`

`ndarray` is the core array data type provided by NumPy. Both `array` (`numpy.ndarray`) and `matrix` (`numpy.matrixlib.defma`
offer a large number of attributes and methods. Attributes are accessed directly from an array, e.g. `x.dtype`, and methods are calls to functions which operate on an array, e.g. `x.sum()`.

Attributes

`T`

`T` returns the transpose of an array.

`dtype`

`dtype` returns the data type of an array.

`flat`

`flat` returns a 1-dimensional iterator for an array.

`imag`

`imag` returns the imaginary portion of an array.

`real`

`real` returns the real portion of an array.

`size`

`size` returns the total number of elements of an array.

`ndim`

`ndim` returns the number of dimensions of an array.

`shape`

`shape` returns the shape of an array as a tuple with `ndim` elements. `shape` can also be used to set the shape using a tuple, e.g. `x.shape=(10,5)` as long as the number of elements does not change.

Methods

`all`

`all` returns `True` if all elements evaluate to `True` (i.e. not `False`, `None` or `0`). `axis` can be used to compute along a particular axis.

`any`

`any` returns `True` if any element evaluates to `True`. `axis` can be used to compute along a particular axis.

`argmax`

`argmax` returns the index of the maximum of an array. `axis` can be used to compute along a particular axis.

`argmin`

`argmin` returns the index of the minimum of an array. `axis` can be used to compute along a particular axis.

`argsort`

`argsort` returns the indices needed to sort an array. `axis` can be used to compute along a particular axis.

`astype`

`astype` allows an array to be viewed as another type (e.g. `matrix` or `recarray`) and copies the underlying.

`conj, conjugate`

`conj` and `conjugate` both return the complex-conjugate element-by-element.

`copy`

`copy` returns a copy of an array.

cumprod

cumprod returns the cumulative product of an array. axis can be used to compute along a particular axis.

cumsum

cumsum return the cumulative sum of an array. axis can be used to compute along a particular axis.

dot

dot computes the dot-product (standard matrix multiplication) or an array with another array.

flatten

flatten returns a copy of an array flattened into a 1-dimensional array.

max

max returns the maximum value of an array. axis can be used to compute along a particular axis.

mean

mean returns the average of an array. axis can be used to compute along a particular axis.

min

min returns the minimum of an array. axis can be used to compute along a particular axis.

nonzero

nonzero returns the indices of the non-zero elements of an array.

prod

prod computes the produce of all elements of an array. axis can be used to compute along a particular axis.

ravel

ravel returns a flattened view of an array without copying data.

repeat

repeat returns an array repeated, element-by-element.

reshape

reshape returns returns an array with a different shape. The number of elements must not change.

`resize`

`resize` changes the size shape of an array in-place. If the new size is larger then new entries are 0 filled. If the new size is smaller, then the elements selected are ordered according too their order in `ravel`.

`round`

`round` returns an array with each element rounded to a provided number of decimals.

`sort`

`sort` sorts an array (in-place). `axis` can be used to compute along a particular axis.

`squeeze`

`squeeze` returns an array with any singleton dimensions removed.

`std`

`std` returns the standard deviation of an array. `axis` can be used to compute along a particular axis.

`sum`

`sum` returns the sum of an array. `axis` can be used to compute along a particular axis.

`tolist`

`tolist` returns a list of an array. If the array has more than 1 dimension, the list will be nested.

`trace`

`trace` returns the sum of the diagonal elements.

`transpose`

`transpose` returns a view of the array transposed. `x.transpose()` is the same as `x.T`.

`var`

`var` returns the variance of an array. `axis` can be used to compute along a particular axis.

`view`

`view` returns a view of an array without copying data.

Methods and Attributes as functions

Many of the `ndarray` methods can be called as functions and behave identically, aside from taking the array as the first input. For example, `sum(x, 1)` is identical to `x.sum(1)`, and `x.dot(y)` is identical to `dot(x,y)`. The following list of functions are identical to their method: `all`, `any`, `argmax`, `argmin`, `argsort`, `conj`, `cumprod`, `cumsum`, `diagonal`, `dot`, `imag`, `real`, `mean`, `std`, `var`, `prod`, `ravel`, `repeat`, `squeeze`, `reshape`, `std`, `var`, `trace`, `ndim`, and `squeeze`.

Functions with different behavior

`round`

`around` is the function name for `round`. `around` is preferred as a function name since `round` is also the name of a built-in function which does not operate on arrays.

`resize`

Using `resize` as a function returns the resized array. Using `resize` as a method performs the resizing in-place.

`sort`

Using `sort` as a function returns the sorted array. Using `sort` as a method performs the sorting in-place.

`size`

Using `size` as a function can take an additional argument `axis` specifying the axis to use. When used without an argument `size(x)` is identical to `x.size`.

`shape`

Using `shape` as a function only returns the shape of an array. The attribute use of `shape` also allows the shape to be directly set, e.g. `x.shape=3,3`.

`max`

`amax` is the function name for the method `max`. `amax` is preferred as a function name since `max` is also the name of a built-in function which does not operate on arrays.

`min`

`amin` is the function name for the method `min`. `amin` is preferred as a function name since `min` is also the name of a built-in function which does not operate on arrays.

23.2.3 `matrix`

`matrix` is a derived class of `ndarray` and so inherits its attributes and methods. Members of the `matrix` class have additional attributes.

Attributes

`I`

`I` returns the inverse of the matrix. This command is equivalent to `inv(x)` for an invertible matrix `x`.

`A`

`A` returns a view of the matrix as a 2-dimensional array.

`A1`

`A1` returns a view of the matrix as a flattened array.

`H`

`H` returns the Hermetian (conjugate-transpose) of a matrix. For real matrices, `x.H` and `x.T` are identical.

Attributes

23.2.4 Array Construction and Manipulation

`linspace`

`linspace` creates an n-element linearly spaced vector between a lower and upper bound.

`logspace`

`logspace` creates a logarithmically spaced (base-10) vector between a lower and upper (log-10) bound.

`arange`

`arange` creates an equally spaced vector between a lower and upper bound using a fixed step size.

`ones`

`ones` creates an array of 1s.

`zeros`

`zeros` creates an array of 0s.

`empty`

`empty` creates an array without initializing the values.

`eye`

eye creates an identity array.

`identity`

`identify` creates an identity array.

`meshgrid`

`meshgrid` creates 2-dimensional arrays from 2 1-dimensional arrays which contain all combinations of the original arrays.

`tile`

`tile` block repeats an array.

`broadcast_arrays`

`broadcast_arrays` produces the broadcasted version of 2 broadcastable arrays.

`vstack`

`vstack` vertically stacks 2 or more size compatible arrays.

`hstack`

`hstack` horizontally stacks 2 or more size compatible arrays.

`vsplit`

`vsplit` splits an array into 2 or more arrays vertically.

`hsplit`

`hsplit` splits an array into 2 or more arrays horizontally.

`split`

`split` splits an array into 2 or more arrays along an arbitrary axis.

`concatenate`

`concetenate` combines 2 or more arrays along an arbitrary axis.

`delete`

`delete` deletes elements from an array. `axis` can be used to delete along a particular axis.

`flipud`

`flipud` flips an array top-to-bottom.

fliplr

fliplrud flips an array left-to-right.

diag

diag returns the diagonal from a 2-dimensional matrix, or returns a diagonal 2-dimensional matrix when used with a 1-dimensional input.

triu

triu returns the upper triangular array from an array.

tril

tril returns the lower triangular array from an array.

23.2.5 Array Functions

kron

kron returns the Kronecker product of 2 arrays.

trace

trace returns the sum of the diagonal elements of an array.

diff

diff returns the 1st difference of an array. An optional second input allows for higher order differencing. axis can be used to compute the difference along a particular axis.

23.2.6 Input/Output

loadtxt

loadtxt loads a rectangular array from a text file. No missing values are allowed. Automatically decompresses gzipped or bzipped text files.

genfromtxt

genfromtxt loads text from a data file and can accept missing values.

load

load loads a npy or npz file.

save

save saves a single array a NumPy data file (npy).

`savez`

`savez` saves an array or set of arrays to a NumPy data file (npz).

`savez_compressed`

`savez_compressed` saves an array or set of arrays to a NumPy data file (npz) using compression.

`savetxt`

`savetxt` saves a single array to a text file.

23.2.7 nan Functions

`nansum`

`nansum` returns the sum of an array, ignoring NaN values. `axis` can be used to compute along a particular axis.

`nanmax`

`nanmax` returns the maximum of an array, ignoring NaN values. `axis` can be used to compute along a particular axis.

`nanargmax`

`nanargmax` returns the index of the maximum of an array, ignoring NaN values. `axis` can be used to compute along a particular axis.

`nanmin`

`nanmin` returns the minimum of an array, ignoring NaN values. `axis` can be used to compute along a particular axis.

`nanargmin`

`nanargmin` returns the index of the minimum of an array, ignoring NaN values. `axis` can be used to compute along a particular axis.

23.2.8 Set Functions

`unique`

`unique` returns the set of unique elements of an array.

`in1d`

`in1d` returns an Boolean array indicating which elements of one array are in another.

intersect1d

intersect1d returns the set of elements of one array which are in another.

union1d

union1d returns the set of elements which are in either of 2 arrays, or both.

setdiff1d

setdiff1d returns the set of elements on one array which are not in another.

setxor1d

setxor1d returns the set of elements which are in either or 2 arrays, but not both.

23.2.9 Logical and Indexing Functions

logical_and

logical_and compute the value of applying and to the elements of two broadcastable arrays.

logical_or

logical_or compute the value of applying or to the elements of two broadcastable arrays.

logical_xor

logical_xor compute the value of applying xor to the elements of two broadcastable arrays.

logical_not

logical_not compute the value of applying not to the elements of an array.

allclose

allclose returns True if all elements of two arrays differ by less than some tolerance.

array_equal

array_equal returns True if two arrays have the same shape and elements.

array_equiv

array_equiv returns True if two arrays are equivalent int eh sense that one array can be broadcast to become the other.

find

find returns the indices of an array where a logical statement is true. The indices returned correspond to the flattened array.

`argwhere`

`argwhere` returns the indices from an array where a logical condition is `True`.

`extract`

`extract` returns the elements from an array where a logical condition is `True`.

`isnan`

`isnan` returns a Boolean array indicating whether the elements of the input are `nan` .

`isinf`

`isinf` returns a Boolean array indicating whether the elements of the input are `inf` .

`isfinite`

`isfinite` returns a Boolean array indicating whether the elements of the input are not `inf` and not `nan`.

`isposinf`

`isposinf` returns a Boolean array indicating whether the elements of the input are `inf`.

`isneginf`

`isneginf` returns a Boolean array indicating whether the elements of the input are `-inf`

`isreal`

`isreal` returns a Boolean array indicating whether the elements of the input are either real or have `0j` complex component.

`iscomplex`

`iscomplex` returns a Boolean array indicating whether the elements of the input are either have non-zero complex component.

`is_string_like`

`is_string_like` returns `True` if the input is a string or similar to a string.

`isscalar`

`isscalr` returns `True` if the input is not an array or matrix.

`is_numlike`

`is_numlike` returns `True` if the input is numeric.

isvector

`isvector` returns `True` if the input has at most 1 dimension which is not unity.

23.2.10 Numerics

nan

`nan` represents Not a Number.

inf

`inf` represents infinity.

finfo

`finfo` can be used along with a data type to return machine specific information about numerical limits and precision.

23.2.11 Mathematics

log

`log` returns the natural logarithm of the elements of an array.

log10

`log10` returns the bast-10 logarithm of the elements of an array.

sqrt

`sqrt` returns the square root of the elements of an array.

exp

`exp` returns the exponential of the elements of an array.

absolute

`absolute` returns the absolute value of the elements of an array.

sign

`sign` returns the sign of the elements of an array.

23.2.12 Rounding

floor

`floor` rounds to next smallest integer.

`ceil`

`ceil` round to next largest integer

23.2.13 Views

`asmatrix`

`asmatrix` returns a view of an array as a `matrix`.

`mat`

`mat` is identical to `asmatrix`.

`asarray`

`asarray` returns a view of a matrix as an `ndarray`.

23.2.14 `rec`

`rec.array`

`rec.array` construct record arrays.

23.2.15 `linalg`

`linalg.matrix_power`

`matrix_power` raises a square array to an integer power.

`linalg.cholesky`

`cholesky` computes the Cholesky factor of a positive definite array.

`linalg.qr`

`qr` computes the QR factorization of an array.

`linalg.svd`

`svd` computes the singular value decomposition of an array.

`linalg.eig`

`eig` computes the eigenvalues and eigenvectors of an array.

`linalg.eigh`

`eigh` computes the eigenvalues and eigenvectors of a Hermitian (symmetric) array.

`linalg.cond`

`cond` computes the conditioning number of an array.

`linalg.det`

`det` computes the determinant of an array.

`linalg.slogdet`

`slogdet` computes the log determinant and the sign of the determinant of an array.

`linalg.solve`

`solve` solves a just-identified set of linear equations.

`linalg.lstsq`

`lstsq` finds the least squares solutions of an over-identified set of equations.

`linalg.inv`

`inv` computes the inverse a of a square array.

23.2.16 `random`

`random.rand`

`rand` returns standard uniform pseudo-random numbers. Uses n inputs to produce an n-dimensional array.

`random.randn`

`randn` returns standard normal pseudo-random numbers. Uses n inputs to produce an n-dimensional array.

`random.randint`

`randing` returns uniform integers on a specified range, exclusive of end point. Uses an n-element tuple to produce an n-dimensional array.

`random.random_integers`

`random_integers` returns uniform integers on a specified range, inclusive of end point. Uses an n-element tuple to produce an n-dimensional array.

`random.random_sample`

`random_sample` returns standard Uniform pseudo-random numbers. Uses an n-element tuple to produce an n-dimensional array.

`random.random`

`random` returns standard Uniform pseudo-random numbers. Uses an n-element tuple to produce an n-dimensional array.

`random.standard_normal`

`standard_normal` returns standard normal pseudo-random numbers. Uses an n-element tuple to produce an n-dimensional array.

`random.sample`

`sample` returns standard Uniform pseudo-random numbers. Uses an n-element tuple to produce an n-dimensional array.

`random.shuffle`

`shuffle` shuffles the elements of an array in-place.

`random.permutation`

`permutation` returns a random permutation of an array.

`random.RandomState`

`RandomState` is a container for the core random generator. `RandomState` is used to initialize and control additional random number generators.

`random.seed`

seed seeds the core random number generator.

`random.get_state`

`get_state` gets the state of the core random number generator.

`random.set_state`

`set_state` sets the state of the core random number generator.

Random Number Generators

Random number generators are available for distribution in the following list: `beta` , `binomial`, `chisquare`, `exponential`, `f`, `gamma`, `geometric`, `laplace`, `logistic`, `lognormal`, `multinomial`, `multivariate_normal`, `negative_binomia`
`normal`, `poisson`, `uniform`.

23.3 SciPy

23.3.1 Statistics (`stats`)

23.3.1.1 Continuous Random Variables

Normal (`norm`), Beta (`beta`), Cauchy (`cauchy`), χ^2 (`chi2`), Exponential (`expon`), Exponential Power (`exponpow`), F (`f`), Gamma (`gamma`), Laplace/Double Exponential (`laplace`), Log-Normal (`lognorm`), Student's t (`t`)

`stats.`*dist*`.rvs`

`rvs` generates pseudo-random variables.

`stats.`*dist*`.pdf`

`pdf` returns the value of the PDF at a point in the support.

`stats.`*dist*`.logpdf`

`logpdf` returns the log of the PDF value at a point in the support.

`stats.`*dist*`.cdf`

`cdf` returns the value of the CDF at a point in the support.

`stats.`*dist*`.ppf`

`ppf` returns the value of the random variable from a point in `(0,1)`. PPF is the same as the inverse CDF.

`stats.`*dist*`.fit`

`fit` estimates parameters by MLE.

`stats.`*dist*`.median`

`median` returns the median of random variables which follow the distribution.

`stats.`*dist*`.mean`

`mean` returns the mean of random variables which follow the distribution.

`stats.`*dist*`.moment`

`moment` returns non-central moments of random variables which follow the distribution.

`stats.`*dist*`.var`

`var` returns the variance of random variables which follow the distribution.

`stats.`*`dist`*`.std`

`std` returns the standard deviation of random variables which follow the distribution.

23.3.1.2 Statistical Functions

`stats.mode`

`mode` returns the empirical mode of an array.

`stats.moment`

`moment` computes non-central moments of an array.

`stats.skew`

`skew` computes the skewness of an array.

`stats.kurtosis`

`kurtosis` computes the *excess* kurtosis of an array.

`stats.pearsonr`

`pearsonr` computes the correlation of an array.

`stats.spearmanr`

`spearmanr` computes the rank correlation of an array.

`stats.kendalltau`

`kendalltau` computed Kendall's τ, which is similar to a correlation, from an array.

`stats.normaltest`

`normaltest` computes a Jarque-Bera like test for normality.

`stats.kstest`

`kstest` computes a Kolmogorov-Smirnov test for a specific distribution.

`stats.ks_2samp`

`ks_2samp` computes a Kolmogorov-Smirnov test from directly from two samples.

`stats.shapiro`

`shapire` computes the Shapiro-Wilks test of normality.

23.3.2 Optimization (`optimize`)

23.3.2.1 Unconstrained Function Minimization

`optimize.fmin_bfgs`

`fmin_bfgs` minimizes functions using the BFGS algorithm.

`optimize.fmin_cg`

`fmin_cg` minimizes functions using a Conjugate Gradient algorithm.

`optimize.fmin_ncg`

`fmin_ncg` minimizes functions using a Newton-Conjugate Gradient algorithm.

23.3.2.2 Derivative Free Unconstrained Function Minimization

`optimize.fmin`

`fmin` minimizes a function using a simplex algorithm.

`optimize.fmin_powell`

`fmin_powell` minimizes a function using Powell's algorithm.

23.3.2.3 Constrained Function Minimization

`optimize.fmin_slsqp`

`fmin_slsqp` minimizes a function subject to inequality, equality and bounds constraints.

`optimize.fmin_tnc`

`fmin_tnc` minimizes a function subject to bounds constraints.

`optimize.fmin_l_bfgs_s`

`fmin_l_bfgs_s` minimizes a function subject to bounds constraints.

`optimize.fmin_colyba`

`fmin_colyba` minimizes a function subject to inequality and equality constraints.

23.3.2.4 Scalar Function Minimization

`optimize.fmin_bound`

`fmin_bound` minimizes a function in a bounded region.

`optimize.golden`

`golden` uses a golden section search to minimize minimize a scalar function.

`optimize.brent`

`brent` uses Brent's method to minimize a scalar function.

23.3.2.5 Nonlinear Least Squares

`optimize.lstsq`

`lstsq` performs non-linear least squares minimization of a function which returns a 1-dimensional array of errors.

23.3.3 Input/Output (`io`)

`io.loadmat`

`loadmat` loads a MATLAB data file.

`io.savemat`

`savemat` saves an array or set of arrays to a MATLAB data file.

23.4 Matplotlib

23.4.1 2D plotting

`plot`

`plot` plots a 2-dimensional line or set of lines.

`pie`

`pie` produces a pie chart.

`hist`

`hist` computes and plots a histogram.

`scatter`

`scatter` produces a scatter plot.

`bar`

`bar` produces a vertical bar chart.

barh

barh produces a horizontal bar chart.

contour

contour produces a 2-dimensional contour representation of 3-dimensional data.

23.4.2 3D plotting

plot

plot using the optional keyword argument zs produces 3-dimensional plots.

plot_wireframe

plot_wireframe plots a 3-dimensional wire-frame surface.

plot_surface

plot_surface plots a 3-dimensional solid surface.

23.4.3 Utility Commands

figure

figure opens a new figure or changes the current figure to an existing figure.

add_axes

add_axes adds a single axes to a figure.

add_subplot

add_subplot adds a subplot to a figure.

show

show updates a figure and pauses the running of a non-interactive program until the figure is closed.

draw

draw updates a figure.

close

close closes figures. `close('all')` closes all figures.

legend

legend adds a legend to a plot or chart using the information in label keyword arguments.

`title`

`title` adds a title to a plot or chart.

`savefig`

`savefig` exports figures to common file formats including PDF, EPS, PNG and SVG.

23.4.4 Input/Output

`csv2rec`

`csv2rec` reads data in a CSV file and returns a NumPy record array. Columns are automatically types based on the first few rows of the input array, and importing dates is supported.

23.5 IPython

?**partial**, ?*module*.**partial**

?**partial** list any known objects – variables, functions or modules – which match the wild card expression **partial** where *partial* can be any string.

function?, *magic*?

?*function*, ?*magic*, ?*module*, *function*?, *magic*? and *module*? all pretty print the docstring for a function, magic word or module.

function??, *magic*??

??*function*, ??*magic*, ??*module*, *function*??, *magic*?? and *module*?? all pretty print the entire function, magic word or module, including both the docstring and the code inside the function.

!*command*

!*command* is used to run a system command as if executed in the terminal. For example, `!copy file.py backup.py` will copy file.by to backup.py. An output can be used to capture the command window text printed by the command to a local variable. Foe example, `dirListing = !dir *.py` will capture the contents to running a directory listing for all py files. This can then be parsed or used for error checking.

`%bookmark`

`%bookmark` allows bookmarks to be created to directories. Manage IPython's bookmark system. For example, the current directory can be bookmarked using `%bookmark currentDir`, and can be returned to using `%cd currentDir` (assuming the current directory doesn't contain a directory named currentDir).

`%cd`

`%cd` changes the current working directory. In Windows, \, \\ or / can be used as directory separators. In Linux, / should be used to separate directories. `%cd` support tab completion, so that only a few letters are needed before hitting the tab key to complete the directory name.

`%clear`, `%cls`

`%clear` and `%cls` both clear the terminal window (but do not remove any variables, functions or modules).

`%edit`

`%edit` opens the editor for py files, and is usually used `%edit` *filename*`.py`.

`%hist`

`%hist` lists command history. `%hist -g` *searchterm* can be used to search for a specific term or wildcard expression.

`%lsmagic`

`%lsmagic` lists all defined magic functions.

`%magic`

`%magic` prints detailed information about all magic functions.

`%pdb`

`%pdb` controls the use of the Python debugger.

`%pdef`

`%pdef` prints only the definition header (the line beginning with `def`) for functions.

`%precision`

`%precision` sets the display precision. For example `%precision 3` will show 3 decimal places.

`%prun`

`%prun` runs a statement in the profiler, and returns timing and function call information. Use `%run -p` *filename.py* to profile a standalone file.

`%psearch`

`%psearch` searches for variables, functions and loaded modules using wildcards.

%pwd

`%pwd` shows the current working directory.

%pycat, %more, %less

`%pycat`, `%more` and `%less` all show the contents of a file in the IPython window.

%pylab

`%pylab` initializes pylab if not initialized from the command line.

%quickref

`%quickref` shows a reference sheet for magic commands.

%reset

`%reset` resets the session by removing all variables, functions and imports defined by the user.

%reset_selective

`%reset_selective` *re* resets all names which match the regular expression *re*.

%run

`%run` *filename.py* executes a file containing Python code in the IPython environment.

%time

`%time` *code* provides the time needed to run the code once. `%timeit` is often more useful.

%timeit

`%timeit` *code* times a segment of code, and is useful for finding bottlenecks and improving algorithms. If *code* is very fast, multiple runs are used to improve timing accuracy.

```
>>> %timeit x=randn(100,100);dot(x.T,x)
1000 loops, best of 3: 758 us per loop
```

%who

`%who` lists all variables in memory.

%who_ls

`%who_ls` returns a sorted list containing the names of all variables in memory.

%whos

`%whos` provides a detailed list of all variables in memory.

`%xdel`

`%xdel` *variable* deletes the variable from memory.

Incomplete

Chapter 24

Parallel

To be completed

24.1 `map` and related functions

`map` is a built-in function which is used to apply a function to a generic iterable. It is used as `map(` *function* `,` *iterable* `)`, and returns a list containing the results of applying the function to each item of *iterable*. Note that the list returned can be either a simple list if the function returns a single item, or a list of tuples if the function returns more than 1 value.

```
def powers(x):
    return x**2, x**3, x**4
```

This function can be called on any iterable, for example a list.

```
>>> y = [1.0, 2.0, 3.0, 4.0]
>>> map(powers, y)
[(1.0, 1.0, 1.0), (4.0, 8.0, 16.0), (9.0, 27.0, 81.0), (16.0, 64.0, 256.0)]
```

The output is a list of tuples where each tuple contains the result of calling the function on a single input. Note that the same result could be achieved using a list comprehension. In general usage, list comprehensions are preferrable to using map.

```
>>> [powers(i) for i in y]
[(1.0, 1.0, 1.0), (4.0, 8.0, 16.0), (9.0, 27.0, 81.0), (16.0, 64.0, 256.0)]
```

`map` can be used with more than 1 iterable, in which case it iterates along the longest iterable. If one of the iterable is shorter than the other(s), then it is extended with `None`. It is usually best practice to ensure that all iterables have the same length before using `map`.

```
def powers(x,y):
    if x is None or y is None:
        return None
    else:
        return x**2, x*y, y**2
```

```
>>> x = [10.0, 20.0, 30.0]
>>> y = [1.0, 2.0, 3.0, 4.0]
>>> map(powers, x, y)
```

```
[(100.0, 10.0, 1.0), (400.0, 40.0, 4.0), (900.0, 90.0, 9.0), None]
```

A related function is `zip`. While `zip` does not apply a function to data, it can be used to combine two or more lists into a single list of tuples. It is similar to calling `map` except that it will stop at the end of the shortest iterable, rather than extending using `None`.

```
>>> x = [10.0, 20.0, 30.0]
>>> y = [1.0, 2.0, 3.0, 4.0]
>>> zip(x, y)
[(10.0, 1.0), (20.0, 2.0), (30.0, 3.0)]
```

24.2 Multiprocess module

The real advantage of `map` over list comprehensions is that it can be combined with the `multiprocess` module to run code on more than 1 (local) processor. `multiprocess` *module does not work correctly in IPython, and so it is necessary to use stand-alone Python programs.*

```
from __future__ import print_function
import multiprocessing as mp
import numpy as np
import matplotlib.pyplot as plt

def compute_eig(arg):
    n = arg[0]
    state = arg[1]
    print(arg[2])
    np.random.set_state(state)
    x = np.random.standard_normal((n))
    m = int(np.round(np.sqrt(n)))
    x.shape=m,m
    w = np.linalg.eigvalsh(np.dot(x.T,x)/m)
    return w


if __name__ == '__main__':
    states = []
    np.random.seed()
    for i in xrange(1000):
        n = 1000000
        states.append((n,np.random.get_state(),i))
        temp = np.random.standard_normal((n))

    # Non parallel map
    # res = map(compute_eig,states)

    # Parallem map
    po = mp.Pool(processes=2)
    res = po.map(compute_eig,states)
    print(len(res))
    po.close()
```

```
maxEig = [r.max() for r in res]
maxEig = np.array(maxEig)
plt.hist(maxEig)
plt.show()
```

Note: You must have a __name__==’__main__ ’ when using multiprocessing.

24.3 Python Parallel

Chapter 25

Other Python Packages

To be completed

25.1 scikits.statsmodels

25.2 pandas

25.3 rpy and rpy2

25.4 PyTables and h5py

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