Cross-Layer Adaptive Techniques for Throughput Enhancement in Wireless OFDM-Based Networks

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***Abstract*—Although independent consideration of layers simplifies wireless system design, it is inadequate since: 1) it does not consider the effect of co-channel user interference on higher layers; 2) it does not address the impact of local adaptation actions on overall performance; and 3) it attempts to optimize performance at one layer while keeping parameters of other layers fixed. Cross-layer adaptation techniques spanning several layers improve performance and provide better quality of service for users across layers. In this study, we consider a synergy between the physical and access layers and address the joint problem of channel allocation, modulation level, and power control in a multicell network. Since performance is determined by channel reuse, it is important to handle co-channel interference appropriately by constructing co-channel user sets and by assigning transmission parameters so that achievable system rate is maximized. The problem is considered for orthogonal frequency-division multiplexing, which introduces novel challenges to resource allocation due to different quality of subcarriers for users and existing transmit power constraints. We study the structure of the problem and present two classes of centralized heuristic algorithms. The first one considers each subcarrier separately and sequentially allocates users from different base stations in the subcarrier based on different criteria, while the second is based on water-filling across subcarriers in each cell. Our results show that the first class of heuristics performs better and quantify the impact of different parameters on system performance.**

***Index Terms*—Cross-layer design, multicell systems, orthogonal frequency-division multiplexing (OFDM), resource allocation.**

I. INTRODUCTION

THE fundamental challenge in a wireless communications system is to satisfy stringent and diverse quality-of-service (QoS) requirements of users in the volatile transmission medium by using the limited available resources. QoS is perceived as: 1) an acceptable signal-to-interference and noise ratio (SINR), outage probability, or bit error rate (BER) at the receiver at the physical layer or 2) a minimum rate or another form of rate guarantee such as fairness or as a maximum delay requirement at higher layers. QoS provisioning at the access layer depends on scheduling, channel access, channel allocation, buffer management, routing, and flow control methods. At the physical layer, adaptation of modulation-level or channel coding rate controls the amount of sustainable interference for a maximum acceptable BER, while transmit power control influences interference levels and ensures acceptable link quality.

Although independent consideration of layers simplifies wireless system design, it constrains performance significantly because: 1) it does not take into account the effect of co-channel interference on higher layer mechanisms; 2) it does not consider the impact of local adaptation actions on overall system performance; and 3) it attempts to optimize performance at one layer while treating parameters of other layers as fixed. The growing consensus about cross-layer design [2] refers to the need for interaction and information exchange between the physical and higher layers and accounts for the volatile and time-varying nature of the wireless medium. Under the premise of cross-layer design, physical and higher layer control decisions reach their full potential when in synergy with each other. The interest in cross-layer design is also manifested by the proposed IEEE 802.11k standard about information exchange between different parts of the network stack for improved performance.

Transmit power control aims at balancing SINRs of co-channel links. In the centralized approach of [3], the maxmin achievable common link SIR is $\gamma^* = 1/(\lambda^* - 1)$, where λ^* is the maximum positive real eigenvalue of a matrix that involves transmitter–receiver link gains. Distributed iterative algorithms for the same purpose have also been proposed, e.g., [4]. In [5], an iterative algorithm for power control and base station (BS) assignment for the uplink is presented. The algorithm converges to a feasible solution, if there exists one, and the total transmit power is minimized. In [6], a heuristic algorithm for BS, power, and channel allocation is proposed, which attempts to provide acceptable link quality to users by using a minimum number of channels. On the other hand, modulation adaptation strives to maintain a certain BER in the presence of channel variations [7]. Qiu *et al.* [8] study joint modulation and power control for a set of co-channel BS-user pairs. Their algorithm maximizes the product of link SINRs, but it is suboptimal in maximizing total rate.

Orthogonal frequency-division multiplexing (OFDM) [9] is a signaling and access technique that is included in IEEE 802.11a [10] and 802.11g standards for WLANs that operate either in distributed coordination function (DCF) mode with users connected in multiple hops or in point coordination function (PCF) mode with single-hop connection to an access point (AP). OFDM is also considered for wireless personal area networks and fixed broadband wireless access and for the evolving standard of WiMAX [11] for broadband wireless access. In OFDM, the wideband spectrum is divided into orthogonal narrowband subcarriers as in frequency-division multiplexing and the user bit stream is split into subsets, the subsymbols. Each subsymbol modulates a subcarrier and several subsymbols of a user are transmitted in parallel over subcarriers. Appropriate

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subcarrier spacing preserves channel orthogonality and leads to high spectral efficiency. OFDM transmission reduces the effective symbol transmission rate and provides immunity to intersymbol interference (ISI).

For one user, the allocation of an amount of power across parallel OFDM subcarriers (each with Gaussian noise) that maximizes total information-theoretic rate is given by water-filling. The bit allocation in each subcarrier follows from power allocation. For the multiuser downlink case, each user has different multipath profile across frequencies and hence different subcarrier gains, since user receivers are not co-located. For a single-cell multiuser system with a certain subcarrier allocation to users and a total power constraint for each user, power water-filling across subcarriers of each user maximizes total rate. However, the problem of subcarrier allocation and bit and power allocation so as to maximize total rate is difficult due to the integer programming nature of subcarrier allocation and the different subcarrier quality for users. In [12], the discrete allocation problem for sum rate maximization subject to a power constraint for each user is relaxed into a continuous convex optimization one that is solved numerically. The same objective with a total power constraint over all users is achieved by assignment of each subcarrier to the user with the largest gain in it and subsequent power water-filling [13]. In [14], a dual problem is studied, namely subcarrier, bit, and power allocation for total power minimization while satisfying rate constraints of users. The continuous relaxation of the integer problem gives rise to an iterative algorithm that leads to a suboptimal solution. In a multicell OFDM system, the problem becomes more difficult even for given subcarrier assignment due to co-channel interference among users from different cells that reuse a subcarrier. In that case, the power allocated to a user in a subcarrier becomes interference for co-channel users. In [15], a distributed heuristic algorithm is presented, which is based on iterating between water-filling on a set of subcarriers and removing subcarriers where SINRs are violated. From an information theoretic view and for given subcarrier allocation, the multicell system resembles that of parallel interference channels, for which no capacity characterization result is available. In our preliminary work in [1], we presented a method for addressing joint channel allocation, modulation level, and power control in a multicell system for generic access schemes with orthogonal channels.

In multicell networks, BSs take independent allocation turns. Each BS assigns a channel to its user with the least interference in it and adjusts transmission parameters. Namely, it selects the highest modulation level for which there exists power within the available power range that gives an acceptable SINR. Channel reassignment occurs if the highest power and lowest modulation cannot provide an acceptable SINR in the current channel. However, with BS coordination through a centralized controller or through signaling over high-speed wire-line or wireless links, information about user channel quality and resource allocation within a cell can become available to other BSs. Then a BS can obtain a network-wide view and compute co-channel interference levels among users in different cells. This leads to a coupling between physical and access layers, in the sense that adaptation decisions in one layer trigger mechanisms at the other layer and affect QoS at both layers.

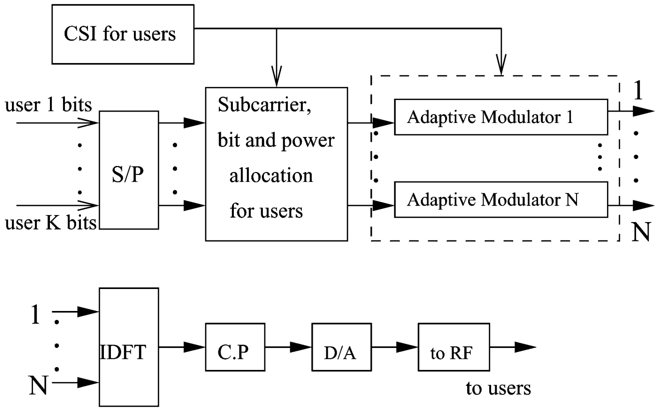

Fig. 1. Single-cell multiuser OFDM transmission diagram.

We address the joint problem of channel allocation, modulation level, and power control in a multicell network. Our objective is to study the impact of these mechanisms on co-channel interference and channel reuse which essentially determine achievable rate. Co-channel interference and user susceptibility to it are controlled by selective insertion of users in subcarriers and transmit parameter control. System rate is high if users are maximally "packed" in co-channel sets in subcarriers and maintain acceptable SINRs. Our contribution to the current literature can be summarized as follows: 1) we address the threefold problem above in a multicell setting where the issue of co-channel interference management arises; 2) we cast the problem in the context of OFDMA which introduces novel challenges in resource allocation due to different quality of subcarriers for different users and transmit power constraints; and 3) we provide explicit characterization of the achievable performance region and introduce two classes of centralized greedy heuristic algorithms that represent different approaches to the problem. The first algorithm treats each subcarrier separately and performs successive insertion of users in each subcarrier and adaptation of transmit parameters, while the second one relies on power water-filling at each BS. Our heuristics are designed so as to be in line with algorithms that solve optimally simple instances of the problem. In addition, our approaches provide a cross-layer perspective to the problem since they involve coordination between physical and access layer mechanisms.

The remainder of this paper is organized as follows. In Section II, we present the model and assumptions. In Section III, we state the problem formally, characterize its performance region and present our algorithms. This section includes optimal solutions to some special cases. Numerical results are shown in Section IV, and Section V concludes the paper.

II. SYSTEM MODEL

We start with downlink OFDM transmission from one BS to K users with N data-carrying subcarriers (Fig. 1). Packetized data from higher layers are decomposed into bits and are transmitted in consecutive time slots of duration T_s of a TDMA scheme. The bit stream of each user is divided into bit groups, each of which constitutes an OFDM symbol for the user. Provided that a user is selected for transmission in a slot, a fixed number of symbols of each selected user $S = T_s/T$ are transmitted in a slot, where T is the symbol (signaling) period. The transmission block diagram consists of the following modules.

1) *Subcarrier, bit, and power allocation module:* This determines the set of subcarriers $\mathcal{S}_k$ that are assigned to each user k and the corresponding bit and power allocations. At most, one user is assigned to a subcarrier, namely sets $\mathcal{S}_k$ are disjoint. The bits of the OFDM symbol of user k, are divided into $|\mathcal{S}_k|$ subgroups. The bits of each subgroup constitute a user subsymbol and will modulate subcarrier $n \in \mathcal{S}_k$. The number of bits $b_{n,k}$ of user k's subsymbol that modulate subcarrier n is selected from a L_0-element set $\mathcal{M}$ of available QAM or QPSK modulation levels with different number of bits per subsymbol, $\{b_i\}_{i=1}^{L_0}$. In general, different numbers of bits of a user can modulate a subcarrier depending on subcarrier quality, as will become evident in the sequel. Also, let $p_k^{(n)}$ denote the power allocated to user k in subcarrier n. A total transmit power constraint at the BS dictates that $\sum_{k=1}^{K} \sum_{n\in\mathcal{S}_k} p_k^{(n)} \le P$.
2) *N modulators:* Each modulator n modulates the corresponding subcarrier with the $b_{n,k}$ bits of the user that is assigned to subcarrier n. The result is a complex subsymbol $d_{n,k}$ at the output of the modulator.
3) *Inverse discrete Fourier transform (IDFT) modules:* Subsymbols of each user are transformed into time-domain samples that form an OFDM user symbol.
4) *Cyclic prefix, D/A, and baseband to RF modules:* These modules add cyclic prefix and perform D/A conversion and upconversion to carrier frequency f_c before the continuous user signals are transmitted to the channel.

Channel (subcarrier) quality for a user remains constant for a slot duration and may change between slots. Thus, each one of the S OFDM symbols of a user, k is split into subsymbols over the same set of subcarriers $\mathcal{S}_k$. User k sends $N_k = S \times \sum_{n\in\mathcal{S}_k} b_{n,k}$ bits in a slot. The rate of user k in a time interval of duration t consisting of $C = \lfloor t/T_s \rfloor$ slots is $(1/t)\sum_{c=1}^{C} N_k^{(c)}$, where $N_k^{(c)} = S \times \sum_{n\in\mathcal{S}_k^{(c)}} b_{n,k}^{(c)}$ is the number of transmitted bits in time slot c and $\mathcal{S}_k^{(c)}$ is the set of utilized subcarriers by user k in slot c. If OFDM symbols do not overlap in time, we can focus on one of the S OFDM symbols for each user. The transmitted baseband signal for user k is

$$x_k(t) = \sum_{n=0}^{N-1} \sqrt{p_k^{(n)}} d_{n,k} e^{j2\pi nt/T}, \quad 0 \le t \le T. \qquad (1)$$

The time-invariant (within a slot) channel between BS and user k has impulse response $h_k(t) = \sum_{\ell=1}^{L} \beta_{k,\ell}\delta(t-\tau_{k,\ell})$, where L is the number of paths in the multipath, $\beta_{k,\ell}$ is the gain of the ℓth path of user k with path loss and shadowing, and $\tau_{k,\ell}$ is path delay.

Consider the received signal at the receiver of user k and fix attention to one OFDM symbol. The signal is downconverted and analog-to-digital (A/D)-converted by being sampled at times $\{iT/N, i = 0, \dots, N-1\}$. Time samples are fed into the DFT module. The nth subsymbol of user k is

$$\begin{aligned} y_{n,k} &= d_{n,k}\sqrt{p_k^{(n)}}\left(\sum_{\ell=1}^{L} \xi_{k,\ell}(n)\right) + z_{n,k} \\ &= d_{n,k}\sqrt{p_k^{(n)}} g_{n,k} + z_{n,k} \end{aligned} \qquad (2)$$

for $n = 1, \dots, N$, where the factor

$$\xi_{k,\ell}(n) = \beta_{k,\ell}\exp\left[-j2\pi(f_c + n/T)\tau_{k,\ell}\right] \qquad (3)$$

captures the different impact of propagation characteristics of path ℓ of the multipath of user k at different subcarriers n and $z_{n,k}$ is a term after sampling of the k receiver's Gaussian noise process.

Channel state information (CSI) at the receiver about the frequency-domain transfer function at each subcarrier can be obtained with pilot symbols which further help in retrieving transmitted data subsymbols. A pilot symbol e consists of known subsymbols $\{e_n\}_{n=0}^{N-1}$. The received pilot subsymbol of user k at subcarrier n is $y_{n,k}^{0} = e_n g_{n,k} + z_{n,k}$. The minimum mean-squared-error (MMSE) estimate of $g_{n,k}$ is $\tilde{g}_{n,k} = y_{n,k}^{0}/e_n$. Subcarrier modulation information is communicated to the receiver. The maximum-likelihood (ML) detector decides on transmitted subsymbol $y_{n,k}$ based on $y_{n,k}/\tilde{g}_{n,k}$. With normalized subsymbols to unit power, the average signal-to-noise ratio (SNR) at the receiver of user k at subcarrier n is $\mathrm{SNR}_{n,k} = p_k^{(n)} G_k^{(n)}/\sigma^2$, where σ^2 is the noise variance and $G_k^{(n)} = |g_{n,k}|^2$ is the link gain between the base station and user k at subcarrier n. In a slowly time-varying channel, the transmitter can obtain CSI for each user and subcarrier via time duplexing.

Consider now downlink OFDM transmission in a multicell system with M BSs and K users, with each user connected to its closest BS. In each slot, each BS sends data to its users with N subcarriers. While a BS must use different sets of subcarriers for each one of its users, different BSs can reuse the same subcarriers. The link between BS i and user j at subcarrier n has gain $G_{ij}^{(n)}$. A user j in a subcarrier n receives useful signal from its serving BS i_j and interference from other BSs that transmit in n. TDMA frames and corresponding slots of different BSs are assumed to be synchronized. Although reception of useful and interfering signals is not synchronized in general, we assume symbol-synchronous reception or equivalently assume that reception delay does not exceed T. Given the order of magnitude of T, this assumption can hold for distances of the order of 1 km among BSs. Frequency synchronization ensures that subcarrier orthogonality is maintained at the receiver.

The average SINR at the matched filter receiver of user j at subcarrier n is

$$\mathrm{SINR}_j^{(n)} = \frac{G_{i_j j}^{(n)} p_{i_j}^{(n)}}{\sum\limits_{i_k \in \mathcal{B}^{(n)}: k \neq j} G_{i_k j}^{(n)} p_{i_k}^{(n)} + \sigma^2} \qquad (4)$$

where $\mathcal{B}^{(n)}$ is the set of BSs that use subcarrier n and $p_i^{(n)}$ is the transmission power of BS i in n. The BER at the output of the detector of a user in a subcarrier should satisfy $\mathrm{BER} \le \epsilon$, where ϵ is a prespecified value. For an M-QAM modulation level with $M = 2^b$, $b \in \mathcal{M}$, the minimum required SINR so that BER $\leq \epsilon$ is $\gamma(b) = -[\ln(5\epsilon)/1.5](2^b - 1)$ as in [8]. If user rate requirements are specified, the problem is to satisfy them with a minimum number of channels. Otherwise, the objective is to maximize total achievable rate. Note that the term "BS" that is used in the paper encompasses the functionality of an AP in a wireless LAN.

III. Joint Channel Allocation, Modulation, and Power Control

In a multicell network, a user receives useful signal in a subcarrier from the serving BS and interference from neighboring BSs which use the subcarrier. A co-channel user set is called *feasible* if BERs at all user receivers do not exceed ϵ. The feasibility of a co-channel user set depends on: 1) the BS-user link gains, which in turn depend on user identities, their proximity to BSs, and their propagation characteristics; 2) modulation levels that are used for transmission by each BS, since these are associated with different SINR thresholds so that BER $\leq \epsilon$; a set of modulation levels that ensures a feasible co-channel user set is called feasible as well; 3) transmit powers, since these control the level of useful signal and interference at receivers; and 4) subcarrier frequency. The multipath characteristics of a user create frequency selectivity that is reflected on different subcarrier gains. Multipath characteristics differ among users, since their receivers are not co-located.

High modulation levels for a user in a subcarrier imply more transmitted bits per user but do not favor large subcarrier reuse since they are vulnerable to interference. On the other hand, low modulation levels can sustain more interference and thus more crowded co-channel sets, but they do not transmit many bits per user. Clearly, the two aspects of the impact of modulation level on achievable rate are conflicting. When power control is also used, the feasibility of the co-channel set and the allocated modulation levels are also affected by transmit powers of BSs, and this creates additional challenges to the tradeoff above. In addition, if a transmit power constraint exists for each BS, sensible use of power through careful subcarrier assignment to users within each BS is also required. The question is whether we can perform jointly subcarrier allocation, modulation-level adaptation, and power control, so as to maximize total rate for each subcarrier individually and for the entire system. This problem involves identifying co-channel user sets with maximum rate for each subcarrier.

A. Characterization of Achievable Performance Region

Consider first the problem of identifying the feasible co-channel set with maximum subcarrier rate in a subcarrier n and fix attention to an instance with gains $G_{ij}^{(n)}$ between BS i and user j. In the next derivation, we drop dependence on n for notational convenience. Let $\mathcal{U}_i$ be the set of users served by BS i for $i = 1, \ldots, M$ and let b_j be the modulation level for user j. An *assignment policy* is a rule for determining a co-channel set of at most M users and their transmission parameters. An assignment policy selects at most one user from each BS and allocates a modulation level (rate) and a power for transmission to each one of the selected users. A BS activation vector $\mathbf{a}$ is a $M \times 1$ binary vector whose ith entry $a_i = 1$ if BS i transmits to a user and 0 otherwise. Let $\mathcal{A}$ be the set of all activation vectors. For activation vector $\mathbf{a}$, define $\mathcal{F}_{\mathbf{a}}$ to be the set of all possible $\prod_{i=1:a_i=1}^{M} |\mathcal{U}_i|$ selections of users from active BSs. A co-channel user set is denoted as $\mathbf{u}(\mathbf{a}, f) = \{u_i(\mathbf{a}, f) : i = 1, \ldots, M$, for some $\mathbf{a} \in \mathcal{A}$ and $f \in \mathcal{F}_{\mathbf{a}}\}$, so that $u_i \in \mathcal{U}_i$. The collection of all co-channel user sets is $\mathcal{U} = \bigcup_{\mathbf{a} \in \mathcal{A}} \mathcal{F}_{\mathbf{a}}$. The user modulation vector for this co-channel set is

$$\mathbf{b}_{u(\mathbf{a},f)} = (\underbrace{0, \ldots, b_{u_1(\mathbf{a},f)}, \ldots, 0}_{|\mathcal{U}_1| \text{ users of BS 1}}, \ldots, \underbrace{0, \ldots, b_{u_M(\mathbf{a},f)}, \ldots, 0}_{|\mathcal{U}_M| \text{ users of BS M}}) \tag{5}$$

where $b_{u_i(\mathbf{a},f)}$ is the modulation level for transmission to the user that is selected from BS i based on selection rule $f \in \mathcal{F}_{\mathbf{a}}$. The power vector $\mathbf{p}_{\mathbf{u}(\mathbf{a},f)}$ is defined similarly. The shorthand notation $\mathbf{b}_{\mathbf{u}} = \mathbf{b}_{\mathbf{u}(\mathbf{a},f)}$ and $\mathbf{p}_{\mathbf{u}} = \mathbf{p}_{\mathbf{u}(\mathbf{a},f)}$ for modulation and power vectors of co-channel set $\mathbf{u}$ will be used. First, assume that power control is not used. For activation vector $\mathbf{a}$, the co-channel interference experienced by a user in a subcarrier is known and the user $u_i^*(\mathbf{a})$ that is selected from BS i is the one with the highest modulation level among users in $\mathcal{U}_i$ such that BER $\leq \epsilon$,

$$u_i^*(\mathbf{a}) = \arg\max_{u \in \mathcal{U}_i} \left\{ \max \left[b \in \mathcal{M} : \frac{G_{iu}}{\sum\limits_{j \neq i : a_j = 1} G_{ju} + \sigma^2} \geq \gamma(b) \right] \right\} \tag{6}$$

for all i such that $a_i \neq 0$. Co-channel set $\mathbf{u}^*(\mathbf{a}) = (u_i^*(\mathbf{a}) : i = 1, \ldots, M,$ with $a_i \neq 0)$ is associated with user modulation vector $\mathbf{b}_{u^*(\mathbf{a})}$ as in (5). The assignment policy that maximizes the subcarrier sum rate uses the BS activation vector that leads to selection of a co-channel set such that the resulting modulation vector has maximum sum of entries, i.e., $\mathbf{u}^* = \arg\max_{\mathbf{a} \in \mathcal{A}} \sum_{i=1}^{M} b_{u_i^*(\mathbf{a})}$. This in turn involves identification of the maximal feasible co-channel set, which is a hard combinatorial optimization problem.

When transmit power control comes into stage, the problem becomes more complicated. With no power control, we could easily specify the user selection rule from each BS and the problem was to find the appropriate BS activation vector. However, when transmit power is controllable, user SINRs depend jointly on powers of all BSs. The assignment rule involves both the BS activation vector and the user selection rule from each BS. In addition, even if the selected user from each BS is given, the computation of transmit powers that maximize subcarrier rate is not straightforward.

Consider $m \leq M$ co-channel users in a subcarrier and set $\sigma^2 = 0$. Let $\mathbf{G} = \{G_{ij}\}$ be the $m \times m$ matrix of link gains from BS i to user j, for $i, j \in \{1, 2, \ldots, m\}$. Let $\mathbf{b} = (b_1, b_2, \ldots, b_m)^T$ be the user modulation level vector and $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_m)^T$ be the corresponding threshold vector. The BS transmit power vector is $\mathbf{p} = (p_1, p_2, \ldots, p_m)^T$. The modulation vector $\mathbf{b}$ is *achievable* for a co-channel set if there exists a power vector $\mathbf{p}$ such that SIR constraints corresponding to modulation vector $\mathbf{b}$ are satisfied for all users, namely, if

$$\mathrm{SIR}_j(\mathbf{p}) = \frac{G_{jj}p_j}{\sum\limits_{i=1,i\neq j}^{m} G_{ij}p_i} \geq \gamma_j, \quad \text{for } j = 1,\ldots,m. \tag{7}$$

The co-channel user set is then *feasible with respect to* $\mathbf{b}$. Condition (7) is written in matrix form as $\mathbf{p} \geq \tilde{\mathbf{G}}(\mathbf{b})\mathbf{p}$, where the $m \times m$ matrix $\tilde{\mathbf{G}}(\mathbf{b})$ has elements

$$\tilde{G}_{ji}(b_j) = \begin{cases} \gamma_j(G_{ij}/G_{jj}), & \text{if } j \neq i \\ 0, & \text{if } j = i. \end{cases} \tag{8}$$

Matrix $\tilde{\mathbf{G}}(\mathbf{b})$ is nonnegative definite and irreducible. From the Perron–Frobenius theorem, it has exactly one positive real eigenvalue $\lambda^* = \max\{|\lambda_i|\}_{i=1}^{m}$, where $\{\lambda_i\}_{i=1}^{m}$ are the eigenvalues of $\tilde{\mathbf{G}}(\mathbf{b})$. Eigenvalue λ^* has an associated eigenvector $\mathbf{p}^*$ with strictly positive entries. Furthermore, the minimum real λ such that inequality $\lambda\mathbf{p} \geq \tilde{\mathbf{G}}(\mathbf{b})\mathbf{p}$ has solutions $\mathbf{p} > 0$ is $\lambda = \lambda^*$. We start by finding the maximum real positive eigenvalue λ^* of $\tilde{\mathbf{G}}(\mathbf{b})$ to guarantee a power vector with positive entries. If $\lambda^* \leq 1$, then $\mathbf{p} \geq \tilde{\mathbf{G}}(\mathbf{b})\mathbf{p}$ holds and modulation vector $\mathbf{b}$ is achievable. The power vector that achieves $\mathbf{b}$ is the eigenvector that corresponds to λ^*.

When receiver noise comes into play, the SINR requirements are written in matrix form as $[\mathbf{I}-\tilde{\mathbf{G}}(\mathbf{b})]\mathbf{p} \geq \mathbf{v}(\mathbf{b})$, where $\mathbf{I}$ is the identity matrix and vector $\mathbf{v}(\mathbf{b}) = \left(\gamma_j\sigma^2/G_{jj} : j = 1,\ldots,m\right)$. If the maximum positive eigenvalue of $\tilde{\mathbf{G}}(\mathbf{b})$ satisfies $\lambda^* < 1$, then matrix $[\mathbf{I}-\tilde{\mathbf{G}}(\mathbf{b})]$ is invertible and any power vector $\mathbf{p}$ with $\mathbf{p} \geq [\mathbf{I}-\tilde{\mathbf{G}}(\mathbf{b})]^{-1}\mathbf{v}(\mathbf{b})$ satisfies the SINR requirements. Moreover, the power vector $\mathbf{p}^* = [\mathbf{I}-\tilde{\mathbf{G}}(\mathbf{b})]^{-1}\mathbf{v}(\mathbf{b})$ minimizes the total transmit power to users in the co-channel set. This fact will be used later.

The overall problem of selecting co-channel user sets, modulations, and powers so as to maximize total rate over all subcarriers can be expressed as

$$\max_{\substack{\mathbf{u}(n)\in\mathcal{U},\mathbf{p}_{\mathbf{u}(n)}\geq\mathbf{0}\\ n=1,\ldots,N}} \sum_{n=1}^{N}\sum_{i=1}^{M} b_{u_i(n)} \tag{9}$$

subject to the constraints

$$\mathrm{SINR}_{u_i(n)}(\mathbf{p}_{\mathbf{u}(n)}) \geq \gamma(b_{u_i(n)}) \qquad \forall\, n \text{ and } \forall\, i \tag{10}$$

$$\sum_{n=1}^{N} p_{u_i(n)} \leq P \qquad \forall\, i \tag{11}$$

where $\mathbf{u}(n)$ denotes a co-channel set at subcarrier n. Constraint (10) states that the co-channel sets should be feasible, while constraint (11) refers to the transmit power constraint for each BS. In one time slot, the set of achievable user rate vectors in a subcarrier n is the set of achievable user modulation vectors for each co-channel set, i.e., $\mathcal{R}^{(n)} = \{\mathbf{b}_{\mathbf{u}(n)} : \exists\mathbf{p}_{\mathbf{u}(n)} > \mathbf{0}$ s.t. $\mathbf{b}_{\mathbf{u}(n)}$ achievable and $\mathbf{u}(n) \in \mathcal{U}\}$. The set of achievable aggregate user rates in all N subcarriers is produced by taking the union of all power splitting rules over subcarriers for each BS.

$$\mathcal{R} = \bigcup_{\substack{\sum_n p_{u_i(n)}\leq P\\ i=1,\ldots,M}} \sum_{n=1}^{N} \mathbf{b}_{\mathbf{u}(n)} \tag{12}$$

such that $\mathbf{b}_{\mathbf{u}(n)}$ is achievable with power vector $\mathbf{p}_{\mathbf{u}(n)}$ for $n = 1,\ldots,N$, $\mathbf{u}(n) \in \mathcal{U}$. In a slotted time schedule, different BS activation vectors and user selections are used in subcarriers of time slots in order to achieve certain aggregate user rate vectors. Although in this work we are concerned with assignment policies within one slot, we implicitly maintain a view towards rate provisioning across multiple time slots.

B. Proposed Heuristic Algorithms

In each slot, the objective is to identify feasible co-channel sets with large rate in each subcarrier, while satisfying the transmit power constraint P at each BS. Since the enumeration of all co-channel user sets is of exponential complexity, we resort to heuristic algorithms. We present three greedy heuristic algorithms that are based on two different principles. In the first two algorithms A and B, each subcarrier is considered separately and users from different BSs are inserted sequentially in the subcarrier based on some greedy criteria. The goal at each step is to create a feasible co-channel set with large rate. Thus, a "vertical" pattern in resource allocation is followed, if we view BSs as rows and (common) subcarriers as columns in a resource allocation array. On the other hand, algorithm C takes a "horizontal" approach, since at each step a BS performs power water-filling across its subcarriers by treating interference from other BSs in each subcarrier as background noise.

Our algorithms are centralized in the sense that they imply BS coordination. This can be achieved by a central controller or by signaling over the backhaul network. Each user j measures the signal received from periodic beacons that are sent from each BS i at each subcarrier n and estimates the downlink path gains $G_{ij}^{(n)}$. The user returns those measurements to its serving BS as feedback and the BS passes them further to other BSs. Each time a BS takes an allocation step and inserts a user in a subcarrier (as in algorithms A and B) or performs water-filling (as in algorithm C), the allocation details are communicated instantaneously to other BSs so that all know the next BS that executes the algorithm.

The algorithms involve subcarrier allocation, transmit power control, and modulation adaptation. The bits of one OFDM symbol of each selected user from a BS are allocated to subcarriers. The allocation decision includes the set of allocated subcarriers, the number of loaded bits and the transmit power to each user. Since subcarrier quality is constant in a slot, this allocation is replicated for the S symbols of each selected user in a slot. Temporal channel quality variations are taken into account by changing the allocation decision in subsequent time slots. The objective is to provide maximum aggregate system rate within a time horizon spanning several time slots or time frames. As a heuristic means of avoiding unfair allocations where users with constantly favorable channel conditions are preferred over users with poor channel conditions, the algorithm could provide minimum rate guarantees. Users that reach their minimum rate levels before the end of the time horizon could be disregarded in allocations at subsequent time slots, while users that have not satisfied minimum rate requirements could be given priority. If all users satisfy minimum rate requirements, the algorithm could then consider all users again for the remainder of the time horizon. Our algorithms can be modified to cope with such scenarios as well.

Algorithms A and B: Algorithm A uses criteria such as induced interference to co-channel users, received interference from co-channel transmissions, and amount of rate increase. In order to maintain reasonable complexity, the algorithm involves sequential user assignment in a subcarrier and no user reassignments. However, modulation-level reassignments for co-channel users are allowed. At each step, an appropriate user is selected for assignment to the subcarrier and modulation levels and transmit powers of other users are adjusted, so that acceptable SINRs are ensured. Fix attention to subcarrier n and let $\mathcal{U}^{(n)}$ be the set of users that are already assigned in n and $\mathcal{B}^{(n)}$ be the set of APs that transmit to users in $\mathcal{U}^{(n)}$.

For each candidate user for insertion k, we do the following reasoning. User k is preferable for assignment in n if:

1) the BS-user gains $\left\{G^{(n)}_{i_k j} : j \in (\mathcal{U}^{(n)} \cup \{k\})\right\}$ and $\left\{G^{(n)}_{i_j k} : i_j \in \left(\mathcal{B}^{(n)} \cup \{i_k\}\right)\right\}$ should be such that interference that is caused by BS i_k serving user k to users in $\mathcal{U}^{(n)}$ is low and interference that is caused by BSs in $\mathcal{B}^{(n)}$ to user k is low.
2) the BS transmit powers should enforce the low interference objectives above.
3) the resulting modulation level vector after insertion of the new user should be such that the incurred rate benefit from user insertion at the subcarrier is maximal.

Let us consider objective 3) first for each candidate user k for insertion. If the BS-user link (i_k, k) is inserted, users in $\mathcal{U}^{(n)}$ receive additional interference from BS i_k. The modulation levels that were used before insertion of user k may need to be reduced to ensure that the new co-channel set is feasible. On the other hand, inserted user k also receives interference from BSs in $\mathcal{B}^{(n)}$. The insertion of a user in a subcarrier is beneficial for subcarrier rate if the total decrease in modulation levels of existing users is less than the number of contributed bits of the new user k. Then, the result of insertion of k is an increase of subcarrier rate. In fact, the most desirable user is the one for which this rate increase is maximal. In order to formalize this rule, let $\mathbf{b}_n^- = \{b_{n,\ell} : \ell \in \mathcal{U}^{(n)}\}$ and $\mathbf{b}_n^+ = \{b_{n,\ell} : \ell \in \mathcal{U}^{(n)} \cup \{k\}\}$ be the modulation level vectors of the co-channel set before and after insertion of user k. Define the rate increment factor (RIF) for each candidate user k as

$$\begin{aligned} T_{n,k} &= \sum_{\ell\in(\mathcal{U}^{(n)}\cup\{k\})} b^+_{n,\ell} - \sum_{\ell\in\mathcal{U}^{(n)}} b^-_{n,\ell} \\ &= b^+_{n,k} + \sum_{\ell\in\mathcal{U}^{(n)},\ell\neq k} \left(b^+_{n,\ell} - b^-_{n,\ell}\right). \end{aligned} \tag{13}$$

Thus, we need to find the modulation vector $\mathbf{b}_n^+$ so that rate increase is maximal and to check whether this vector is achievable through a transmit power vector. Namely, check whether condition $\lambda^* \leq 1$ holds for the $(|\mathcal{U}^{(n)}|+1) \times (|\mathcal{U}^{(n)}|+1)$ matrix $\tilde{\mathbf{G}}(\mathbf{b}_n^+)$ that corresponds to co-channel set $(\mathcal{U}^{(n)} \cup \{k\})$. There exist $b_{L_0}\left(\prod_{\ell=1}^{|\mathcal{U}^{(n)}|} b^-_{n,\ell}\right)$ options of possible modulation levels for modulation vector $\mathbf{b}_n^+$. We can either check all modulation vectors or proceed heuristically by starting from vector $(\mathbf{b}_n^-, b_{L_0})$ and decreasing the modulation level in one entry until we find an achievable vector.

If $T_{n,k} > 0$, we compute the signal-interference factor (SIF) of k as

$$S_{n,k} = \frac{p_{i_k} G^{(n)}_{i_k k}}{\max\left\{p_{i_k} \sum\limits_{j\in\mathcal{U}^{(n)}} G^{(n)}_{i_k j},\ \sum\limits_{i_j\in\mathcal{B}^{(n)}} p_{i_j} G^{(n)}_{i_j k}\right\}} \tag{14}$$

where the transmit powers are computed from power vector $\mathbf{p} = \left[\mathbf{I} - \tilde{\mathbf{G}}\left(\mathbf{b}_n^+\right)\right]^{-1} \mathbf{v}\left(\mathbf{b}_n^+\right)$ that is associated with the achievable modulation vector $\mathbf{b}_n^+$ we found. It can be seen that SIF captures objectives 1) and 2) above. All three objectives are taken into account if we define an assignment preference factor (APF) $A_{n,k}$ as $A_{n,k} = S_{n,k} T_{n,k}$. The user that is inserted in the subcarrier is the one with maximum APF. This multiplicative form stands for a rate increment factor that is scaled by the ratio of useful and interference powers. Among users that cause the same rate increase, the one with the smallest amount of received or induced interference is preferred. Among users that cause or receive the same amount of interference, the more preferable is the one that incurs larger rate benefit. By allowing the least interference increase in the system, we also implicitly favor future assignments.

If k is the first user to be inserted in n, then $S_{n,k} = G^{(n)}_{i_k k}$ and $T_{n,k} = b_{L_0}$ or, more precisely, $T_{n,k}$ is the maximum modulation level for which SNR exceeds the corresponding threshold. Since user assignment should only increase the already achieved subcarrier rate, the sequential assignment of users terminates when $T_{n,k} \leq 0$ for all remaining candidate users k.

Algorithm B uses the expression for the RIF in (13), but it uses a different SIF factor. The preferable user assignment is the one that maximizes minimum SINR of users in the subcarrier over all possible assignments. Since users can have different modulation levels, SINRs are scaled by the corresponding thresholds. First, RIF values $T_{n,k}$ are computed for each user k. If $T_{n,k} > 0$, the SIF factor is computed as

$$S_{n,k} = \min\left\{\frac{\mathrm{SINR}_{n,k}}{\gamma_{n,k}},\ \min_{j\in\mathcal{U}^{(n)}} \frac{\mathrm{SINR}_{n,j}}{\gamma_{n,j}}\right\} \tag{15}$$

where $\gamma_{n,k}$ and $\gamma_{n,j}$ are the SINR thresholds corresponding to modulations of users k and $j \in \mathcal{U}^{(n)}$. By attempting to balance the scaled SINRs, Algorithm B aims at increasing the number of users with SINRs above thresholds. The APF factor is $A_{n,k} = S_{n,k} T_{n,k}$, and the user with the maximum APF is selected for assignment.

A note about the transmit power constraints at each BS is in place here. Each BS has a total power budget P to use across all its subcarriers in a slot. Each time a user from a BS is assigned to a subcarrier, an amount of transmit power is used for this transmission. A BS can use a subcarrier for transmission in a slot as long as it has sufficient residual power to use. The main steps of algorithms A and B are as follows.

- **Step 0**: Consider the first subcarrier n. Initialize candidate user list $\mathcal{L}$ to $\bigcup_{i=1}^{M} \mathcal{U}_i$.
- **Step 1**: Compute APF $A_{n,k}$ for users $k \in \mathcal{L}$ with $T_{n,k} > 0$. If for some users k_0 the APF and subsequent transmit power levels result in a co-channel set such that some BSs do not have enough residual power to carry out the transmission, remove users k_0 from $\mathcal{L}$. Select user $k^* \in \mathcal{L}$ with maximum APF and assign k^* to subcarrier n. Update power budgets of BSs. Remove k^* and all users served by BS i_{k^*} from $\mathcal{L}$.
- **Step 2**: Compute the RIF each for $k \in \mathcal{L}$. If $\exists k_0 \in \mathcal{L}$ with $T_{n,k_0} > 0$, go to step 1. Else go to step 3.
- **Step 3**: If at least one of the following conditions hold, (i) $\mathcal{L}$ is empty or (ii) $T_{n,k} \le 0, \forall\, k \in \mathcal{L}$, the assignment for subcarrier n is terminated. Go to next step.
- **Step 4**: If at least one BS has positive residual power, proceed to subcarrier $n+1$. Go to step 0 and repeat steps 0–3. Repeat the procedure for each subcarrier.
- **Step 5**: Replicate the subcarrier, modulation, and power assignment for the S symbols of each user in a slot.
- **Step 6**: Obtain a new CSI report (if any change has occurred) and repeat the procedure for each slot in the specified time interval. If a user reaches its minimum rate requirements, do not consider it for allocation until all users reach their rate requirements.

The complexity of the algorithms depends on the method of finding the achievable modulation vector. If this is done heuristically, then we have complexity of $O\left(L_0 K M^5\right)$ per subcarrier. The computationally intensive part is the $O(M^3)$-complexity eigenvalue computation of an $M \times M$ matrix that may be required up to ML_0 times. The complexity can be decreased to $O\left(L_0 K M^2\right)$ if power control is not performed to the expense of loss in performance. Such orders of complexity are not prohibitive for small or moderate-sized systems. Although our approach is directly applicable in WLANs with some tens of subcarriers (e.g., 52 as in 802.11a), it can also be applied in systems with more subcarriers, such as 802.16a with 2048 subcarriers and channel bandwidth of 20 MHz. Since the subcarrier bandwidth in that case is small, the set of subcarriers can be divided in subbands, each with some subcarriers. Since neighboring subcarriers in a subband are very likely to exhibit similar characteristics, each subband is treated in the allocation as a carrier with one channel gain.

Algorithm C : Algorithms A and B focused on deriving an achievable modulation vector of large total rate at each subcarrier and on maintaining a feasible co-channel set at each step. Algorithm C takes a "horizontal" view and considers one BS at each step, while keeping interference from other BSs fixed. The BS allocates each subcarrier to the user that can make best use of it. Then, it splits the power across subcarriers with water-filling. In order to have a fair basis of comparison with algorithms A and B, no user reallocations are assumed.

Fix attention to a BS i and assume there exists receiver noise of variance σ^2 at each subcarrier. BS i allocates each subcarrier n to the user with the larger gain in that subcarrier, namely user $j_{n,i} = \arg\max_{j \in \mathcal{U}_i} G_{ij}^{(n)}$. Given this subcarrier allocation, the total information-theoretic rate achieved by BS i across all subcarriers, $\sum_{n=1}^{N} \log\left(1 + (G_{ij_{n,i}}^{(n)} p_i^{(n)})/\sigma^2\right)$ is maximized if the total power P is allocated to subcarriers with water-filling, that is,

$$p_i^{(n)} = \left(\frac{1}{\lambda} - \frac{\sigma^2}{G_{ij_{n,i}}^{(n)}}\right)^+, \qquad \text{for } n = 1, \ldots, N \tag{16}$$

where λ is the Lagrange multiplier that emerges from power constraint $\sum_{n=1}^{N} p_i^{(n)} \le P$ and can be computed with this constraint and $x^+ = x$ if $x > 0$ and 0 otherwise. The maximum modulation level that can be supported by user $j_{n,i}$ is $b_{j_{n,i}} = \max\left\{b \in \mathcal{M} : G_{ij_{n,i}}^{(n)} p_i/\sigma^2 \ge \gamma(b)\right\}$. We define the modulation vector of BS i as $\mathbf{b}_i = (b_i(n) : n = 1, \ldots, N) = (b_{j_{n,i}} : n = 1, \ldots, N)$.

BSs take turns in running the algorithm. The first BS $i(1)$ that is selected to perform water-filling is the one that achieves largest rate, i.e.,

$$i(1) = \arg \max_{i=1,\ldots,M} \sum_{n=1}^{N} b_i(n). \tag{17}$$

Next, another BS $i(2)$ will be selected to perform subcarrier allocation and water-filling. Each candidate BS $i(2)$ will again allocate each subcarrier to the user that will make best use of it. BS $i(2)$ allocates subcarriers by taking into account the existing co-channel interference in each subcarrier from the already allocated power $p_{i(1)}^{(n)}$ of $i(1)$. Thus, from $i(2)$, user

$$j_{n,i(2)} = \arg \max_{j \in \mathcal{U}_{i(2)}} \frac{G_{i(2)j}^{(n)}}{G_{i(1)j}^{(n)} p_{i(1)}^{(n)} + \sigma^2} \tag{18}$$

is allocated to subcarrier n. BS $i(2)$ then performs water-filling across subcarriers and computes modulation levels for users in subcarriers. However, it may happen that modulations in co-channel users at some subcarriers are not achievable for the current power allocations, since these were performed independently by each BS. For each subcarrier, we can find the achievable modulation vector of co-channel users in it for the assigned power levels. For each BS, we find the BS modulation vector that consists of modulation levels of users across its subcarriers.

Let $\mathcal{B}$ be the set of selected BSs until a certain stage of the algorithm. At the next stage, the selection of the new BS is based on a rate increment criterion similar in flavor to the RIF of algorithms A and B. Let $\mathbf{b}^- = \left\{\mathbf{b}_\ell^- : \ell \in \mathcal{B}\right\}$ and $\mathbf{b}^+ = \left\{\mathbf{b}_\ell^+ : \ell \in \mathcal{B} \cup \{i\}\right\}$ be the ensembles of BS modulation vectors before and after selection (and subsequent water-filling) of the new BS i. For algorithm C, the RIF for each candidate BS i is defined as

$$T_i = \sum_{\ell \in \mathcal{B} \cup \{i\}} \sum_{n=1}^{N} b_\ell^+(n) - \sum_{\ell \in \mathcal{B}} \sum_{n=1}^{N} b_\ell^-(n). \tag{19}$$

At each step, we select the BS i with the maximum RIF and remove it from the list of candidate BSs. Similarly to algorithms A and B, algorithm C terminates when no further rate increase is possible, namely when $T_i \leq 0$ for all remaining candidate BSs i. Finally, we note that another version of the algorithm could be as follows. After finding the achievable modulation vector at each subcarrier, reduce the powers to the necessary levels and distribute the total excess power to subcarriers in an effort to increase the rate in some of them. We will not consider this option further.

C. Optimal Solution for Special Cases

Here, we provide optimal solutions for some simple special cases of the problem.

Consider one subcarrier and two BS-user links. Let rates be continuous variables and assume no receiver noise and no power constraints. The goal is to assign transmit powers p_1 and p_2 and rates b_1 and b_2 to links so as to maximize total continuous rate. Note that this is an upper bound on the rate with discrete rate variables. The problem is formulated as follows:

$$\max_{(p_1,p_2,b_1,b_2)} (b_1 + b_2) \tag{20}$$

subject to

$$\frac{G_{11}p_1}{G_{21}p_2} \geq c(2^{b_1} - 1) \text{ and } \frac{G_{22}p_2}{G_{12}p_1} \geq c\left(2^{b_2} - 1\right) \tag{21}$$

with $p_i, b_i \geq 0$, for $i = 1, 2$, and $c = -\ln(5\epsilon)/1.5$ from the BER requirement. By using the standard method of Lagrange multipliers, we obtain the optimal solution

$$\begin{aligned} b_1^* &= b_2^* \\ &= \frac{1}{ln2} \ln\left(1 + \frac{1}{c}\sqrt{\frac{G_{11}G_{22}}{G_{12}G_{21}}}\right) \end{aligned} \tag{22}$$

$$\frac{p_1}{p_2} = \sqrt{\frac{G_{21}G_{22}}{G_{11}G_{12}}}. \tag{23}$$

Consider $M = 2$ BSs and let $\mathcal{U}_i$ be the set of users in BS i, for $i = 1, 2$. Let N subcarriers and L_0 modulation levels be to the disposal of BSs and let $\gamma(b)$ be the SIR threshold for modulation level b. Also, assume no transmit power constraints at each BS. Let $G_{ij}^{(n)}$ be the path gain from BS i to user $j \in \mathcal{U}_1 \bigcup \mathcal{U}_2$ in subcarrier n. Construct a bipartite graph $G = (V_1 \bigcup V_2, E)$ as follows. Node set V_i consists of N nodes for each user $j \in \mathcal{U}_i$, for $i = 1, 2$. Let $v_j(n)$ be the nth node of user j corresponding to use of subcarrier n, for $j = 1, \ldots, |\mathcal{U}_1|$ and $j = 1, \ldots, |\mathcal{U}_2|$ and $n = 1, \ldots, N$.

Between each node $v_j(n) \in V_1$ and $v_k(n) \in V_2$, we add three kinds of edges. These correspond to subcarrier reuse by users j, k or to subcarrier use by one user j or k. First, consider subcarrier reuse by both users. For user pair (j, k) and subcarrier n we can find the achievable modulation vector with the maximum sum of components in two ways: either solve the continuous optimization problem above and round down the derived b_i's, or check each of the $b_{L_0}^2$ possible modulation vectors for achievability through power control with the eigenvalue condition. The derived modulation vector is then used to assign a weight $w_{j,k}(n)$ to this edge, equal to the sum of components of the vector. The weight of each one of the other two kinds of edges is the maximum modulation level that can be supported for each user alone.

A matching $\mathcal{K}$ in graph G is a subset of edges of G, such that no two edges in $\mathcal{K}$ share the same node. An edge in $\mathcal{K}$ is called a matched edge. A maximum-weight matching $\mathcal{K}^*$ is a matching that has the largest total weight of edges. It can be seen that the problem of finding the maximum total rate of this system is equivalent to finding a maximum-weight matching in the bipartite graph above. The principle of the algorithm for finding a maximum-weight matching is to start with an empty matching and at each stage to find an augmenting path with the maximal increase in weight [16]. Thus, the algorithm for solving optimally this special case can be viewed as the basis of the rationale of our greedy heuristics, even if for more than two BSs the appropriate co-channel set does not reduce to an edge of the matching.

IV. PERFORMANCE RESULTS

We consider a 8 km × 8 km area with 16 BSs. Square cells are arranged in four rows and four columns, and each BS is located at the center of a cell. Users are located at fixed but random positions, uniformly distributed in the area. A user is served by its closest BS and does not have minimum rate requirements. All BS use OFDM with the same set of 20 subcarriers. Path loss causes a decay of $1/d^4$ in received power with distance d from a BS. Multipath is modeled with two paths. The complex gain of each path is an independent log-normal random variable with zero-mean and standard deviation 10 dB, which accounts for shadow fading. The delay of each path is uniformly distributed in $[0, T]$, where T is the symbol period. A target BER of 10^{-3} at the receiver is assumed. The gains for each BS-user link at each subcarrier are generated with this model. We assume additive white Gaussian noise (AWGN) with $\sigma = 0.01$ at each receiver. The power constraint per BS is such that the BS can transmit with maximum modulation level in a hypothetical scenario with no co-channel users and 100 channels.

Performance is measured in terms of average subcarrier rate, namely number of loaded bits per subcarrier. The objectives of the simulations are: 1) to establish the relative impact of modulation level and power control on performance of algorithms A and B and 2) to assess performance of algorithms A, B, and C and compare to a near-optimal solution. For each experiment, we generate random user locations. For each set of locations, we create a different instance of gain matrices per user per slot by changing multipath parameters and we find the average over 10 000 such instances. Unless otherwise stated, each experiment is repeated for 100 random location sets. The outcome is the average of these experiments.

First, we discuss the relative significance of modulation and power control in algorithms A and B. We consider the following schemes: 1) modulation and power control; 2) modulation control only, where the powers do not appear in the SIF expressions; and 3) power control only. In the latter case, SIFs and RIFs are

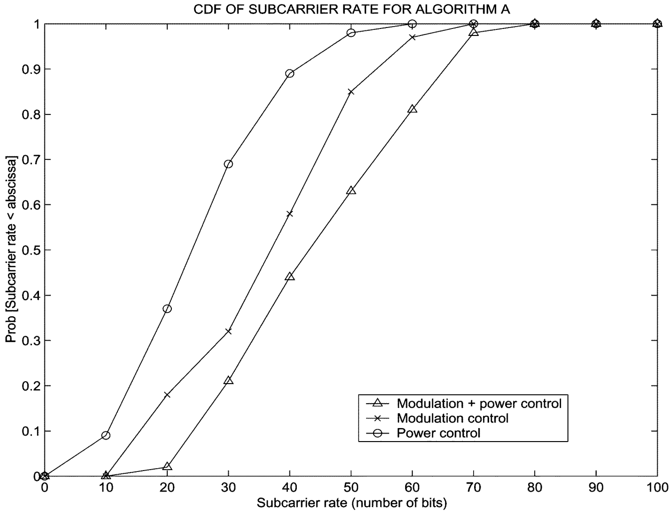

Fig. 2. CDF of total rate per subcarrier for algorithm A and different adaptation schemes.

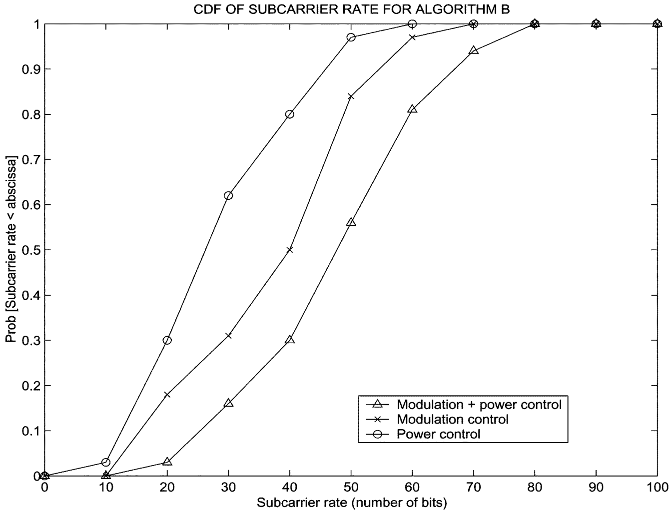

Fig. 3. CDF of total rate per subcarrier for algorithm B and different adaptation schemes.

computed as usual. The user that is inserted into the channel is the one with maximum APF, subject to the constraint that all co-channel users use a common modulation level (which may change with each user insertion).

Fig. 2 depicts the cumulative distribution function (cdf) of subcarrier rate for algorithm A and the three methods above. When the modulation level is controllable, six modulation levels were used, while in power control alone a common modulation level was used for all co-channel users. For 16 BSs, the maximum rate is 96 bits, since at most one user per BS uses a subcarrier. The achievable rate is further limited by fading. We observe that power control provides the lowest average rate, while joint modulation and power control yields the best performance. For example, with joint modulation and power control almost 40% of subcarriers achieve or exceed a rate of 50 bits, which implies that subcarriers are reused efficiently. For power control alone, this percentage is 1%. Modulation control alone provides satisfactory performance and could be used so as to avoid high complexity of joint modulation and power control.

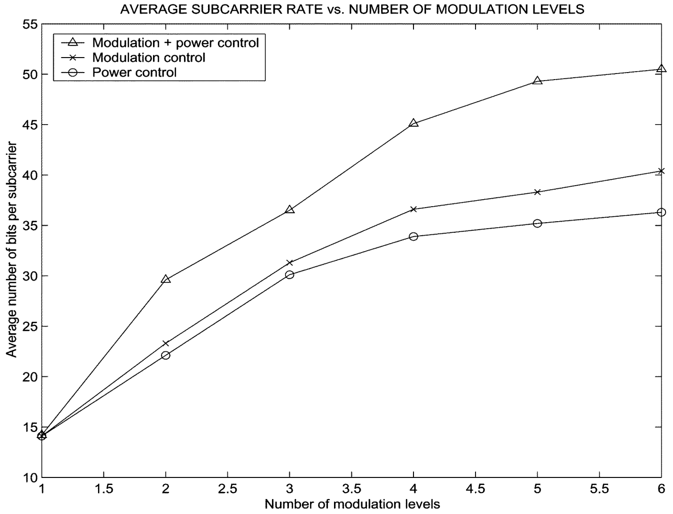

Fig. 4. Average rate per subcarrier versus number of available modulation levels.

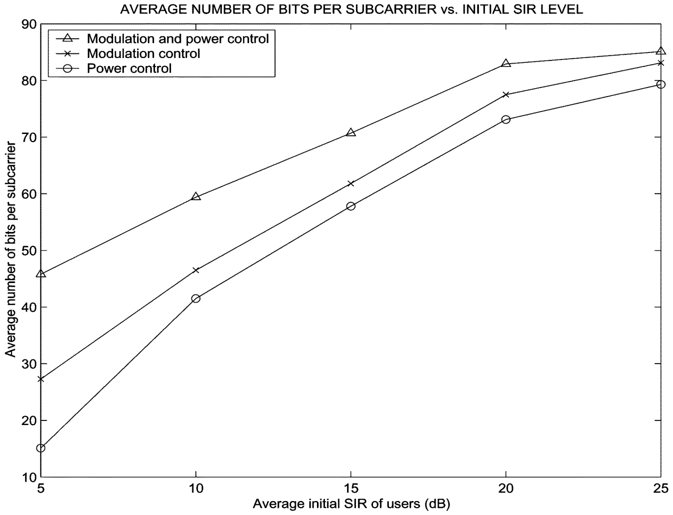

Fig. 5. Average rate per subcarrier for different initial SIR values.

Similar trends appear in Fig. 3 for algorithm B. In addition, algorithm B yields significant gains compared to A because of the different SIF expression. An improvement of about 10% is observed in percentages of subcarriers that exceed a certain rate. We also note here the similarity of the plots in our results with those obtained in [8] for one channel.

Fig. 4 shows the average subcarrier rate as a function of number of modulation levels for algorithm B. When k modulation levels are used, these are $b_1, \ldots, b_k$. It is shown that addition of power control in adaptive modulation is beneficial up to five modulation levels. For instance, for four modulation levels, the performance gain of joint modulation and power control over modulation control alone is double than that with three modulation levels. The use of more than four modulation levels has marginal improvement on performance. In Fig. 5, the performance is depicted as a function of BS-user proximity for algorithm B. Sets of random locations were generated and for each set the average BS-user proximity was computed and mapped to path loss of the BS-user link. Assuming that BSs

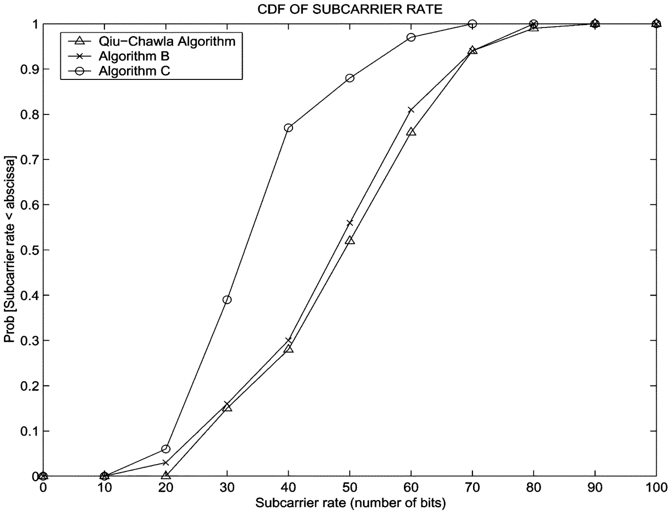

Fig. 6. CDF of total rate per subcarrier for algorithms B and C and the algorithm in [8].

transmit with fixed power, the initial SIR of user j is $\mathrm{SIR}_j^0 = \left(\sum_{i=1,i\neq j}^{M} (d_{jj}/d_{ij})^4\right)^{-1}$, where d_{ij} is the distance between BS i and user j and d_{jj} is the distance between user j and its serving BS. Locations were generated until a sufficient number of scenarios with some average initial SIR was collected. A point of x dB in the horizontal axis corresponds to user sets with initial SIRs in $[x, x+1]$ dB. Low initial SIRs denote users located far from serving BSs or close to interfering BSs.

Modulation control is shown to alleviate interference better than power control and to achieve higher subcarrier rate. Power control does not provide good performance for low initial SIRs, since SIR balancing is not effective at high interference regimes. For milder interference conditions (higher SIR values), the difference in performance becomes less evident. Joint modulation and power control always achieves the best performance.

Finally, we show the performance of heuristics B and C and of the approach in [8] in Fig. 6. For the latter, we run the algorithm that maximizes the product of SINRs for the final co-channel set. Algorithm B performs well in the sense that achievable subcarrier rate of the co-channel set is close to that obtained by the algorithm in [8]. Furthermore, “vertical” allocation algorithm B outperforms “horizontal” water-filling-based algorithm C. This implies that subcarriers are more efficiently reused or equivalently that transmit power of each BS is more efficiently allocated. This is anticipated since in algorithms A and B the transmit power vector that ensures a feasible co-channel set is the one with minimum sum of entries.

V. DISCUSSION

We addressed the joint subcarrier allocation, modulation adaptation, and power control problem in a multicell OFDM system with the objective to maximize aggregate achievable rate. In such a system, our main concern is to manage the arising co-channel interference from neighboring cells. We found that the “vertical” approach with successive user assignment in subcarriers based on a scaled SINR balancing criterion achieves the best performance. The rationale of the algorithm relies on maximizing rate increase at each step. In that sense, it can be considered as an extension of the approach that solves optimally a special case by finding the maximum-weight matching of a graph. Our results provide valuable insights about the structure of an efficient heuristic and further constitute the basis for devising efficient distributed algorithms that would be applicable either in the uplink of one cell or in multicell systems. In these cases, each user or each BS respectively could take allocation decisions separately by attempting to maximize their own incurred benefit.

Our approach is extensible to other multiple access schemes such as TDMA or CDMA with deterministic codes. In the latter, the channels are nonorthogonal codes with pairwise cross correlation that creates cross-channel interference as well. Similar tradeoffs arise here as well, since codes with low spreading gain have higher rates, but they also have higher cross correlation with other codes and they are associated with lower SINRs.

In the presented snapshot model, we did not capture system dynamics such as arrival rates, rate adaptation, and channel variation and their impact on buffer status. It is meaningful to study subcarrier allocation and user scheduling policies that maximize rate and maintain a stable system in terms of bounded buffer lengths. Furthermore, our algorithms admittedly treat users with poor channel conditions unfairly and do not provide any rate guarantees. It would be interesting to address rate allocation fairness. This problem is challenging in OFDM, since rate allocation is performed both in the frequency domain with subcarrier allocation and in the time domain with scheduling. In a multicell system, an additional degree of freedom is the user selection rule from BSs.

Finally, we have used the expression of BER requirement that applies to uncoded modulation so as to have an analytically tractable model at hand and capture the susceptibility of the modulation level to interference. Our model can accommodate the case of independent coding at each subcarrier, where the BER expression would depend on coding rate as well. If coding across subcarriers is used, the encoder performs the coding in the time domain and subsequently bits are loaded to subcarriers. Then, analytical tractability becomes difficult since the number of coded bits that are distributed in each subcarrier is not known. Under the assumption of Gaussian interference and certain assumptions on the decoder and the interleaver that distributes coded bits into subcarriers, there exist upper bound expressions on BER [17]. The BER requirement would then correspond to a constraint on allocated bits and powers collectively across subcarriers.

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