ΘΕΩΡΙΑ ΣΗΜΑΤΩΝ ΚΑΙ ΣΥΣΤΗΜΑΤΩΝ

ΛΥΣΕΙΣ $3^{ΗΣ}$ ΣΕΙΡΑΣ ΑΣΚΗΣΕΩΝ

1.

$$X(\omega) = \frac{1}{(a+j\omega)^2} = \left(\frac{1}{a+j\omega}\right)\left(\frac{1}{a+j\omega}\right)$$

But,

$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$$

and

$$x_1(t) * x_2(t) \longleftrightarrow X_1(\omega)\,X_2(\omega)$$

Therefore we have:

$$\begin{aligned} x(t) &= e^{-at}u(t) * e^{-at}u(t) \\ &= \int_{-\infty}^{\infty} e^{-a\tau}u(\tau)e^{-a(t-\tau)}u(t-\tau)\,d\tau \\ &= e^{-at}\int_0^t d\tau = te^{-at}u(t) \end{aligned}$$

Hence,

$$te^{-at}u(t) \longleftrightarrow \frac{1}{(a+j\omega)^2}$$

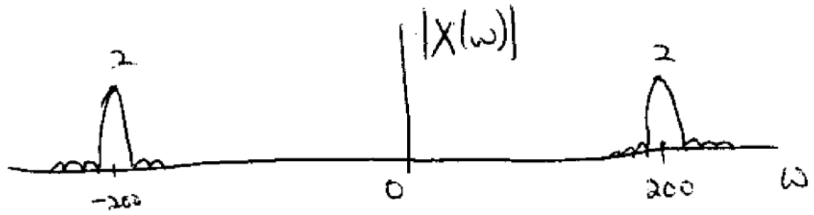
2.

a.

$$x(t) = \cos(200t)\, p_4(t)$$

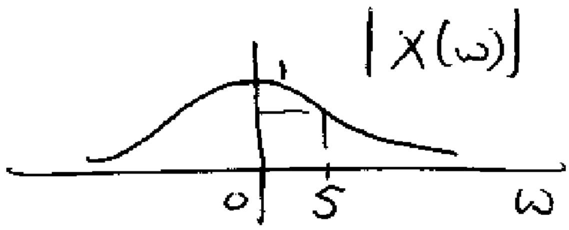
$$p_4(t) \longleftrightarrow 4\,\text{sinc}\left(\frac{2\omega}{\pi}\right)$$

$$\cos(200t)\, p_4(t) \longleftrightarrow 2\,\text{sinc}\left(\frac{2}{\pi}(\omega-200)\right) + 2\,\text{sinc}\left(\frac{2}{\pi}(\omega+200)\right)$$



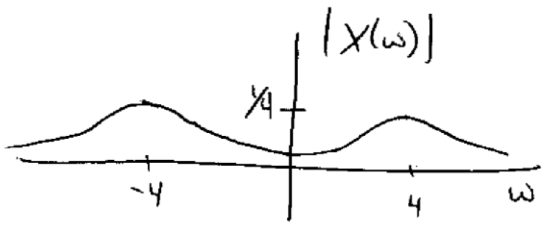
b.

$$X(\omega) = \frac{5}{5+j\omega}$$



c.

$$X(\omega) = \frac{1}{2}\left[\frac{1}{j(\omega-4)+2} + \frac{1}{j(\omega+4)+2}\right]$$



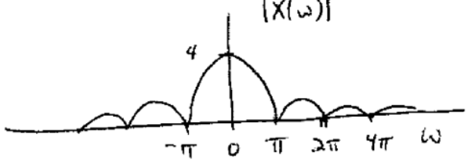
3.

1. $x(t) = 2\,p_2(t-1)$

$$X(\omega) = 2(2)\,\mathrm{sinc}\left(\frac{2\omega}{2\pi}\right) e^{-j\omega}$$

$$= 4\,\mathrm{sinc}\left(\frac{\omega}{\pi}\right) e^{-j\omega}$$

$$|X(\omega)| = 4\left|\mathrm{sinc}\left(\frac{\omega}{\pi}\right)\right|$$



2. $x(t) = 3\,p_2(t-3) + 3\,p_2(t+3)$

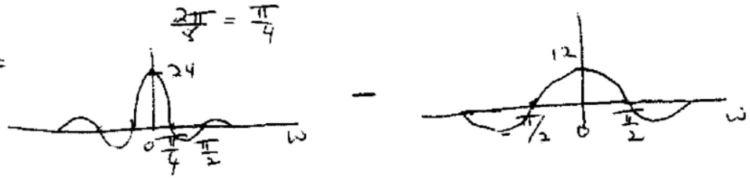
or $\quad = 3\,p_8(t) - 3\,p_4(t)$

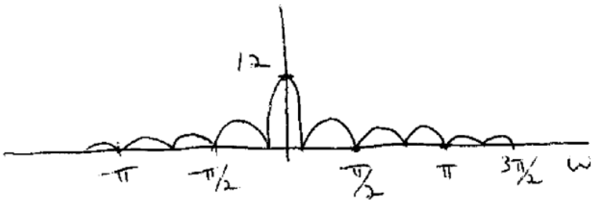
$$X(\omega) = 3(8)\,\mathrm{sinc}\left(\frac{8\omega}{2\pi}\right) - 3(4)\,\mathrm{sinc}\left(\frac{4\omega}{2\pi}\right)$$

$$= 24\,\mathrm{sinc}\left(\frac{4\omega}{\pi}\right) - 12\,\mathrm{sinc}\left(\frac{2\omega}{\pi}\right)$$

zero crossings at $\frac{2\pi}{8} = \frac{\pi}{4}$ zero crossings at $\frac{2\pi}{4} = \pi/2$

$|X| =$





$$x(t) = t2p_4(t-2)$$

$$p_4(t-2) \longleftrightarrow 4\ \mathrm{sinc}\left(\frac{2w}{\pi}\right) e^{-2jw}$$

$$2t\,p_4(t-2) \longleftrightarrow j\,2\frac{d}{dw}\left[4\frac{\sin(2w)}{2w}e^{-2jw}\right]$$

$$= 4j\left[\frac{w(2\cos(2w)e^{-2jw} - 2j\ \sin(2w)e^{-2jw})}{w^2} - \frac{\sin(2w)e^{-2jw}}{w^2}\right]$$

$$= \frac{4j}{w^2}e^{-2jw}\left(w2\cos(2w) - 2jw\sin(2w) - \sin(2w)\right)$$

$$= \frac{4j}{w^2}e^{-2jw}\left(w2e^{-2jw} - \sin(2w)\right)$$

4.

a) $X(\underline{\Omega}) = 2\pi\left(\delta(\underline{\Omega}-1) - \delta(\underline{\Omega}+1)\right) + 3\left(\delta(\underline{\Omega}-2\pi) + \delta(\underline{\Omega}+2\pi)\right)$

<u>Ιδιότητες Fourier</u>

$2\pi\,\delta(\underline{\Omega}) \leftrightarrow 1$

$X(\underline{\Omega}-\underline{\Omega}_0) \leftrightarrow x(t)\,e^{j\underline{\Omega}_0 t}$

$2\pi\,\delta(\underline{\Omega}-1) \leftrightarrow e^{jt}$

$2\pi\,\delta(\underline{\Omega}+1) \leftrightarrow e^{-jt}$

$\delta(\underline{\Omega}-2\pi) \leftrightarrow \frac{1}{2\pi}\,e^{j2\pi t}$

$\delta(\underline{\Omega}+2\pi) \leftrightarrow \frac{1}{2\pi}\,e^{-j2\pi t}$

άρα

$$x(t) = e^{jt} - e^{-jt} + \frac{3}{2\pi}e^{j2\pi t} + \frac{3}{2\pi}e^{-j2\pi t}$$

$$= 2j\,\frac{e^{jt}-e^{-jt}}{2j} + \frac{3}{\pi}\,\frac{e^{j2\pi t}+e^{-j2\pi t}}{2}$$

$$= 2j\sin(t) + \frac{3}{\pi}\cos(2\pi t)$$

β) $\frac{j\underline{\Omega}}{1+\underline{\Omega}^2}$ όταν $e^{-|t|} \leftrightarrow \frac{2}{1+\underline{\Omega}^2}$

<u>Ιδιότητα</u>: $\frac{d^n x(t)}{dt^n} \leftrightarrow (j\underline{\Omega})^n X(\underline{\Omega})$

άρα $j\underline{\Omega}\cdot X(\underline{\Omega}) \leftrightarrow \frac{dx(t)}{dt}$

Έστω $X(\Omega) = \dfrac{1}{1+\Omega^2}$ για το $\dfrac{j\Omega}{1+\Omega^2}$ με $e^{-|t|} \longleftrightarrow \dfrac{2}{1+\Omega^2}$

άρα $\dfrac{j\Omega}{1+\Omega^2} \longleftrightarrow \dfrac{1}{2}\dfrac{d}{dt}\left(e^{-|t|}\right)$

$$\frac{d}{dt}\left(e^{-|t|}\right) = \frac{d}{dt}\left(e^{-t}u(t) + e^{t}u(-t)\right)$$

$$= -e^{-t}u(t) + e^{t}u(-t)$$

άρα $\dfrac{j\Omega}{1+\Omega^2} \longleftrightarrow \dfrac{1}{2}\left(e^{t}u(-t) - e^{-t}u(t)\right)$

5.

(1) $R(\Omega) \stackrel{?}{=} -\frac{2}{\pi} \int_0^{+\infty} \int_0^{+\infty} I(\varphi)\sin(\varphi t)\cos(\Omega t)\, d\varphi\, dt$

Έχουμε $X(\Omega) = \int_0^{+\infty} x(t) e^{-j\Omega t} dt$

και $x(t) = -\frac{2}{\pi} \int_0^{+\infty} I(\Omega)\sin(\Omega t)\, d\Omega$

οπότε:

$$X(\Omega) = \int_0^{+\infty} \left(-\frac{2}{\pi} \int_0^{+\infty} I(\varphi)\sin(\varphi t)\, d\varphi \right) e^{-j\Omega t} dt$$

$$= -\frac{2}{\pi} \int_0^{+\infty} \int_0^{+\infty} I(\varphi)\sin(\varphi t) \left(\cos(\Omega t) - j\sin(\Omega t) \right) d\varphi\, dt$$

$$= -\frac{2}{\pi} \int_0^{+\infty} \int_0^{+\infty} I(\varphi)\sin(\varphi t)\cos(\Omega t)\, d\varphi\, dt$$

$$+ j\frac{2}{\pi} \int_0^{+\infty} \int_0^{+\infty} I(\varphi)\sin(\varphi t)\sin(\Omega t)\, d\varphi\, dt$$

$X(\Omega) = R(\Omega) + jI(\Omega) \Rightarrow R(\Omega) = -\frac{2}{\pi} \int_0^{+\infty} \int_0^{+\infty} I(\varphi)\sin(\varphi t)\cos(\Omega t)\, d\varphi\, dt$