ΑΣΚΗΣΗ 1.3:

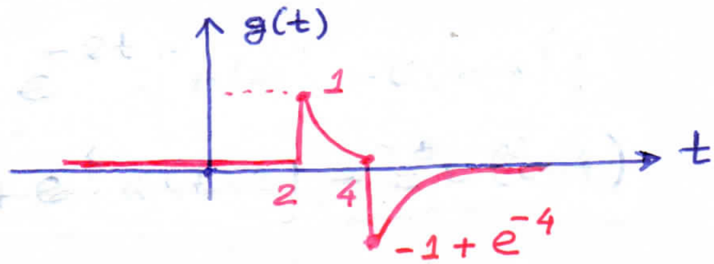
$$\boxed{\begin{aligned} x(t) &= u(t-2) - u(t-4) \\ h(t) &= e^{-2t} u(t) \end{aligned}}$$

(B) $\boxed{g(t) = \frac{dx(t)}{dt} * h(t)}$

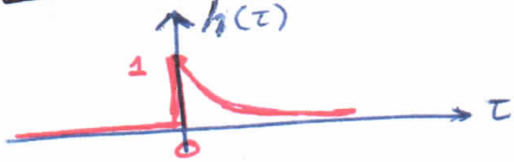
$$\frac{dx(t)}{dt} = \frac{d}{dt}\Big(u(t-2) - u(t-4)\Big) = \delta(t-2) - \delta(t-4)$$

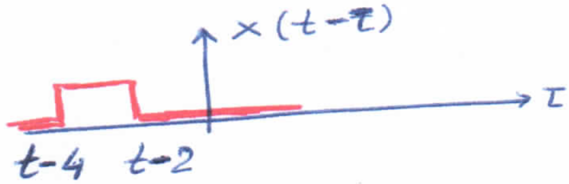
$$\Longrightarrow g(t) = h(t-2) - h(t-4)$$

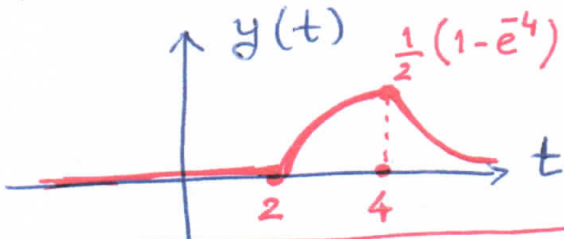
$$= e^{-2(t-2)} u(t-2) - e^{-2(t-4)} u(t-4)$$



A $\boxed{x(t) * h(t)} = y(t)$







ΠΕΡΙΠΤΩΣΗ

- $t-2 \leq 0 \Leftrightarrow \boxed{t \leq 2}$

  $\hookrightarrow \Rightarrow \boxed{y(t) = 0}$

- $\left.\begin{matrix} t-2 \geq 0 \\ t-4 \leq 0 \end{matrix}\right\} \Leftrightarrow \boxed{2 \leq t \leq 4}$

  $\hookrightarrow y(t) = \int_0^{t-2} e^{-2\tau} d\tau$

  $= -\frac{1}{2} e^{-2\tau} \Big|_0^{t-2} = \boxed{\frac{1}{2}\left(1 - e^{-2(t-2)}\right)}$

- $t-4 \geq 0 \Leftrightarrow \boxed{t \geq 4}$

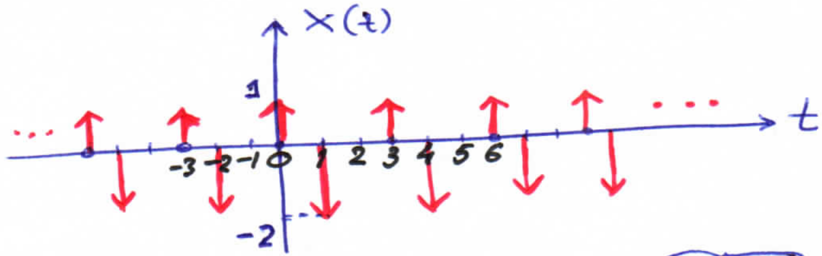
  $\hookrightarrow y(t) = \int_{t-4}^{t-2} e^{-2\tau} d\tau = \boxed{\frac{1}{2}\left(e^{-2(t-4)} - e^{-2(t-2)}\right)}$

Άρα:

$$\boxed{y(t) = \begin{cases} 0, & t \leq 0 \\ \frac{1}{2}\left[1 - e^{-2(t-2)}\right], & 2 \leq t \leq 4 \\ \frac{1}{2}(e^{8} - e^{4})\, e^{-2t}, & t \geq 4 \end{cases}}$$

ΑΣΚΗΣΗ 1.4:

$$\boxed{x(t) = \sum_{k=-\infty}^{+\infty}\Big(\delta(t-3k) - 2\,\delta(t-1-3k)\Big)}$$



Το σήμα είναι περιοδικό με περίοδο $\boxed{T=3}$, κατά συνέπεια προχωρούμε στην αναπαράστασή του ως σειρά Fourier:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\Omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{2\pi}{3}t}$$

με συντελεστές

$$a_k = \frac{1}{3}\int_{3} x(t)\, e^{-jk\frac{2\pi}{3}t}\,dt = \frac{1}{3}\int_{-\varepsilon}^{3-\varepsilon}\left[\delta(t) - 2\delta(t-1)\right] e^{-jk\frac{2\pi}{3}t}\,dt$$

$-\varepsilon \rightsquigarrow$ ε θετικό $\cong 0$

$$= \frac{1}{3}\left[1 - 2e^{-jk\frac{2\pi}{3}}\right]$$

$$= \frac{1}{3}\left[1 + \left(-\frac{1}{2}\right)^{k-1}\left(1+\sqrt{3}\right)^{k}\right]$$