Π. Θ. ΤΗΜΜΥ | ΗΥ 241: ΣΗΜΑΤΑ & ΣΥΣΤΗΜΑΤΑ | ΕΞ. 2016-17 ΕΑΡΙΝΟ

ΤΕΛΙΚΗ ΕΞΕΤΑΣΗ — Α

1A $x[n] = \dfrac{1-\cos^2(\pi n/4)}{n^2}$

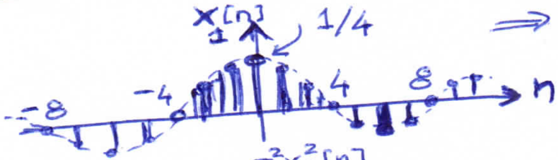
ΣΧΕΔΙΟ? ΕΝΕΡΓΕΙΑΣ? ΙΣΧΥΟΣ?

• Από τριγωνομετρία:

$$x[n] = \frac{\sin^2(\pi n/4)}{n^2} = \pi^2 \cdot \Big[\underbrace{\frac{\sin(\pi n/4)}{\pi n}}_{x_1[n]}\Big]^2$$

$$\Rightarrow \boxed{x[n] = \pi^2 x_1^2[n]} \quad (1)$$

• ΣΧΕΔΙΟ:

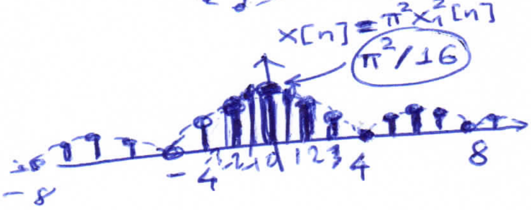


ZERO CROSSINGS @: $\frac{\cancel{\pi} n}{4} = k\cancel{\pi} \Leftrightarrow$ $\Leftrightarrow n = 4k$

MAX VAL @ 0 by L'HOSPITAL

$$\frac{(\sin(\pi t/4))'}{(\pi t)'} = \frac{\cancel{\pi}}{4}\frac{1}{\cancel{\pi}}\cancel{\cos \pi t/4}^{\,1}$$

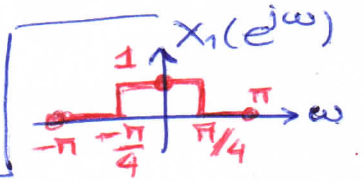
Άρα:



• Θα χρησιμοποιήσουμε το Θ. Parseval, υπολογιζόμενοι ότι πρόκειται για σήμα ενέργειας,

$$\boxed{\sum_{n=-\infty}^{+\infty} x^2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega} \quad (2)$$

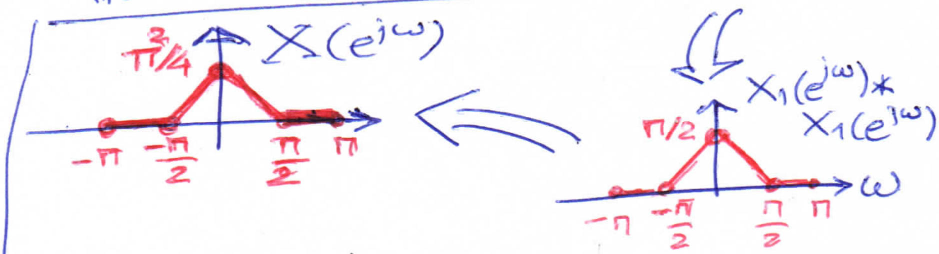
• Από ιδιότητα γινομένου (λόγω (1)):



$$X(e^{j\omega}) = \frac{1}{2\cancel{\pi}} \cdot \pi^{\cancel{2}} \cdot X_1(e^{j\omega}) * X_1(e^{j\omega})$$

$$= \begin{cases} \pi^2/4 \cdot (1 - 2|\omega|/\pi) & \text{για } |\omega| \le \pi/2 \\ 0, & \text{για } \frac{\pi}{2} < |\omega| \le \pi \end{cases} \quad (3)$$

(επαναλαμβανόμενο ανά 2π)



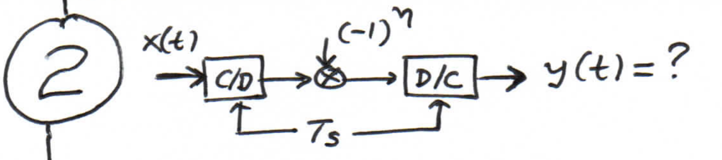
• Συνεπώς από (2) & (3) [και λόγω συμμετρίας]:

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \frac{1}{\pi}\int_{-\pi/2}^{0} \frac{\pi^2}{4}\left(1 + \frac{2\omega}{\pi}\right) d\omega = \frac{\pi}{4}\left[\frac{\pi}{2} + \frac{\cancel{2}\,\omega^2/\cancel{2}}{\pi}\Big|_{-\pi/2}^{0}\right] =$$

$$= \frac{\pi}{4}\left[\frac{\pi}{2} - \frac{\pi^{\cancel{2}}/4}{\cancel{\pi}}\right] = \frac{\pi}{4}\cdot\frac{\pi}{4} = \boxed{\frac{\pi^2}{16}}$$

1B $FT^{-1}\left\{\dfrac{4j\Omega}{1+2\Omega^2+\Omega^4}\right\} = ?$

• $X(j\Omega) = j\dfrac{4\Omega}{(1+\Omega^2)^2} = -j\dfrac{d}{d\Omega}\Big[\underbrace{\dfrac{2}{1+\Omega^2}}_{\overset{\mathcal{F}}{\longleftrightarrow}\, e^{-|t|}}\Big] \Longrightarrow \boxed{x(t) = -t e^{-|t|}}$

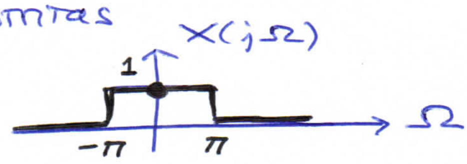


$x(t) = \dfrac{\sin(\pi t)}{\pi t}$, $\Omega_s = 3\pi$

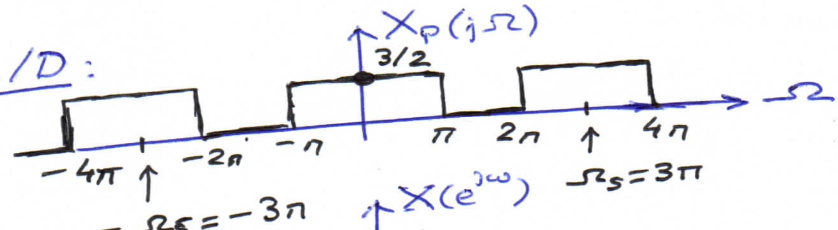
$T_s = \dfrac{2\pi}{3\pi} = \dfrac{2}{3}$

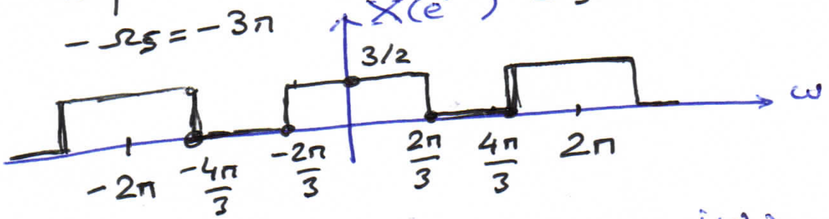
- Δουλεύουμε στο πεδίο της συχνότητας

$x(t) = \dfrac{\sin(\pi t)}{\pi t} \Longrightarrow X(j\Omega)$:



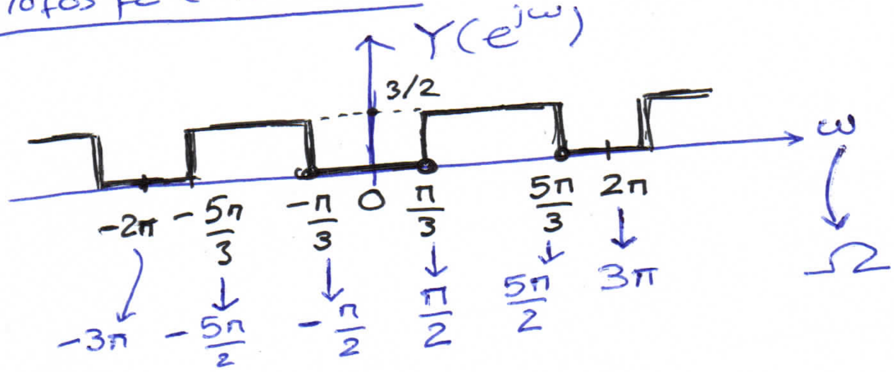
- C/D:



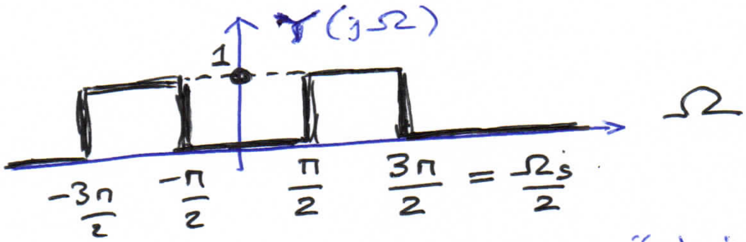


- Πολ/σμός με $(-1)^n = e^{j\pi n}$

$Y(e^{j\omega}) = X(e^{j(\omega-\pi)})$



- D/C



- Το φάσμα αυτό μπορεί να θεωρηθεί ως διαφορά δύο "παλμών" ή άθροισμα δύο μετατοπισμένων παλμών. Άρα:

$$y(t) = \frac{\sin(3\pi t/2) - \sin(\pi t/2)}{\pi t}$$

Ισοδύναμα:

$$y(t) = \frac{\sin(\pi t/2)}{\pi t} \cdot \left(e^{j\pi t} + e^{-j\pi t}\right) = \frac{2\sin(\pi t/2)\cos\pi t}{\pi t}$$

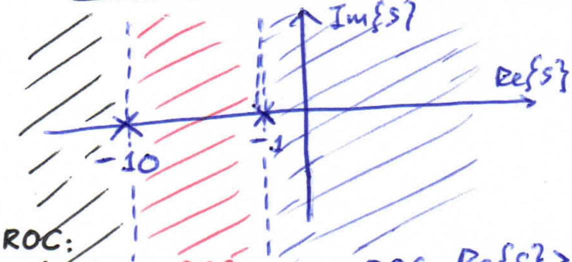
3 $\ddot{y}(t)+11\dot{y}(t)+10y(t)=10x(t)$ (1)

A $H(s)=?$

(1) $\Rightarrow$ $\boxed{H(s)=\dfrac{10}{s^2+11s+10}}$ (2)

B POLES, ZEROS ROC/PROPERTIES

(2) $\Rightarrow$ $H(s)=\dfrac{10}{(s+1)(s+10)}$ (3)



ΠΟΛΟΙ: $\{-1,-10\}$
ΜΗΔΕΝΙΚΑ: $\{\infty,\infty\}$

ROC: $Re\{s\}<-10$ ΑΣΤΑΘΕΣ ΜΗ ΑΙΤΙΑΤΟ

ROC: $-10<Re\{s\}<-1$ ΑΣΤΑΘΕΣ ΜΗ ΑΙΤΙΑΤΟ

ROC: $Re\{s\}>-1$ ΕΥΣΤΑΘΕΣ ΑΙΤΙΑΤΟ (4)

C $x(t)=u(t-2)$ ΕΥΣΤΑΘΕΣ $\Longrightarrow y(t)=?$

• Λόγω ΓΧΑ, αρκεί να βρούμε την έξοδο στο $u(t)$.

$$\left.\begin{array}{l} x'(t)=u(t)\Rightarrow X'(s)=\dfrac{1}{s},\quad Re\{s\}>0 \\ (3),(4)\Rightarrow H(s)=\dfrac{10}{(s+1)(s+10)},\quad Re\{s\}>-1 \end{array}\right\}\Rightarrow Y'(s)=\dfrac{10}{s(s+1)(s+10)},\quad Re\{s\}>0$$

$$\Rightarrow Y'(s)=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+10}$$

$$A=\left.\frac{10}{(s+1)(s+10)}\right|_{s=0}=\frac{10}{1\cdot 10}=1$$

$$B=\left.\frac{10}{s(s+10)}\right|_{s=-1}=\frac{10}{(-1)(+9)}=-\frac{10}{9}$$

$$C=\left.\frac{10}{s(s+1)}\right|_{s=-10}=\frac{10}{(-10)(-9)}=\frac{1}{9}$$

$$\Rightarrow Y'(s)=\frac{1}{s}-\frac{10}{9}\frac{1}{s+1}+\frac{1}{9}\frac{1}{s+10}\Rightarrow \quad Re\{s\}>0$$

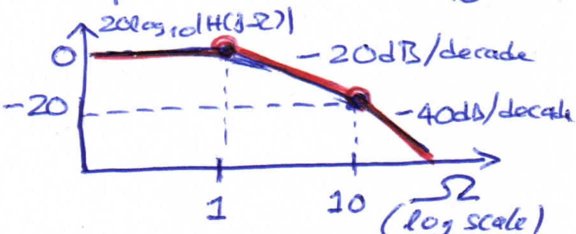
$$\Rightarrow y'(t)=\dots \Rightarrow \left(\overset{\text{ΛΟΓΩ:}}{y(t)=y'(t-2)}\right)\Rightarrow$$

$$\boxed{y(t)=\left[1-\frac{10}{9}e^{-(t-2)}+\frac{1}{9}e^{-10(t-2)}\right]u(t-2)}$$

D BODE PLOT $20\log_{10}|H(j\Omega)|$

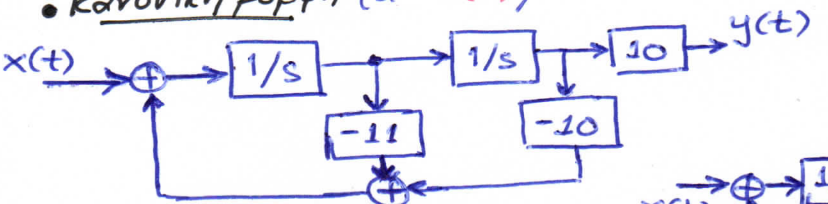
Ευστάθεια (3),(4) $\Rightarrow H(j\Omega)=\dfrac{10}{(j\Omega+1)(j\Omega+10)}=\dfrac{1}{\left(\frac{j\Omega}{1}+1\right)\left(\frac{j\Omega}{10}+1\right)}$

$$\Rightarrow 20\log_{10}|H(j\Omega)|=-20\log_{10}\left|\frac{j\Omega}{1}+1\right|-20\log_{10}\left|\frac{j\Omega}{10}+1\right|$$



E IMPL. DIAGRAMS

• Κανονική μορφή (από (2))



• Εν σειρά → (από (3))