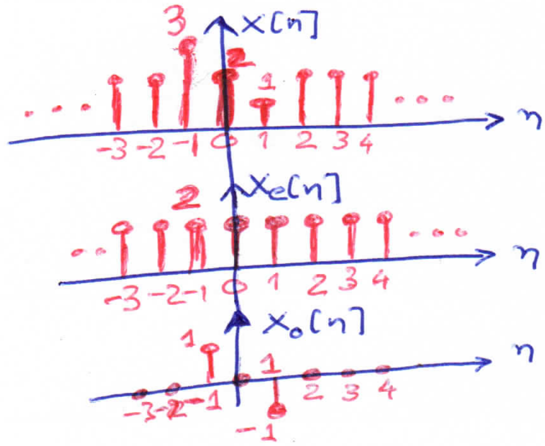
2016-17 ΕΑΡ | ΗΥ 241 | ΠΡΟΟΔΟΣ

Β | ΣΗΜΑΤΑ & ΣΥΣΤΗΜΑΤΑ | 5-4-17

1 (A) $x[n] = 2 - \delta[n-1] + \delta[n+1]$
$= x_e[n] + x_o[n]$



Εύκολα βλέπουμε ότι:

$$x_e[n] = 2$$
$$x_o[n] = \delta[n+1] - \delta[n-1],$$

καθώς, προφανώς,

$$x_e[n] + x_o[n] = x[n]$$
$$x_e[n] = x_e[-n]$$
$$x_o[n] = -x_o[-n]$$

1 (B) $y[n] = \exp\{-x[n]\}$

ΓΧΑ; Ευστάθεια;

Αντιστρέψιμο;

- Δεν είναι γραμμικό, άρα ούτε και ΓΧΑ, αφού: $e^{-x_1[n]} + e^{-x_2[n]} \underset{\text{εν γένει}}{\neq} e^{-x_1[n]} \cdot e^{-x_2[n]}$

- Είναι αντιστρέψιμο, καθώς $y[n] = e^{-x[n]} \Rightarrow \ln y[n] = -x[n] \Rightarrow$
$\Rightarrow x[n] = -\ln y[n]$

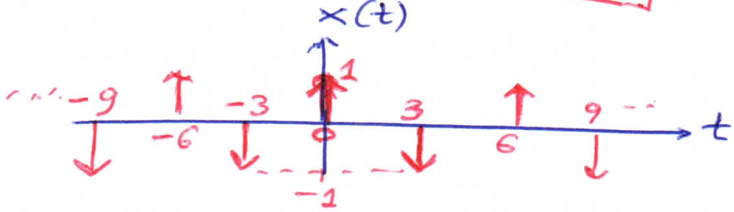
$$x[n] \rightarrow \boxed{y[n] = e^{-x[n]}} \xrightarrow{y[n]} \boxed{w[n] = -\ln y[n]} \rightarrow w[n] = x[n]$$

- Είναι ευσταθές, γιατί:

$$|x[n]| < B_x \Rightarrow |-x[n]| < B_x \Rightarrow |y[n]| < e^{B_x} \equiv B_y$$

② $x(t)=\sum_{n=-\infty}^{+\infty}(-1)^n\,\delta(t-3n) \Rightarrow F.S.\,?$

- Σχέδιο σήματος:

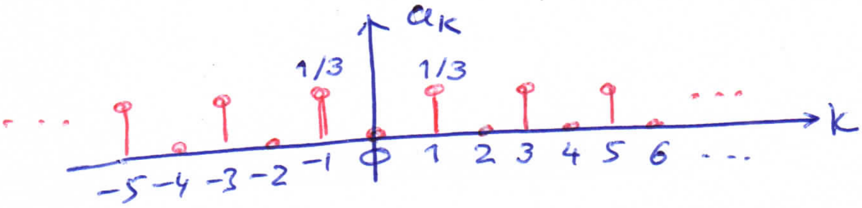


- Το σήμα είναι περιοδικό, με $\boxed{T=6} \Rightarrow \Omega_0=\frac{2\pi}{6}=\frac{\pi}{3}$

- Άρα F.S: $x(t)=\sum_{k=-\infty}^{+\infty} a_k\, e^{jk\frac{\pi}{3}t}$

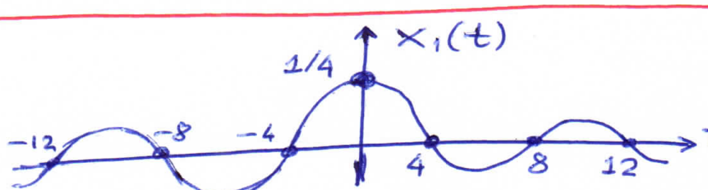
με: $a_k=\frac{1}{6}\int_{-1}^{5} x(t)\,e^{-jk\frac{\pi}{3}t}\,dt=\frac{1}{6}\int_{-1}^{5}\left[\delta(t)-\delta(t-3)\right]e^{-jk\frac{\pi}{3}t}\,dt$

$$=\frac{1}{6}\left[1-e^{-jk\pi}\right]=\frac{1}{6}\left[1-(-1)^k\right]=\begin{cases}\frac{1}{3}, & k \text{ ΠΕΡΙΤΤΟ}\\ 0, & k \text{ ΑΡΤΙΟ}\end{cases}$$

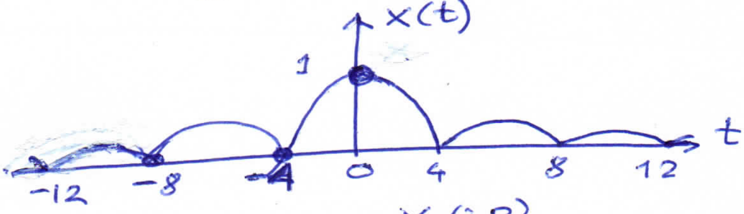


$$\textcircled{3} \quad x(t)=\left[\frac{\sin(\pi t/4)}{\pi t/4}\right]^2 \Longrightarrow \text{ΣΧΕΔΙΟ},\ X(j\Omega),\ \int_{-\infty}^{+\infty} x^2(t)\,dt$$

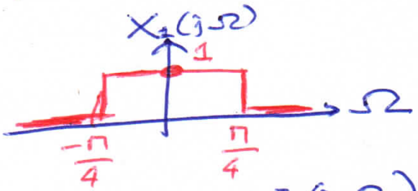
• Έστω $\boxed{x_1(t)=\left[\frac{\sin(\pi t/4)}{\pi t}\right]}$ ①



Τότε $\boxed{x(t) = 16\,x_1^2(t)}$ ②



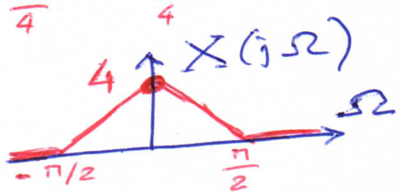
• Από ① $\Rightarrow X_1(j\Omega)=\begin{cases}1, & |\Omega|<\frac{\pi}{4}\\ 0, & \text{αλλού}\end{cases}$ $\xrightarrow{\text{ΣΧΗΜΑ}}$



• Από ② & ιδιότητα πολ/σμού στον χρόνο:

$$X(j\Omega)=\frac{16}{2\pi}\cdot X_1(j\Omega) * X_1(j\Omega) \quad ③$$

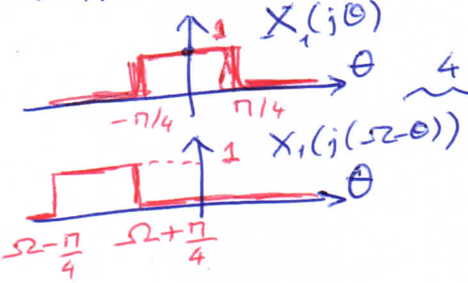
$$\frac{\cancel{16}}{\cancel{2\pi}} \to \frac{8}{\pi}$$



• Από πράξεις συνέλιξης:

$$X_1(j\Omega) * X_1(j\Omega) = \begin{cases}\frac{\pi}{2}-|\Omega|; & |\Omega|<\frac{\pi}{2}\\ 0; & \text{αλλού}\end{cases} \overset{③}{\Longrightarrow} \boxed{X(j\Omega)=\begin{cases}4\left(1-\frac{2|\Omega|}{\pi}\right); & |\Omega|<\frac{\pi}{2}\\ 0; & \text{αλλού}\end{cases}}$$

• Πιο λεπτομερώς:



$$\left.\begin{array}{l}\Omega+\frac{\pi}{4}<-\frac{\pi}{4} \Leftrightarrow \Omega<-\frac{\pi}{2}\\ \Omega-\frac{\pi}{4}>\frac{\pi}{4} \Leftrightarrow \Omega>\frac{\pi}{2}\end{array}\right\} \Rightarrow \text{NO OVERLAP}$$

$\xrightarrow{4\ \text{ΠΕΡΙΟΧΕΣ}}$

$$-\frac{\pi}{4}\le \Omega+\frac{\pi}{4}\le\frac{\pi}{4} \Leftrightarrow -\frac{\pi}{4}\le\Omega\le 0 \Rightarrow \int_{-\pi/4}^{\Omega+\frac{\pi}{4}} d\theta = \Omega+\frac{\pi}{2}$$

$$-\frac{\pi}{4}\le \Omega-\frac{\pi}{4}\le\frac{\pi}{4} \Leftrightarrow 0\le\Omega\le \frac{\pi}{4} \rightsquigarrow \int_{\Omega-\frac{\pi}{4}}^{\pi/4} d\theta = \frac{\pi}{2}-\Omega$$

• Από M/Σ Fourier: $\int_{-\infty}^{+\infty} x(t)\,dt = X(j\Omega)\Big|_{\Omega=0} = \boxed{4}$

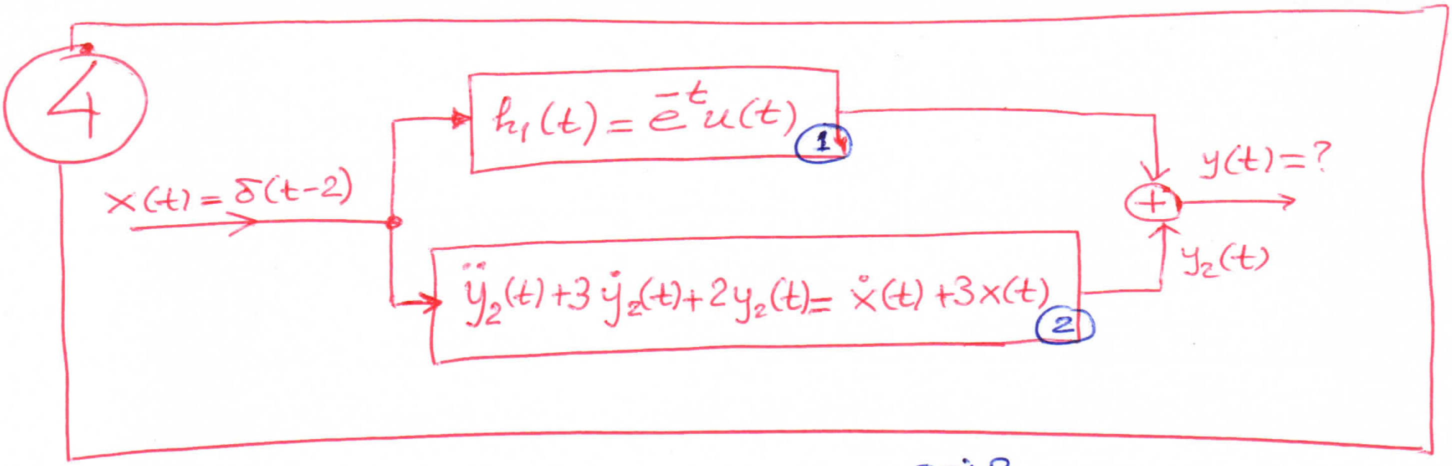
• Από Θ. PARSEVAL: $\int_{-\infty}^{+\infty} x^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\Omega)|^2\,d\Omega =$

$$= \frac{16}{2\pi}\left[\int_{-\pi/2}^{0}\left(1+\frac{2\Omega}{\pi}\right)^2 d\Omega + \int_{0}^{\pi/2}\left(1-\frac{2\Omega}{\pi}\right)^2 d\Omega\right] =$$

$$\frac{\pi}{2}-\frac{2}{\pi}\frac{\pi^2}{4}+\frac{4}{3\pi^2}\frac{\pi^3}{8}=\frac{\pi}{6}$$

$$= \frac{16}{2\pi}\left[\int_{-\pi/2}^{0}\left(1+\frac{4\Omega}{\pi}+\frac{4\Omega^2}{\pi^2}\right)d\Omega + \cdots\right] = \frac{16}{2\pi}\left[\left(\Omega+\frac{2\Omega^2}{\pi}+\frac{4\Omega^3}{3\pi^2}\right)\Big|_{-\pi/2}^{0} + \cdots\right]$$

$$= \frac{32}{2\cancel{\pi}}\frac{\cancel{\pi}}{6} = \boxed{\frac{8}{3}}$$



- Από (1): $H_1(j\Omega) = \dfrac{1}{j\Omega + 1}$ (γιατί $1>0$) $\overset{s=j\Omega}{=} \dfrac{1}{s+1}$ (3)

- Από (2): $H_2(j\Omega) = \dfrac{j\Omega + 3}{(j\Omega)^2 + 3j\Omega + 2} \overset{s=j\Omega}{=} \dfrac{s+3}{(s+2)(s+1)}$ (4)

- Παράλληλη συνδεσμολογία $\Rightarrow H(j\Omega) = H_1(j\Omega) + H_2(j\Omega)$ (5)

- Από (3),(4),(5) $\Rightarrow H(s) = \dfrac{1}{s+1} + \dfrac{s+3}{(s+1)(s+2)} = \dfrac{2s+5}{(s+1)(s+2)} \Rightarrow$

$$\Rightarrow H(s) = \frac{A}{s+1} + \frac{B}{s+2} \quad (6)$$

- Από μεθοδολογία ανάλυσης σε μερικά κλάσματα:

$$A = \left.\frac{2s+5}{s+2}\right|_{s=-1} = \frac{3}{1} = 3 \quad (7)$$

$$B = \left.\frac{2s+5}{s+1}\right|_{s=-2} = \frac{1}{-1} = -1 \quad (8)$$

- Από (6),(7),(8) $\Rightarrow H(s) = \dfrac{3}{s+1} - \dfrac{1}{s+2} \xRightarrow{s \to j\Omega}$

$$\Rightarrow H(j\Omega) = \frac{3}{j\Omega+1} - \frac{1}{j\Omega+2} \Rightarrow$$

$$\Rightarrow h(t) = (3e^{-t} - e^{-2t})u(t) \xRightarrow[\text{λόγω Γ.Χ.Α.}]{x(t)=\delta(t-2)}$$

$$\Rightarrow \boxed{y(t) = (3e^{-(t-2)} - e^{-2(t-2)})u(t-2)}$$