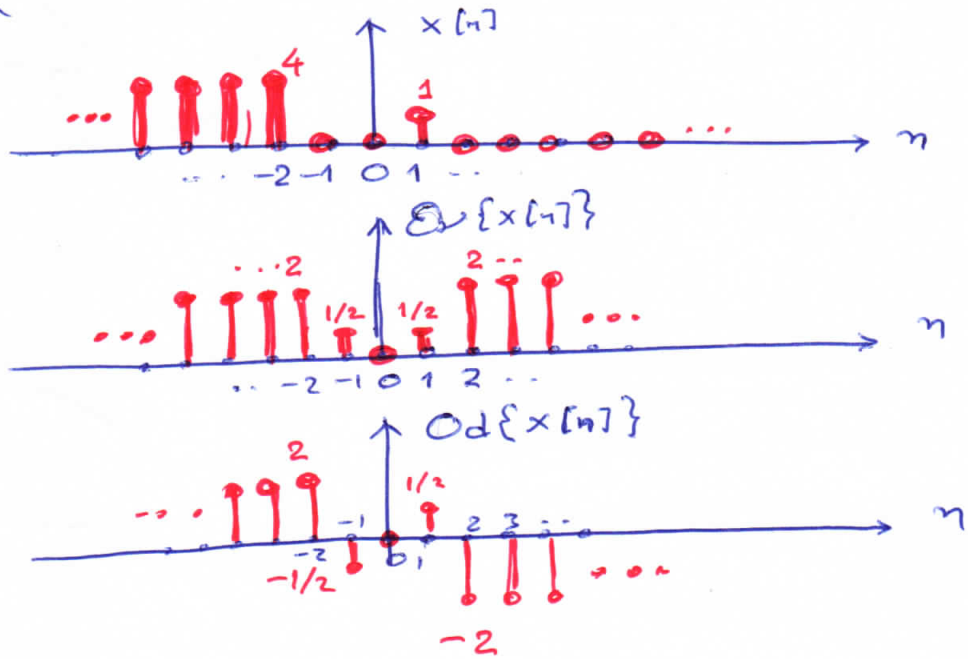
HY 241 — 13/14 — ΠΡΟΟΔΟΣ — Δ

Δ1a

$$\boxed{x[n] = \delta[n-1] + 4u[-n-2]}$$

$$Ev\{x[n]\} = \frac{x[n]+x[-n]}{2} = \frac{\delta[n-1]}{2} + \frac{\delta[-n-1]}{2} + 2u[-n-2] + 2u[n-2]$$

$$Od\{x[n]\} = \frac{x[n]+x[-n]}{2} = \frac{\delta[n-1]}{2} - \frac{\delta[-n-1]}{2} + 2u[-n-2] - 2u[n-2]$$



Δ1b

Για $N \geq 2$: $\dfrac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2 = \dfrac{1}{2N+1}\left(1 + 4^2(N-2)\right)$

$$= \frac{16N - 31}{2N+1} \xrightarrow[N\to\infty]{} 8$$

$$\boxed{\begin{array}{l} \text{POWER} = 8 \\ \text{ENERGY} = \infty \end{array}}$$

ΙΣΧΥΟΣ

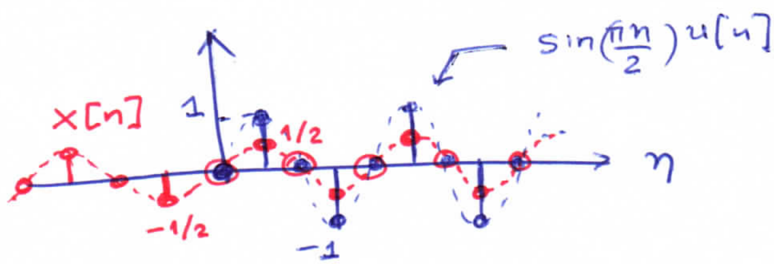
Δ1c

$$x[n] = Od\left\{ \sin\left(\frac{\pi n}{2}\right) u[n] \right\}$$

$$= \frac{1}{2}\left(\sin\left(\frac{\pi n}{2}\right) u[n] - \sin\left(\frac{-\pi n}{2}\right) u[-n] \right)$$

$$= \frac{1}{2}\left(\sin\left(\frac{\pi n}{2}\right) u[n] + \sin\left(\frac{\pi n}{2}\right) u[-n] \right) =$$

$$= \frac{1}{2} \sin\left(\frac{\pi n}{2}\right)$$ ⇝ ΠΕΡΙΟΔΙΚΟ ΜΕ ΠΕΡΙΟΔΟ $N_0 = \frac{2\pi}{\pi/2} = 4$



Δ2a

$y[n] = n^2 x[n+1]$	ΓΡΑΜΜΙΚΟ	?

Ναί, είναι!

$$\left.\begin{array}{l} x_1[n] \to y_1[n] \\ x_2[n] \to y_2[n] \end{array}\right\} \Longrightarrow \alpha x_1[n] + \beta x_2[n] \to$$

$$\to n^2 (\alpha x_1[n+1] + \beta x_2[n+1]) = \alpha n^2 x_1[n+1] + \beta n^2 x_2[n+1] =$$

$$= \alpha y_1[n] + \beta y_2[n]$$

Δ2β

$y(t) = 3\int_{-\infty}^{t} x(\tau)\,d\tau$	ΑΝΤΙΣΤΡΕΨΙΜΟ	?

Ναί, το αντίστροφο του ολοκληρωτή είναι ο διαφοριστής

$$\boxed{w(t) = \frac{1}{3}\frac{dy(t)}{dt}} = \frac{1}{\cancel{3}}\cancel{3}\frac{d}{dt}\int_{-\infty}^{t} x(\tau)\,d\tau = x(t)$$

Δ4

$$\boxed{\frac{d^2y(t)}{dt^2} + \frac{4\,dy(t)}{dt} + 3y(t) = 3\frac{dx(t)}{dt} + 5x(t)}$$

$$\xRightarrow{s=j\Omega} H(s) = \frac{3s+5}{s^2+4s+3} = \frac{3s+5}{(s+1)(s+3)} = \frac{1}{s+1} + \frac{2}{s+3}$$

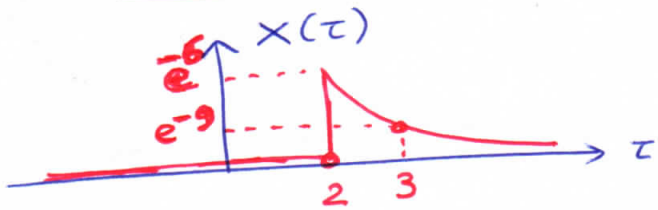
$$\left.\frac{3s+5}{s+3}\right|_{s=-1} = \frac{2}{2} = 1$$

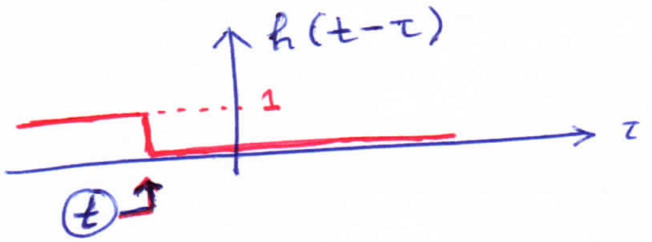
$$\left.\frac{3s+5}{s+1}\right|_{s=-3} = \frac{-4}{-2} = 2$$

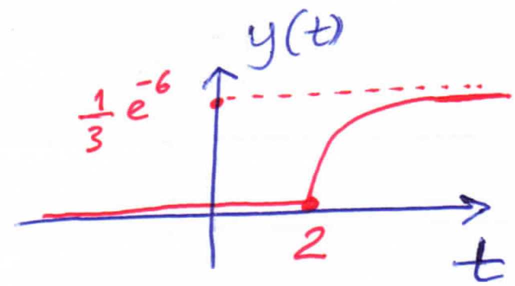
$$\Longrightarrow \boxed{h(t) = \left[e^{-t} + 2e^{-3t}\right]u(t)}$$

Δ3

$$\boxed{\begin{aligned} x(t) &= e^{-3t}u(t-2) \\ h(t) &= u(t) \end{aligned}}$$







$$t \le 2 \Rightarrow y(t) = 0$$

$$t \geq 2: \quad y(t) = \int_2^t e^{-3\tau}\,d\tau = -\frac{1}{3}e^{-3\tau}\Big|_2^t =$$

$$= \frac{1}{3}e^{-6} - \frac{1}{3}e^{-3t}$$

Άρα: $\boxed{y(t) = \left[\frac{1}{3}e^{-6} - \frac{1}{3}e^{-3t}\right]u(t-2)}$