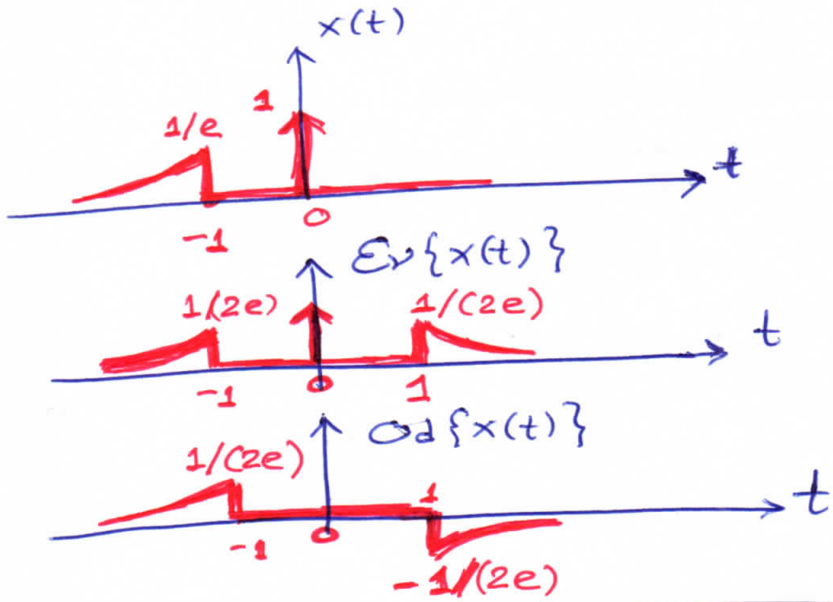
ΗΥ 241 – 13/14 – ΠΡΟΟΔΟΣ – Β

B1a

$$\boxed{x(t) = \delta(t) + e^{t} u(-t-1)}$$

$$\Rightarrow \begin{cases} \mathcal{E}v\{x(t)\} = \dfrac{x(t)+x(-t)}{2} = \dfrac{1}{2} e^{t} u(-t-1) + \dfrac{1}{2} e^{-t} u(t-1) + \delta(t) \\[2ex] \mathcal{O}d\{x(t)\} = \dfrac{x(t)-x(-t)}{2} = \dfrac{1}{2} e^{t} u(-t-1) - \dfrac{1}{2} e^{-t} u(t-1) \end{cases}$$



B1b

$$x(t) - \delta(t) = \boxed{e^{t} u(-t-1) = y(t)}$$

$$\text{Energy} = \int_{-\infty}^{+\infty} |y(t)|^2 dt = \int_{-\infty}^{-1} e^{2t} dt = \frac{1}{2} e^{2t} \Big|_{-\infty}^{-1} = e^{-2}/2.$$

$$\text{Power} = 0$$

B1c

$$x(t) = [\cos(2t - \pi/6)]^2 \rightsquigarrow \quad \cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\rightsquigarrow x(t) = \frac{1}{2} + \frac{1}{2} \cos(4t - \pi/3)$$

↳ ok

↳ ΠΕΡΙΟΔΙΚΟ $\frac{2\pi}{4} = \frac{\pi}{2}$ → $T = \pi/2$

B2a

$$\boxed{y[n] = 3x[n] + 2}$$ ΓΡΑΜΜΙΚΟ ?

ΟΧΙ λόγω του σταθερού όρου 2

πχ
$$\left.\begin{array}{l} x_1[n] \longrightarrow y_1[n] \\ x_2[n] \longrightarrow y_2[n] \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow x_1[n] + x_2[n] \longrightarrow 3(x_1[n] + x_2[n]) + 2$$

$$\neq$$

$$3x_1[n] + 3x_2[n] + 4$$

$$\parallel$$

$$y_1[n] + y_2[n]$$

B2b

$$\boxed{y(t) = [\cos(3t)]\, x(t)}$$ ΕΥΣΤΑΘΕΣ ?

ΝΑΙ, γιατί:

$$|x(t)| < B, \ \forall t \Longrightarrow$$

$$\Longrightarrow |y(t)| \leq |\cos(3t)|\,|x(t)| \leq |x(t)| = B \quad \forall t$$

B4

$$\boxed{\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{3dx(t)}{dt} + 7}$$

$$\overset{s=j\Omega}{\Longrightarrow} H(s) = \frac{3s+7}{s^2+4s+3} = \frac{3s+7}{(s+1)(s+3)} = \frac{2}{s+1} + \frac{1}{s+3}$$

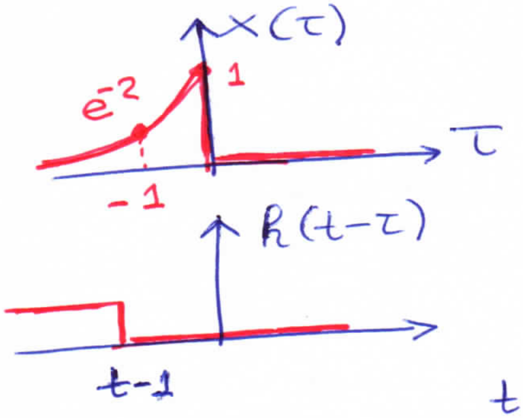
$$\left.\frac{3s+7}{s+3}\right|_{s=-1} = \frac{4}{2}$$

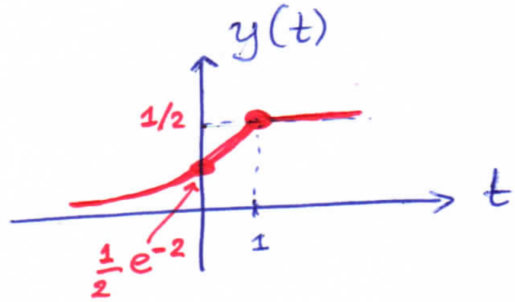
$$\left.\frac{3s+7}{s+1}\right|_{s=-3} = \frac{-2}{-2}$$

$$\Longrightarrow \boxed{h(t) = [2e^{-t} + e^{-3t}]\,u(t)}$$

B.3

$$\boxed{\begin{aligned} x(t) &= e^{2t}u(-t) \\ h(t) &= u(t-1) \end{aligned}}$$





$t-1 \leq 0 \Leftrightarrow t \leq 1$ $\quad \displaystyle\int_{-\infty}^{t-1} e^{2\tau}\,d\tau = \frac{1}{2} e^{2\tau}\Big|_{-\infty}^{t-1} =$

$$= \frac{1}{2} e^{2(t-1)}$$

$t-1 \geq 0 \Leftrightarrow t \geq 1$ $\quad \displaystyle\int_{-\infty}^{0} e^{2\tau}\,d\tau = \frac{1}{2} e^{2\tau}\Big|_{-\infty}^{0} = \frac{1}{2}$

<u>Αρα:</u> $y(t) = \begin{cases} \frac{1}{2} e^{2(t-1)}, & t \leq 1 \\ \frac{1}{2}, & t \geq 1 \end{cases}$