Π.Θ. ΤΗΜΜΥ

ΗΥ241: ΣΗΜΑΤΑ & ΣΥΣΤΗΜΑΤΑ

ΛΥΣΕΙΣ $2^{ου}$ ΣΕΤ ΑΣΚΗΣΕΩΝ

ΕΞ. 2018-19 ΕΑΡΙΝΟ

2.1.Α

$x[n] = a^n u[n] * a^n u[n-1] * a^n u[n+1] = ?$ (με $|a|<1$)

- Έστω $x_1[n] = a^n u[n]$, $x_2[n] = a^n u[n-1]$, $x_3[n] = a^n u[n+1]$

Τότε $X_1(e^{j\omega}) = \dfrac{1}{1-ae^{-j\omega}}$

$$x_2[n] = a\cdot a^{n-1}u[n-1] \underset{DTFT}{\Longrightarrow} X_2(e^{j\omega}) = \frac{a e^{-j\omega}}{1-ae^{-j\omega}}$$

$$x_3[n] = \frac{1}{a} a^{n+1} u[n+1] \underset{DTFT}{\Longrightarrow} X_3(e^{j\omega}) = \frac{1}{a}\,\frac{e^{j\omega}}{1-ae^{-j\omega}}$$

$$\xRightarrow{\text{ΙΔΙΟΤΗΤΑ ΣΥΝΕΛΙΞΗΣ}} X(e^{j\omega}) = X_1(e^{j\omega})X_2(e^{j\omega})X_3(e^{j\omega}) = \frac{1}{(1-ae^{-j\omega})^3} \Longrightarrow$$

$$\xRightarrow{DTFT^{-1}} x[n] = \frac{(n+2)!}{n!\,2!}a^n u[n] \Longrightarrow \boxed{x[n] = \frac{(n+1)(n+2)}{2}a^n u[n]}$$

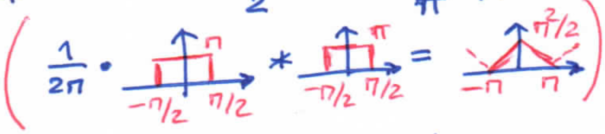
2.1.Β

$x[n] = \begin{cases} 1/n^2, & n \text{ περιττό} \\ 0, & n \text{ άρτιο} \end{cases} \Longrightarrow$ ΕΝΕΡΓΕΙΑ ή ΙΣΧΥΣ

- Έστω $x_1[n] = \dfrac{\sin(\pi n/2)}{n} = \begin{cases} (-1)^{\frac{n-1}{2}}/n, & n \text{ περιττό} \\ 0, & n \text{ άρτιο}, n\neq 0 \\ \pi/2, & n=0 \end{cases} \overset{DTFT}{\longleftrightarrow} X_1(e^{j\omega}) = \begin{cases} \pi, & |\omega|\leq \pi/2 \\ 0, & \frac{\pi}{2}<|\omega|<\pi \end{cases}$

- Τότε $x_2[n] = x_1^2[n] = \begin{cases} 1/n^2, & n \text{ περιττό} \\ 0, & n \text{ άρτιο} \neq 0 \\ \pi^2/4, & n=0 \end{cases} \overset{DTFT}{\longleftrightarrow} X_2(e^{j\omega}) = \dfrac{1}{2\pi} X_1(e^{j\omega}) \underset{\text{PERIODIC}}{*} X_1(e^{j\omega})$

$$= \frac{\pi^2}{2}\left(1-\frac{|\omega|}{\pi}\right), \ |\omega|\leq\pi$$



- Τότε:

$$\sum_{-\infty}^{+\infty}|x[n]|^2 = \sum_{-\infty}^{+\infty} x_2^2[n] - \frac{\pi^4}{16} = \frac{1}{2\pi}\int_{-\pi}^{\pi}|X_2(e^{j\omega})|^2 d\omega - \frac{\pi^4}{16}$$

$x[n] = x_2[n]$ για $n \neq 0$ — PARSEVAL

$$= \frac{1}{\pi}\frac{\pi^4}{4}\int_0^{\pi}\left(1-\frac{\omega}{\pi}\right)^2 d\omega - \frac{\pi^4}{16} = \frac{\pi^3}{4}\int_0^{\pi}\left(1+\frac{\omega^2}{\pi^2}-\frac{2\omega}{\pi}\right)d\omega - \frac{\pi^4}{16} =$$

ΛΟΓΩ ΣΥΜΜΕΤΡΙΑΣ

$$= \frac{\pi^3}{4}\left(\omega\Big|_0^{\pi} + \frac{\omega^3}{3\pi^2}\Big|_0^{\pi} - \frac{\omega^2}{\pi}\Big|_0^{\pi}\right) - \frac{\pi^4}{16} =$$

$$= \frac{\pi^3}{4}\left(\pi + \frac{\pi}{3} - \pi\right) - \frac{\pi^4}{16} = \frac{\pi^4}{12} - \frac{\pi^4}{16} = \frac{\pi^4}{4}\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{\pi^4}{48}$$

$$\Longrightarrow \boxed{E_\infty = \pi^4/48}$$

2.2.A $\boxed{x[n] = n\,2^{-|n|} \Rightarrow X(e^{j\omega}) = ?}$

- Από τις σημειώσεις του κεφ. 5, έχουμε:

$$x[n] = a^{|n|}, \quad |a|<1 \xleftrightarrow{DTFT} X(e^{j\omega}) = \frac{1-a^2}{1-2a\cos\omega + a^2}$$

- Με $a = 1/2$, έχουμε λοιπόν

$$x_1[n] = \left(\frac{1}{2}\right)^{|n|} \longleftrightarrow X_1(e^{j\omega}) = \frac{1-1/4}{1-2\frac{1}{2}\cos\omega + \frac{1}{4}} = \frac{3}{4}\,\frac{1}{\frac{5}{4}-\cos\omega}$$

- Από την ιδιότητα της παραγώγισης:

$$x[n] = n\,x_1[n] \longleftrightarrow X(e^{j\omega}) = j\,\frac{dX_1(e^{j\omega})}{d\omega} = \boxed{-j\,\frac{3}{4}\,\frac{\sin\omega}{\left(\frac{5}{4}-\cos\omega\right)^2}}$$

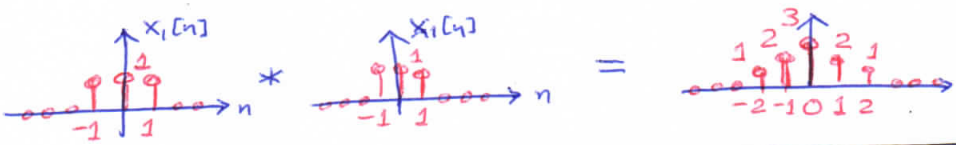
2.2.B $\boxed{X(e^{j\omega}) = \frac{1-\cos^2(3\omega/2)}{1-\cos^2(\omega/2)}} \Rightarrow \boxed{x[n] = ?}$

- Παρατηρούμε ότι $X(e^{j\omega}) = \dfrac{\sin^2(3\omega/2)}{\sin^2(\omega/2)} = \left[X_1(e^{j\omega})\right]^2$

όπου $X_1(e^{j\omega}) = \dfrac{\sin(\omega(1+1/2))}{\sin(\omega/2)} \xleftrightarrow{DTFT^{-1}} x_1[n] = \begin{cases} 1, & |n| \le 1 \\ 0, & \text{αλλού} \end{cases}$

- Από ιδιότητα συνέλιξης

$$x[n] = x_1[n] * x_1[n] = (\delta[n] + \delta[n-1] + \delta[n+1]) * (\delta[n] + \delta[n-1] + \delta[n+1])$$



$$\Rightarrow \boxed{x[n] = \delta[n+2] + 2\delta[n+1] + 3\delta[n] + 2\delta[n-1] + \delta[n-2]}$$

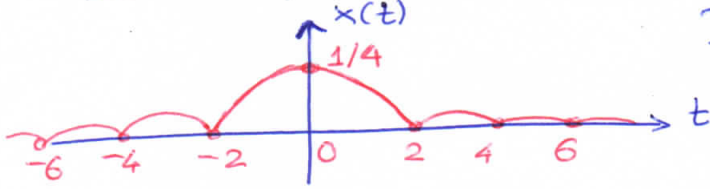
2.3

$x(t) \rightarrow \boxed{C/D} \xrightarrow{x_d[n]} \boxed{H(e^{j\omega})} \xrightarrow{y_d[n]} \boxed{D/C} \rightarrow y(t)$, with T_s into C/D and T_s into D/C

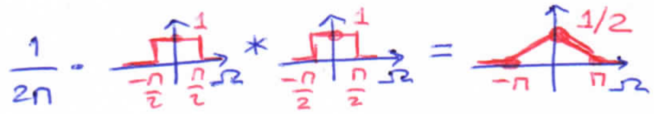
$x(t) = \left[\dfrac{\sin(\pi t/2)}{\pi t}\right]^2, \quad \Omega_s = \pi$

$y_d[n] = \dfrac{x_d[n-2] + x_d[n+2]}{2}$

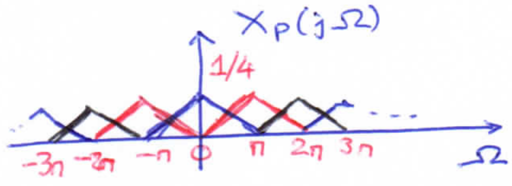
$\Rightarrow \quad y(t) = ? \quad \int_{-\infty}^{+\infty} y(t)\,dt = ? \quad \int_{-\infty}^{+\infty} y^2(t)\,dt = ?$



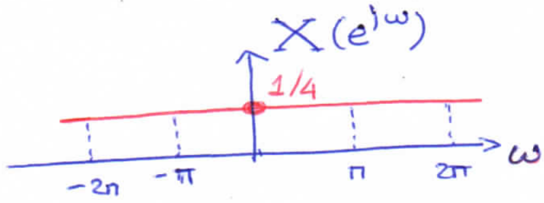
F.T. $\Longrightarrow$



C/D $\Longrightarrow$ $T_s = \dfrac{2\pi}{\Omega_s} = 2$

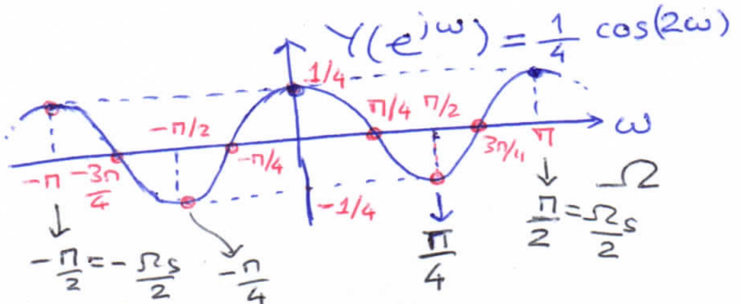


$=$



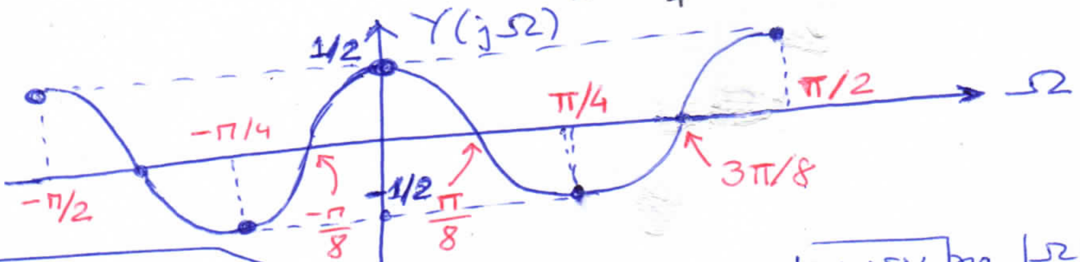
$H(e^{j\omega}) = \dfrac{e^{-2j\omega} + e^{2j\omega}}{2} = \cos(2\omega)$

$\Longrightarrow$



D/C $\Longrightarrow$

LOWPASS FILTERING
GAIN = 2
CUTOFF $= \dfrac{\Omega_s}{2} = \dfrac{\pi}{2}$



• Άρα $\boxed{Y(j\Omega) = \frac{1}{2}\cos(4\Omega)}$, για $|\Omega| < \pi/2$, & $\boxed{\text{ΜΗΔΕΝ}}$ για $|\Omega| \geq \frac{\pi}{2}$

• $y(t) = \mathcal{F}^{-1}\{Y(j\Omega)\} = \dfrac{1}{8\pi}\displaystyle\int_{-\pi/2}^{\pi/2} \left(e^{4j\Omega} + e^{-4j\Omega}\right) e^{j\Omega t}\,d\Omega =$

$= \dfrac{1}{8\pi}\displaystyle\int_{-\pi/2}^{\pi/2} e^{j\Omega(4+t)}\,d\Omega + \dfrac{1}{8\pi}\displaystyle\int_{-\pi/2}^{\pi/2} e^{j\Omega(t-4)}\,d\Omega =$

$= \dfrac{1}{8\pi j}\dfrac{1}{4+t}\left(e^{j\frac{\pi}{2}(4+t)} - e^{-j\frac{\pi}{2}(4+t)}\right) + \dfrac{1}{8\pi j}\dfrac{1}{t-4}\left(e^{j\frac{\pi}{2}(t-4)} - e^{-j\frac{\pi}{2}(t-4)}\right)$

$\Longrightarrow \boxed{y(t) = \dfrac{\sin\left[\frac{\pi}{2}(4+t)\right]}{4\pi(4+t)} + \dfrac{\sin\left[\frac{\pi}{2}(t-4)\right]}{4\pi(t-4)}}$

• $\displaystyle\int_{-\infty}^{+\infty} y(t)\,dt = Y(j\Omega)\Big|_{\Omega=0} = \boxed{\frac{1}{2}}$

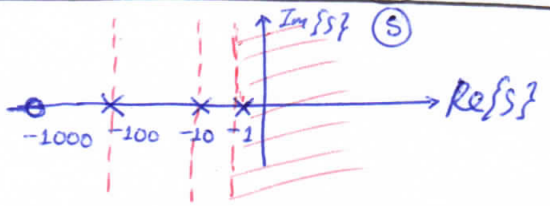
2.4

$$H(s)=\frac{e^{3s}(s+1000)}{(s^2+11s+10)(s+100)} \Rightarrow$$

(a) POLE-ZERO DIAG; ROCs?
(β) ΑΙΤΙΑΤΟΤΗΤΑ?
(c) Bode diagram (για ΕΥΣΤΑΘΕΣ)
(d) Εξοδος στο $x(t)=\delta(t-1)$
(e) ΔΙΑΓΡΑΜΜΑΤΑ ΥΛΟΠΟΙΗΣΗΣ της $h(t-3)$

(a) POLES $=\{-1,-10,-100\}$

ZEROS $=\{-1000,\infty,\infty\}$



Πιθανές ΠΣ: $Re\{s\}>-1$ ⇒ ΕΥΣΤΑΘΕΣ (ΑΙΤΙΑΤΟΤΗΤΑ ΠΙΘΑΝΗ – ΒΛΕΠΕ (β))

$-10<Re\{s\}<-1$ ⇒ ΑΣΤΑΘΕΣ, ΜΗ ΑΙΤΙΑΤΟ

$-100<Re\{s\}<-10$ ⇒ >> , >>

$Re\{s\}<-100$ ⇒ >> , >>

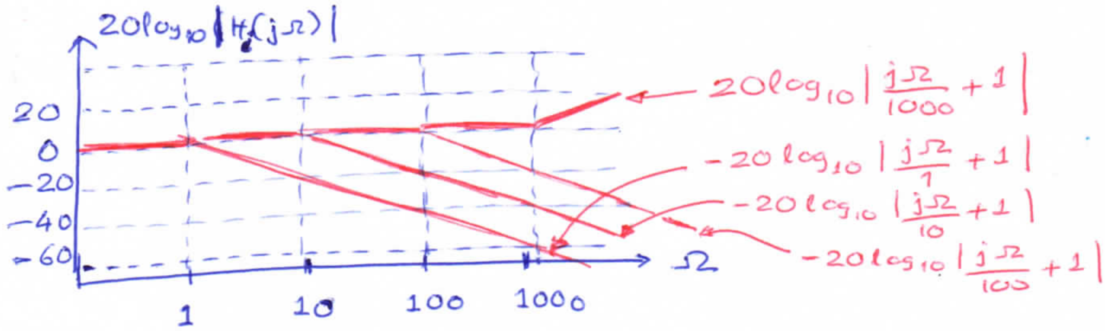
(β) $$H(s)=\frac{(s+1000)e^{3s}}{(s^2+11s+10)(s+100)}=\frac{(s+1000)e^{3s}}{(s+1)(s+10)(s+100)}=\frac{111}{99}\frac{e^{3s}}{s+1}-\frac{11}{9}\frac{e^{3s}}{s+10}+\frac{10}{99}\frac{e^{3s}}{s+100}$$

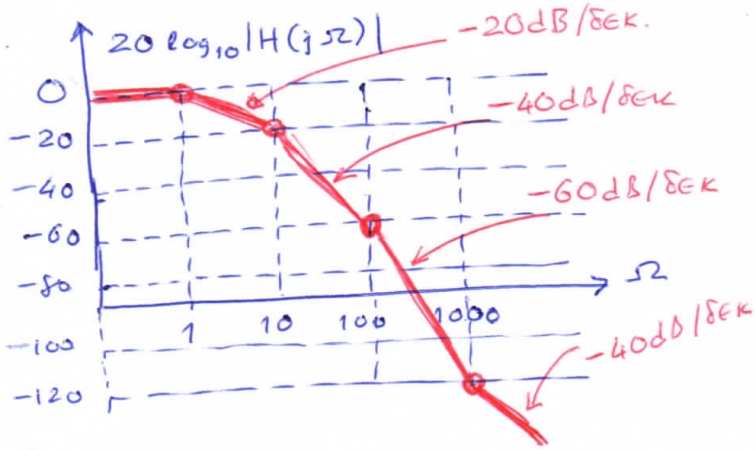
$\xRightarrow{\text{Π.Σ.}}$ $Re\{s\}>-1$ (ΜΟΝΗ ΠΕΡΙΠΤΩΣΗ ΠΙΘΑΝΗΣ ΑΙΤΙΑΤΟΤΗΤΑΣ)

$$h(t)=\left[\frac{111}{99}e^{-(t+3)}-\frac{11}{9}e^{-10(t+3)}+\frac{10}{9}e^{-100(t+3)}\right]u(t+3)$$

ΑΡΑ, $\boxed{\text{ΜΗ ΑΙΤΙΑΤΟ}}$

(c) $$20\log_{10}|H(j\Omega)|=\underbrace{20\log_{10}|e^{3j\Omega}|}_{0}+20\log_{10}|(j\Omega/1000)+1|$$
$$-20\log_{10}|(j\Omega/1)+1|-20\log_{10}|(j\Omega/10)+1|-20\log_{10}|(j\Omega/100)+1|$$

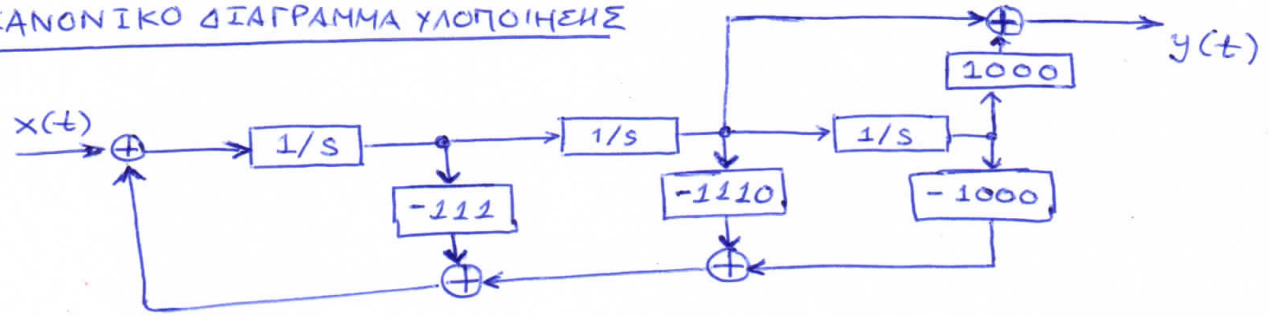




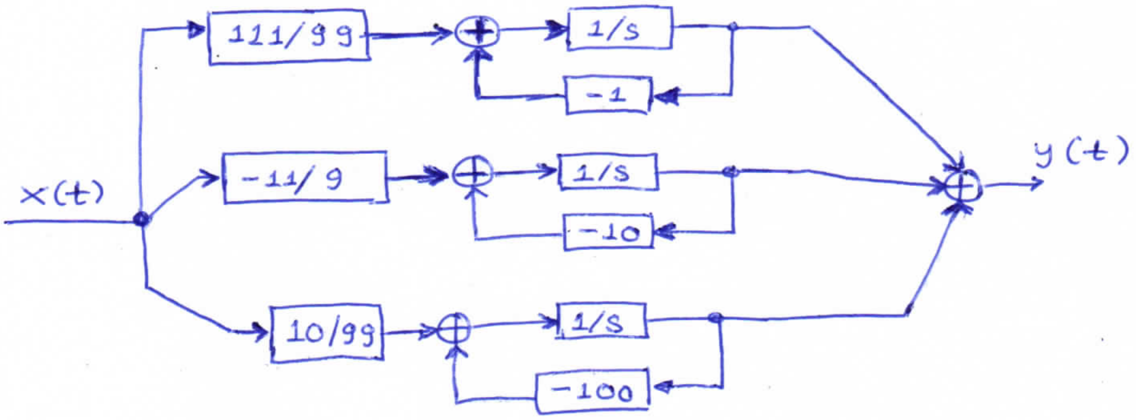
(δ) $$y(t)=h(t)*\delta(t-1)=$$
$$=\left[\frac{111}{99}e^{-(t+2)}-\frac{11}{9}e^{-10(t+2)}+\frac{10}{9}e^{-100(t+2)}\right]u(t+2)$$

(e) Το σύστημα με κρουστική απόκριση $h_1(t) = h(t-3)$ θα έχει συνάρτηση μεταφοράς την $H_1(s) = e^{-3s}H(s) = \frac{s+1000}{(s+1)(s+10)(s+100)} = \frac{s+1000}{s^3+111s^2+1110s+1000}$

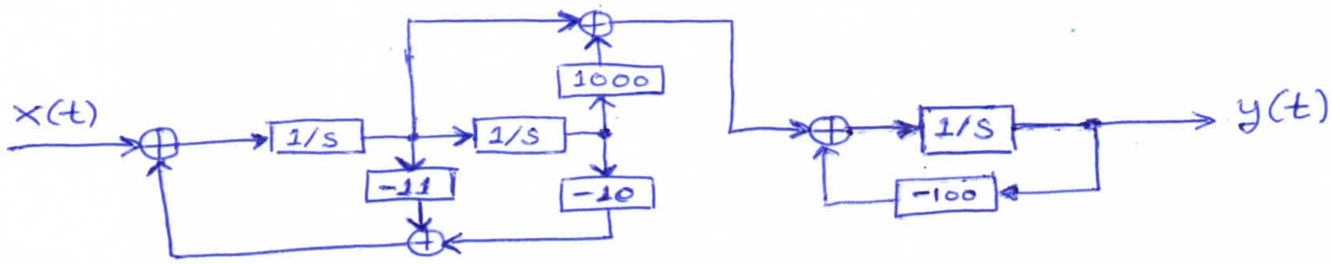
ΚΑΝΟΝΙΚΟ ΔΙΑΓΡΑΜΜΑ ΥΛΟΠΟΙΗΣΗΣ



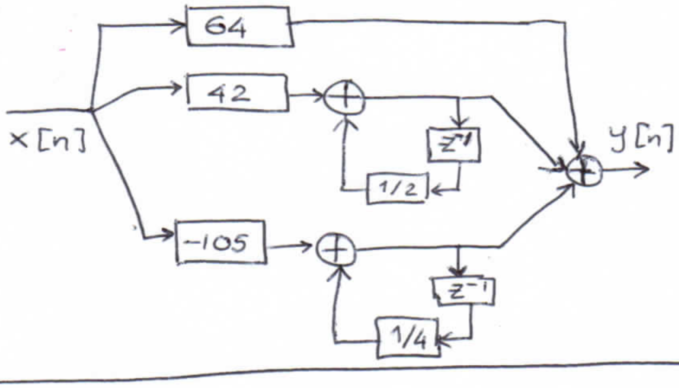
ΠΑΡΑΛΛΗΛΟ ΔΙΑΓΡΑΜΜΑ ΥΛΟΠΟΙΗΣΗΣ



ΕΝ-ΣΕΙΡΑ ΥΛΟΠΟΙΗΣΗ (υπάρχουν πολλαπλές δυνατότητες συνδυασμών---)



2.5



ⓐ $H(z)$, diff eqns

ⓑ ZERO-POLE DIAG, ROCs

ⓒ $h[n]$

ⓓ $|H(e^{j\omega})|$ (if stable)

ⓔ DIRECT FORM + CASCADE IMPLEMENTATIONS

A

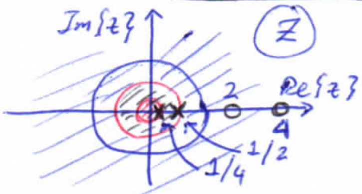
$$H(z) = 64 + \frac{42}{1-\frac{1}{2}z^{-1}} - \frac{105}{1-\frac{1}{4}z^{-1}} = \frac{1-6z^{-1}+8z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{(1-2z^{-1})(1-4z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

ΕΞΙΣΩΣΗ ΔΙΑΦΟΡΩΝ / ΣΧΕΣΗ ΕΙΣΟΔΟΥ-ΕΞΟΔΟΥ: $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] - 6x[n-1] + 8x[n-2]$

B

ΠΟΛΟΙ $= \{1/2, 1/4\}$

ΜΗΔΕΝΙΚΑ $= \{2, 4\}$



ΠΙΘΑΝΕΣ ROC:

(i) $|z| > 1/2$ ΕΥΣΤΑΘΕΣ & ΑΙΤΙΑΤΟ

(ii) $\frac{1}{4} < |z| < 1/2$ ΑΣΤΑΘΕΣ & ΜΗ ΑΙΤΙΑΤΟ

(iii) $|z| < 1/4$ >> & >>

C

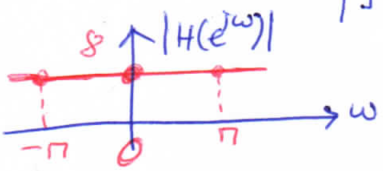
$$h[n] = 64\delta[n] + 42\left(\frac{1}{2}\right)^n u[n] - 105\left(\frac{1}{4}\right)^n u[n]$$

D

$$|H(e^{j\omega})| = \frac{|1-2e^{-j\omega}||1-4e^{-j\omega}|}{|1-\frac{1}{2}e^{-j\omega}||1-\frac{1}{4}e^{-j\omega}|} = \frac{|-2e^{-j\omega}||1-\frac{1}{2}e^{j\omega}||-4e^{-j\omega}||1-\frac{1}{4}e^{j\omega}|}{|1-\frac{1}{2}e^{-j\omega}||1-\frac{1}{4}e^{-j\omega}|} = 8$$

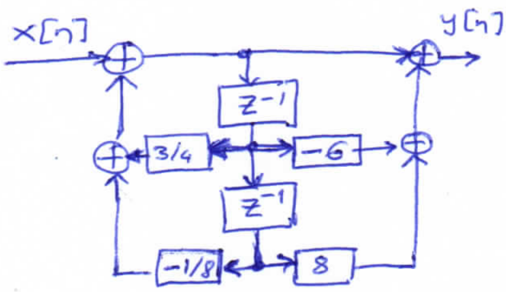
ΣΥΖΗΓΕΙΣ

ΣΥΖΗΓΕΙΣ



E

ΚΑΝΟΝΙΚΗ ΥΛΟΠΟΙΗΣΗ



ΕΝ-ΣΕΙΡΑ ΥΛΟΠΟΙΗΣΗ