

# A Unified Mechanism Design Framework for Networked Systems

Tansu Alpcan, Holger Boche, and Siddharth Naik

**Abstract**—Mechanisms such as auctions and pricing schemes are utilized to design strategic (noncooperative) games for networked systems. Although the participating players are selfish, these mechanisms ensure that the game outcome is optimal with respect to a global criterion (e.g. maximizing a social welfare function), preference-compatible, and strategy-proof, i.e. players have no reason to deceive the designer. The mechanism designer achieves these objectives by introducing specific rules and incentives to the players; in this case by adding resource prices to their utilities. In auction-based mechanisms, the mechanism designer explicitly allocates the resources based on bids of the participants in addition to setting prices. Alternatively, pricing mechanisms enforce global objectives only by charging the players for the resources they have utilized. In either setting, the player preferences represented by utility functions may be coupled or decoupled, i.e. they depend on other player's actions or only on player's own actions, respectively. The unified framework and its information structures are illustrated through multiple example resource allocation problems from wireless and wired networks.

**Index Terms**—Game theory, mechanism design, auctions, pricing, interference coupling

## I. INTRODUCTION

**Game theory** has been enjoying widespread adoption by the engineering community as a distributed optimization and control framework for networked systems, partly for taking into account preferences of individual users, who share and compete for system resources. Resting upon a rich mathematical foundation, game theoretical approaches, especially strategic (noncooperative) games, have been valuable for analysis and design of various resource allocation protocols in wireless and wired networks. Problems such as rate control, interference management, and power control (e.g. in wireless and optical networks) have been investigated extensively by the research community using game theoretical methods [1]–[4].

Game theory models nodes of networked systems as independent and autonomous decision makers with limited global information, and studies incentives of individual players and effects of their preferences on the overall outcome. The Nash equilibrium (NE), where no player has an incentive to deviate from the NE while others adopt it, is known to be useful solution concept for such games. It is widely adopted for development of distributed and dynamic algorithms assuming some mild existence and uniqueness conditions [5], [6].

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Given the broad applicability of game theoretic frameworks, it is not surprising to observe an increasing interest in **mechanism design**, which studies rules and structure of games such that their outcome achieve certain objectives [7]–[12]. This is especially relevant in development of distributed control schemes for networks where satisfying certain global properties such as **efficiency** are as important as the solution's compatibility with user incentives.

A game designer can impose rules and incentives, e.g. in the form of prices, to players of such that the outcome of a strategic game, for example, the unique Nash equilibrium solution is **preference-compatible** and at the same time maximizes a certain global objective function such as the sum of player utilities or quality-of-service (QoS) constraints. However, this interaction between the designer and players of the game may create now a separate incentive for the players to misrepresent their utilities to the designer with the purpose of selfishly benefiting from it. Therefore, the mechanism designer has a third objective called **strategy-proofness** (or *truth dominance*), in addition to the goals of efficiency and preference-compatibility.

This paper builds upon earlier work [13], [14], which has presented a decision and control theoretic approach to game design taking into account only efficiency and preference-compatibility objectives while assuming that players are honest toward the game designer in terms of their preferences. Here, we present an optimization framework for mechanism design that satisfies all three objectives, adding strategy-proofness to the previous two.

The difficulty facing a mechanism designer trying to achieve all three objectives can be best appreciated with a specific example. Consider maximization of the sum of player utilities as the efficiency criterion of a specific problem. Assume that the designer can impose a pricing scheme on users for their actions as an enforcement method. However, individual player utilities are not directly revealed to anyone. Assume in addition that the underlying strategic game admits a unique NE solution. The task of the designer is then to find such a mechanism that it moves the NE of the game to a point, which maximizes the sum of these unknown player utilities (Figure 1), while the players try to mislead the designer by misrepresenting their actual utility functions. In addition, the designer may not *observe* players actions completely bringing additional restrictions to the information flow within the system.

Due to the difficulty of the above described task, there are naturally many impossibility results in the mechanism design literature [15]–[18]. In contrast, this paper adopts a more constructive engineering approach and focuses on schemes that achieve all three objectives, albeit in some cases only approximately. The algorithms presented and analyzed here are examples of market clearance schemes, where all participants

have an incentive to reveal their true preferences, and leading to solutions satisfactory to both designer and players from global and local points of view, respectively. Most of these mechanisms can be intuitively explained by the old adages of “*actions speak louder than words*” (designer deducing players’ true preferences by observing their actions) and “*you get what you paid for*” (designer charging players for their actions).

However, we also note that the presented results are obtained only in very specific settings with various assumptions on player preferences (smooth and convex utility functions), on the underlying game (existence and uniqueness of the NE), and on a certain degree of observability of player actions by the designer. While these restrictions may decrease applicability of the results to certain areas of economics, the presented optimization framework is of value in engineering settings, especially for the purpose of analyzing and developing distributed optimization and control schemes for networks.

The main contributions of this paper include:

- Development of an unifying optimization framework for mechanism design, which encompasses both auction-based and pricing mechanisms.
- Extension of earlier results on game design [13], [14] to mechanism design by taking into account the strategy-proofness criterion.
- Application of the mechanism design framework to resource allocation problems in networks such as rate control and interference management (power control).

The rest of the paper is organized as follows. The next section presents the underlying model and assumptions of the unified framework developed. Section III studies auction-based mechanisms. Subsequently, Section IV investigates pricing mechanisms. Section V provides an overview of relevant literature on mechanism design. The paper concludes with remarks of Section VI.

## II. UNIFIED FRAMEWORK

This section discusses the underlying model and assumptions of the unified framework for mechanism design.

### A. Model

At the center of the game and mechanism design model is the *designer*  $\mathcal{D}$  who influences  $N$  *players*, denoted by the set  $\mathcal{A}$ , and participating in a **strategic (noncooperative) game**. These players are autonomous and independent decision makers, who share and compete for limited resources under the given constraints of the environment. Concurrently, the designer tries to ensure that the outcome of the game satisfies the desirable properties of efficiency, preference-compatibility, and strategy-proofness. This setup is applicable to a variety of problems in networking (wireless spectrum and bandwidth management) and economics (auctions).

Let us define an  $N$ -player strategic game,  $\mathcal{G}$ , where each player  $i \in \mathcal{A}$  has a respective **decision variable**  $x_i$  such that

$$x = [x_1, \dots, x_N] \in \mathcal{X} \subset \mathbb{R}^N,$$

where  $\mathcal{X}$  is the decision space of all players. As a starting point, this paper assumes scalar decision variables and a

compact and convex decision space. The decision variables may represent, depending on the specific problem formulation, player flow rate, power level, investment, or bidding in an auction. Due to the inherent coupling between the players, the decisions of players directly affect each other’s performance as well as the aggregate allocation of limited resources.

The **preferences** of the players are captured by utility functions

$$U_i(x) : \mathcal{X} \rightarrow \mathbb{R}, \quad \forall i \in \mathcal{A},$$

which are chosen to be continuous and differentiable for analytical tractability. In many cases, the utility functions have special properties such as concavity or monotonicity due to the underlying problem formulation, or these can be assumed to simplify the analysis.

The designer  $\mathcal{D}$  devises a **mechanism**  $M$ , which can be represented by the mapping  $M : \mathcal{X} \rightarrow \mathbb{R}^N$ , implemented by introducing incentives in the form of *rules and prices* to players. The latter can be formulated by adding it as a cost term such that the player  $i$  has the cost function

$$J_i(x) = c_i(x) - U_i(x). \quad (1)$$

Thus, the **player objective** is to solve the following individual optimization problem in the strategic game

$$\min_{x_i} J_i(x), \quad (2)$$

under the given constraints of the strategic game, and rules and prices imposed by the designer. Specific properties and variants of these rules and prices will be discussed in the subsequent sections.

The **Nash equilibrium** (NE) is a widely-accepted and useful solution concept in strategic games, where no player has an incentive to deviate from it while others play according to their NE strategies. It plays an important role here since if it is unique, then the NE outcome automatically satisfies the **preference-compatibility** criterion, which basically states that the mechanism outcome must coincide with the solution of the players’ individual optimization problems (2).

The NE  $x^*$  of the game  $\mathcal{G}$  is formally defined as

$$x_i^* := \arg \min_{x_i} J_i(x_i, x_{-i}^*), \quad \forall i \in \mathcal{A},$$

where  $x_{-i}^* = [x_1^*, \dots, x_{i-1}^*, x_{i+1}^*, \dots, x_N^*]$ . The NE is at the same time the intersection point of players’ best responses obtained by solving (2) individually. If some special convexity and compactness conditions are imposed to the game  $\mathcal{G}$ , then it admits a unique NE solution, which simplifies mechanism and algorithm design significantly. For a detailed discussion on these conditions and properties of NE, we refer to [5], [6].

Similar to player preferences, the **designer objective**, e.g. maximization of aggregate user utilities or social welfare, can be formulated using a smooth objective function  $V$  for the designer:

$$V(x, U_i(x), c_i(x)) : \mathcal{X} \rightarrow \mathbb{R},$$

where  $c_i(x)$  and  $U_i(x)$ ,  $i = 1, \dots, N$  are user-specific pricing terms and player utilities, respectively. Hence, the global optimization problem of the designer is simply  $\max_x V(x, U_i(x), c_i(x))$ , which it solves *indirectly* by setting

rules and prices. In some cases, the objective function  $V$  characterizes the desirability of an outcome  $x$  from the designers perspective. In other cases when the designer objective is to satisfy certain minimum performance constraints such as players achieving certain quality-of-service levels, the objective can be characterized by a region (a subset of the game domain  $\mathcal{X}$ ). Thus, the designer objective represents and corresponds to the **efficiency** criterion of the mechanism.

It is important to note that the designer can only influence the outcome of the game indirectly and cannot dictate actions of players (which would have immediately negated preference-compatibility). It has been shown in [14] that a function linear in  $x_i$ , such as  $c_i(x) = \alpha_i x_i$ , is sufficient for the designer to (indirectly) manipulate the unique NE outcome in the ideal full information case where the players are honest and open about their preferences. Figure 1 visualizes this process.

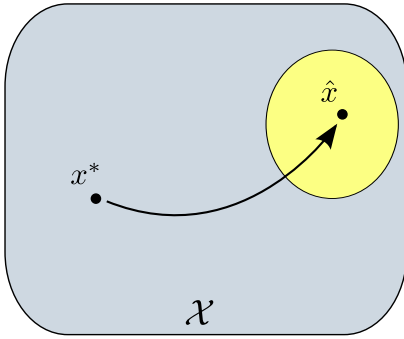


Fig. 1. The manipulation of the unique Nash equilibrium,  $x^*$  of the game by the mechanism designer  $\mathcal{D}$  to a desirable region or point,  $\hat{x} = \arg \max V$ .

The third and an important criterion of mechanism design is **strategy-proofness**, which is also referred to as *incentive-compatibility* or *truth dominance*. If a mechanism does not possess this property, then the players have an incentive to misrepresent their utilities to the designer and “cheat” in order to possibly obtain a larger share of the resources. Within the context of the presented model, this criterion can be formally expressed as:

$$J_i(x^*) < J_i(\tilde{x}) \Leftrightarrow c(x^*) - U_i(x^*) < c(\tilde{x}) - \tilde{U}_i(\tilde{x}) \quad \forall i \in \mathcal{A},$$

where  $\tilde{U}_i$  is the misrepresented utility,  $x^*$  is the original NE solution, and  $\tilde{x}$  is the distorted NE under  $\tilde{U}_i$ . The interaction between the players of the underlying strategic game,  $\mathcal{A}$ , and the mechanism designer,  $\mathcal{D}$  is depicted in Figure 2.

### B. Assumptions

Taking into account the breadth of the field mechanism design, it is useful to clarify the underlying assumptions of the model studied in this paper. The **environment** where the players and designer interact is characterized by the following properties:

- The available resources, which the players share and compete for, are limited.
- The environment imposes restrictions on available information to players and communication between them. Hence, it imposes a certain information structure to

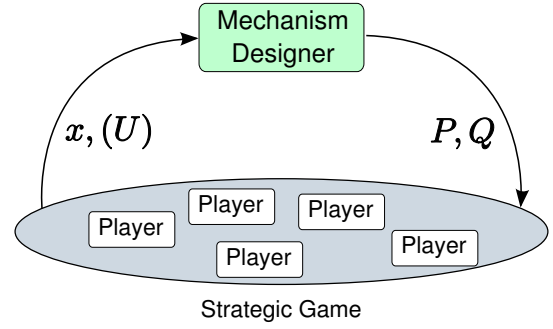


Fig. 2. The interaction between the players of the underlying strategic game and the mechanism designer, who observes players actions  $x$  and utilities  $U$ , while imposing prices  $P$  and in auctions an allocation rule  $Q$ .

distributed mechanisms and sometimes makes it difficult to deploy centralized ones.

- The designer may not fully observe the player actions and has often limited information about their preferences.

The players share and compete for limited resources in the given environment under its information and communication constraints. Three basic types of resource sharing and coupling are often encountered in a variety of problems in networking:

- 1) *Additive resource sharing*: the players share a finite resource  $C$  such that

$$\sum_{i=1}^N x_i = C.$$

This type of coupling is encountered in bandwidth sharing and rate control in networks.

- 2) *Interference coupling* (linear interference): the resource allocated to player  $i$ ,  $\gamma_i$ , is inversely proportional to interference generated others such that

$$\gamma_i(x) = \frac{h_i x_i}{\sum_{j \neq i} h_j x_j + \sigma},$$

where  $h_i \forall i$  and  $\sigma$  denote some system parameters. Interference coupling occurs in wireless networks where  $\gamma$  represents signal-to-interference ratio.

- 3) *Multiplicative coupling*: the resource  $y_i$  of player  $i$  is affected multiplicatively by the decisions of others such that

$$y_i = x_i \prod_{j \neq i} (1 - x_j).$$

This type of coupling is seen in random multiple access schemes, e.g. slotted Aloha scheme in wireless networks [19].

It is possible to extend these definitions, for example, by making the finite resource  $C$  time varying or changing the interference function. Couple of axiomatic frameworks for the second case exist in the literature [20], [21]. The examples in this paper are of types 1 and 2.

The following assumptions are made on the designer and players:

- The designer is honest, i.e. does not try to deceive the players.

- Each player acts independently and rationally according to its own self interests.
- The players may try to deceive the designer by hiding or misrepresenting their individual preferences.
- Both players and designer follow the rules of the mechanism.

Within the scope of the model discussed in the previous subsection, specific formulations of the three criteria of mechanism design are summarized as:

TABLE I  
THREE CRITERIA OF MECHANISM DESIGN

Criterion	Formulation in the Model
Efficiency	Designer objective
Preference Compatibility	Player minimizing own cost or existence of a unique NE
Strategy-Proofness	No player gains from cheating

### III. AUCTION-BASED MECHANISMS

In auction-based mechanisms, the designer uses an allocation rule in addition to pricing. Hence, the designer *explicitly allocates* the players their share of resources based on their bids. The players decide on their bids or actions by minimizing their cost which is a combination of their own utilities and prices imposed by the designer. Specifically, the designer  $\mathcal{D}$  imposes on a player  $i \in \mathcal{A}$  a user-specific

- resource allocation rule,  $Q_i(x)$ ,
- resource pricing,  $P_i(x)$ ,

where  $x$  denotes the vector of player actions or bids. The specific properties of these functions will be discussed later as part of individual mechanisms.

As presented in Section II-A, each player  $i$  aims to minimize its own cost  $J_i(Q_i(x), P_i(x))$ , as in (1), while the designer tries to achieve the objectives summarized in Table I. In some cases, the designer may only observe the bids imperfectly as a function of the actual bids,  $y = f(x)$ . However, in this paper, we assume that all bids are perfectly observable and  $y = x$  for simplicity. Figure 3 visually depicts the auction-based mechanisms described.

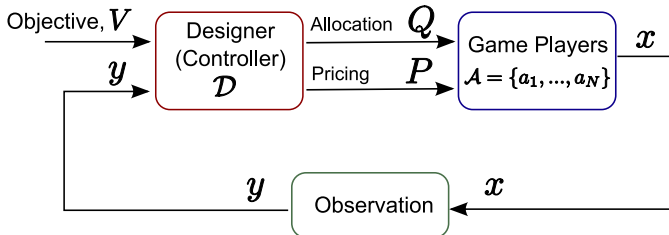


Fig. 3. An auction-based mechanism, where the designer  $\mathcal{D}$  imposes a resource allocation rule as well as pricing on players  $\mathcal{A}$  of the underlying strategic game, whose bids  $x$  may be observed imperfectly as  $y$ , with the purpose of satisfying a global objective  $V$ .

#### A. Auctions for Separable Utilities

Consider, as a starting point, an additive resource sharing scenario where the players bid for a fixed divisible resource  $C$  and are allocated their share captured by the vector  $Q = [Q_1, \dots, Q_N]$  such that at full utilization  $\sum_i Q_i = C$ .

The  $i^{\text{th}}$  player's individual cost function  $J_i(x)$  in terms of player bids  $x$  is defined as

$$J_i(x) = c_i(x) - U_i(Q_i(x)).$$

The pricing term has the general form of

$$c_i(x) = \int_0^{Q_i(x)} P_i(\xi) d\xi, \quad (3)$$

where  $P_i$  denotes the unit price. In accordance with the earlier results [13], [14] and due to the nature of the auction-based mechanism, it is sufficient for the purposes of the designer to choose a pricing function linear in  $Q_i$ , i.e.  $c_i(x) = P_i(x)Q_i(x)$ . The player utility function  $U_i$  is separable, i.e. it depends only on the individual allocation of the player. It is also assumed to be continuous, strictly concave, and twice differentiable in terms of its argument  $Q_i$ . Thus, the cost function of player  $i$  can be written as

$$J_i(x) = P_i(x)Q_i(x) - U_i(Q_i(x)), \quad (4)$$

which is strictly convex with respect to  $Q_i$  under the assumptions made.

From a player's perspective, who tries to minimize its cost in terms of the actual resources obtained, the condition

$$\frac{\partial J_i}{\partial Q_i} = \frac{\partial c_i}{\partial Q_i} - \frac{\partial U_i}{\partial Q_i} = c'_i - U'_i$$

is necessary and sufficient for optimality. Thus suppressing the dependence of user cost on bids  $x$ , in order for the auction-based mechanism to be **preference-compatible**, it has to satisfy

$$P_i(Q) = U'_i(Q_i) \quad \forall i \in \mathcal{A}. \quad (5)$$

Furthermore, if additional assumptions are made on  $J_i(x)$ , it can be shown that the game admits a unique NE,  $Q^*$  (or  $x^*$ ) [6].

Different from players, the designer  $\mathcal{D}$  has two objectives: maximizing the sum of utilities of players and allocating all of the existing resource  $C$ , i.e. its full utilization. Hence, the designer  $\mathcal{D}$  solves the constrained optimization problem

$$\max_Q V(Q) \Leftrightarrow \max_Q \sum_i U_i(Q_i) \text{ such that } \sum_i Q_i = C, \quad (6)$$

in order to find a globally optimal allocation  $Q$  that satisfies this **efficiency criterion**. The associated Lagrangian function is then

$$L(Q) = \sum_i U_i(Q_i) + \lambda \left( C - \sum_i Q_i \right),$$

where  $\lambda > 0$  is a scalar Lagrange multiplier. Under the convexity assumptions made, this leads to

$$\frac{\partial L}{\partial Q_i} \Rightarrow U'_i(Q_i) = \lambda, \quad \forall i \in \mathcal{A}, \quad (7)$$

and the efficiency constraint

$$\frac{\partial L}{\partial \lambda} \Rightarrow \sum_i Q_i = C. \quad (8)$$

*Remark III.1.* It is important to note that sum of utility maximization as designer objective, i.e.  $V = \sum_i U_i$  is **only one** possible global objective among many others such as ensuring a certain QoS to players (see [13], [14] for a more detailed discussion).

The interaction between the designer and players (see Figure 2) is through a *bidding/allocation process* in auction-based mechanisms. Since the players cannot obtain the resource  $Q$  directly, they make a bid for their own total cost, which is denoted by the vector  $x$ . The pricing  $P(x)$  and allocation  $Q(x)$  rules of the auction-based mechanism should satisfy the efficiency and preference-compatibility criteria discussed above.

A player's bid (or action),  $x_i$ , is an indicator of the player's willingness to pay and plays a crucial role in devising a mechanism that is **strategy-proof**. Formally, a mechanism is strategy-proof, if no player has an incentive to deviate from its truthful bid

$$J_i(x_i^* + \delta) \geq J_i(x_i^*) \quad \forall i \in \mathcal{A}, \delta,$$

where  $\delta \in \mathbb{R}$  is a scalar and  $x^*$  is the outcome (NE) of the underlying strategic game.

*Example 1:*

In the specific resource sharing setting defined, an auction-based mechanism,  $\mathcal{M}^a$ , can be defined based on the bid of player  $i$ ,

$$x_i := P_i(x)Q_i(x), \quad (9)$$

the pricing function

$$P_i := \frac{\sum_{j \neq i} x_j + \omega}{C}, \quad (10)$$

for a scalar  $\omega > 0$  sufficiently large such that  $\sum_i Q_i \leq C$ , and the resource allocation rule

$$Q_i := \frac{x_i}{\sum_{j \neq i} x_j + \omega} C. \quad (11)$$

It is also possible to interpret the scalar  $\omega$  as a *reserve bid* [22]. The next theorem establishes that this mechanism is preference-compatible, strategy-proof, and asymptotically efficient.

**Theorem III.2.** *The auction-based mechanism  $\mathcal{M}^a$  defined by (9), (10), and (11) allocates the fixed divisible resource  $C$  to a set of selfish rational players  $\mathcal{A}$  with respective cost functions (4) in such a way that the mechanism is preference-compatible, strategy-proof, and asymptotically efficient, if*

$$U_i(x_i^* + \delta) - U_i(x_i^*) \leq \delta, \quad \forall i, \forall \delta \in \mathbb{R},$$

where  $x^*$  denotes the truthful bid of player  $i$  at the NE outcome. In other words, the outcome of the mechanism ensures that

- optimal allocation obtained,  $Q^*$ , satisfies  $Q_i^* = \arg \min_{Q_i} J_i(Q) \Rightarrow P_i(Q^*) = U_i'(Q_i^*) \quad \forall i$ ,

- no player has an incentive to deviate from its truthful bid,  $J_i(x_i^* + \delta) \geq J_i(x_i^*), \quad \forall i, \delta$
- $Q^*$  solves the constrained optimization problem in (6) asymptotically, i.e. as  $\lim N \rightarrow \infty$ .

*Proof:*

The mechanism  $\mathcal{M}^a$  is defined by the bidding process (9), unit prices (10), and allocation rule (11) for each player  $i \in \mathcal{A}$ . Substituting these into the player cost function (4) results in

$$J_i(x) = x_i - U_i \left( \frac{x_i}{\sum_{j \neq i} x_j + \omega} C \right).$$

Due to the convexity of  $J_i$  in  $x_i$ , the first order necessary condition for optimality is also sufficient:

$$\frac{\partial J_i(x)}{\partial x_i} = 1 - \frac{\partial U_i(Q_i)}{\partial Q_i} \left( \frac{C}{\sum_{j \neq i} x_j + \omega} \right) = 0.$$

From definition of  $P_i$  in (10) follows  $P_i(Q) = U_i'(Q_i)$  for each player  $i$ . Hence, the mechanism  $\mathcal{M}^a$  is *preference-compatible*. Furthermore, it is straightforward to show that this game admits a unique NE,  $x^*$  [6].

Assume a player  $i$  deviates from its truthful bid  $x_i$  by an amount  $\delta \in \mathbb{R}$  such that  $\tilde{x}_i = x_i + \delta$ . Then, the player cost under  $\mathcal{M}^a$  becomes

$$\tilde{J}_i = \tilde{x}_i - U_i \left( \frac{\tilde{x}_i}{\sum_{j \neq i} x_j + \omega} C \right).$$

In order  $\mathcal{M}^a$  to be *strategy-proof*,

$$\tilde{J}_i - J_i = \delta - (U_i(Q_i(x_i + \delta)) - U_i(Q_i(x_i))) > 0,$$

which immediately holds under the assumption in the theorem.

Although it is preference-compatible and strategy-proof, the mechanism  $\mathcal{M}^a$  is not fully efficient as it does not exactly solve the designer optimization problem (6). To see this, let us solve (7) and (8) using  $x_i = P_i Q_i$  to obtain

$$P_i = \frac{\sum_i x_i}{C} \quad \text{and} \quad Q_i = \frac{x_i}{\sum_i x_i} C.$$

These optimal solutions (with respect to designer objective) are only approximated by the pricing (10) and allocation (11) rules. Hence,

$$P_i = \frac{\sum_{j \neq i} \alpha_j + \omega}{C} \neq \frac{\sum_i \alpha_i}{C}$$

and

$$\sum_i Q_i = \sum_i \frac{\alpha_i}{\sum_{j \neq i} \alpha_j + \varepsilon} C \approx C.$$

The choice of suboptimal (in the sense of efficiency) rules is due to the fact that  $\mathcal{M}^a$  has to achieve strategy-proofness at the same time as efficiency and preference-compatibility. However, as the number of players increases,  $N \rightarrow \infty$ , and by choosing  $\omega$  accordingly small, the approximation becomes more precise. Thus, the mechanism  $\mathcal{M}^a$  is *asymptotically efficient*. ■

*Example 2:*

As a special case of the auction-based mechanism  $\mathcal{M}^a$ , consider a setup where, the player utility functions are logarithmic and respectively weighted by a positive scalar parameter  $\alpha$  such that

$$U_i = \alpha_i \log Q_i \quad \forall i \in \mathcal{A}.$$

Then, the following result holds as a special case of Theorem III.2.

**Corollary III.3.** *The auction-based mechanism  $\mathcal{M}^a$  defined by (9), (10), and (11) allocates the fixed divisible resource  $C$  to a set of selfish rational players  $\mathcal{A}$  with respective cost functions (4) and utilities  $U_i = \alpha_i \log Q_i \quad \forall i \in \mathcal{A}$  in such a way that the mechanism is preference-compatible, strategy-proof, and asymptotically efficient.*

*Proof:* The proofs of preference-compatibility and asymptotic efficiency follow directly from the ones of Theorem III.2. Furthermore, the mechanism is strategy-proof under logarithmic player utilities since they satisfy the sufficient condition in Theorem III.2. The condition in this case is

$$\alpha_i \log(Q_i(x_i + \delta)) - \alpha_i \log(Q_i(x_i)) \leq \delta,$$

leading to

$$\log\left(1 + \frac{\delta}{x_i}\right) \leq \frac{\delta}{\alpha_i}.$$

The player's truthful bid is  $x_i = \alpha_i$  from its cost function (4). Thus, we obtain

$$\exp\left(\frac{\delta}{\alpha_i}\right) > 1 + \frac{\delta}{\alpha_i},$$

which holds by definition, and completes the proof.  $\blacksquare$

### B. Auctions for Non-separable Utilities

In many problem formulations, the player utilities are non-separable, i.e. they depend also on other player's actions. This is the case, for example, in interference coupled systems such as a cellular wireless system with a base station (acting as the designer) and mobile devices or users as players who bid to achieve a certain QoS level. Let  $x_i$  denote the bid of a mobile device and the  $q_i(x)$  the transmission power assigned to it by the base station. Then, the signal-to-interference ratio (SIR) of the received signal by the mobile is

$$\gamma_i = \frac{q_i(x)}{\sum_{j \neq i} q_j(x) + \sigma}, \quad (12)$$

where  $\sigma > 0$  is an independent noise term. Notice that this is essentially a centralized scheme similar to the ones currently deployed. A decentralized version will be discussed in Section IV.

This interference management and power control formulation has been discussed extensively in the literature, e.g. [20], [21], [23]. However, such mechanisms do not necessarily need to be limited to wireless networks and apply to any system with linear interference coupling [24] under the assumption that the player utilities are  $U_i(\gamma_i)$  continuous, strictly concave, and twice differentiable in their arguments  $\gamma_i$  (12).

*Example 3*

Consider an auction-based mechanism for an interference-coupled system where players have non-separable and logarithmic utilities and a linear pricing scheme, which make the problem more tractable. Then, each player  $i$  minimizes its respective cost

$$J_i(x) = P_i(x)q_i(x) - \alpha_i \log(\gamma_i(q(x))), \quad (13)$$

which is strictly convex in player power level  $q_i$ . Consequently, the general condition for player preference-compatibility is  $P_i = \alpha_i/q_i, \forall i \in \mathcal{A}$ , as in Examples 1 and 2.

The global objective of the designer is to maximize sum of utilities of players while trying to limit the total interference effect to an upper-bound  $C$ . This approximate formulation is motivated by, for example, limiting the aggregate inter-cell interference created by the mobile devices in a wireless network, where base stations have no means of communication among themselves. Hence, the designer  $\mathcal{D}$  solves the constrained convex optimization problem

$$\max_q V(q) \Leftrightarrow \max_q \sum_i \alpha_i \log(\gamma_i(q)) \quad \text{such that} \quad \sum_i q_i \leq C.$$

The resulting necessary and sufficient conditions for optimality are

$$\frac{\alpha_i}{q_i} - \sum_{j \neq i} \frac{\alpha_j}{\bar{C} - q_j} = \lambda \quad \text{and} \quad \sum_i q_i = C,$$

where  $\bar{C} = C + \sigma$ .

In the specific resource sharing setting defined, an auction-based mechanism,  $\mathcal{M}^b$ , is defined based on the bid of player  $i$ ,

$$x_i := P_i(x)Q_i(x). \quad (14)$$

and the allocation rule

$$Q_i := \frac{x_i}{P_i(x)} = q_i(x), \quad (15)$$

which assigns users power levels based on their bids and computed prices.

Under the preference-compatibility condition, the bids have to match the utility parameter,  $x_i = \alpha_i$ . Then, the optimality conditions for the global problem become

$$\frac{x_i}{q_i} - \sum_{j \neq i} \frac{x_j}{\bar{C} - q_j} = \lambda \quad \text{and} \quad \sum_i q_i = C. \quad (16)$$

which are solved to obtain  $(q^*, \lambda^*)$ . Accordingly, the pricing function is

$$P_i(x) := \lambda^* + \sum_{j \neq i} \frac{x_j}{\bar{C} - q_j^*}. \quad (17)$$

As a result of this design, the auction-based mechanism  $\mathcal{M}^b$  is clearly efficient and preference-compatible.

We next show that mechanism  $\mathcal{M}^b$  is asymptotically strategy-proof. Assume that a player  $i$  deviates from its truthful bid  $x_i$  by an amount  $\delta \in \mathbb{R}$  such that  $\tilde{x}_i = x_i + \delta$ . The strategy-proofness is then equivalent to

$$\tilde{J} - J = \delta - \alpha_i \log\left(\frac{\gamma_i(x_i + \delta)}{\gamma_i(x_i)}\right) > 0.$$



As in the previous example, this leads to

$$\frac{\gamma_i(x_i + \delta)}{\gamma_i(x_i)} < \exp\left(\frac{\delta}{\alpha_i}\right),$$

or

$$\frac{\tilde{q}_i \bar{C} - q_i \lambda}{q_i \bar{C} - \tilde{q}_i \lambda} < \exp\left(\frac{\delta}{\alpha_i}\right),$$

where  $\tilde{\lambda}$  is the solution of (16) under  $\tilde{x}_i$ . Note that, as the number of players goes to infinity,<sup>1</sup> we have

$$\lim_{N \rightarrow \infty} \frac{\bar{C} - q_i \lambda}{\bar{C} - \tilde{q}_i \lambda} = 1.$$

Thus, it asymptotically holds that

$$1 + \frac{\delta}{\alpha_i} < \exp\left(\frac{\delta}{\alpha_i}\right),$$

which establishes the result summarized in the following theorem.

**Theorem III.4.** *Consider a set selfish rational players  $\mathcal{A}$  with respective cost functions (13) and non-separable utilities  $U_i = \alpha_i \log(\gamma(q(x))) \quad \forall i \in \mathcal{A}$  in an interference-coupled system (12). The auction-based mechanism  $\mathcal{M}^b$  defined by (14), (15), and (17) maximizes the sum of utilities of players while limiting the total interference effect to an upper-bound  $C$  in such a way that the mechanism is preference-compatible, efficient, and asymptotically strategy-proof.*

#### IV. PRICING MECHANISMS

Pricing mechanisms differ from auction-based ones by the property that the designer does not allocate the resources explicitly, i.e. there is no allocation rule  $Q$ . The players obtain resources directly as a result of their actions but are charged for them by the designer observing these actions (Figure 4). Hence, the designer has relatively less leverage in this case compared to auctions.

Pricing mechanisms are applicable to many networked systems where an explicit allocation of resources brings a prohibitively expensive overhead or simply not feasible, e.g. due to participating players being selfish or located in a distributed manner. Example problems include rate control in wired networks, interference management in wireless networks, and power control in optical networks [1]–[4].

##### A. Pricing Mechanisms for Separable Utilities

We study an additive resource sharing scenario, where the players compete for a fixed divisible resource  $C$  as in Section III. The players' individual cost functions, which they minimize, have the general form

$$J_i(x) = P_i(x)x_i - U_i(x_i). \quad (18)$$

Here,  $x_i$  denotes the player's action of obtaining that specific amount of the resource directly, in contrast to bidding for it and receiving an allocation from the designer. It is sufficient

<sup>1</sup>We remind here the underlying assumption that each player acts individually and there is no coordination among players. This assumption is applicable to many networked systems with information flow constraints.

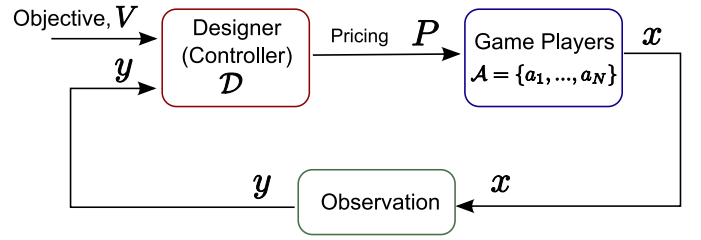


Fig. 4. The block diagram of a generic pricing mechanism. The designer sets the prices  $P$  to achieve a global objective  $V$  based on the observations  $y$  of player actions  $x$ . The players choose their actions  $x$  independently according to their utilities  $U$  (preferences) and prices  $P$ . The overall mechanism aims to ensure efficiency, preference-compatibility, and strategy-proofness.

for the purposes of the designer to choose a pricing function linear in  $x_i$ . A more general form of pricing is provided in (3). The player utility function  $U_i$  is assumed to be continuous, strictly concave, and twice differentiable. At the same time it only takes the player's own action as its argument, i.e. the player utilities are separable in this formulation.

In order for a pricing mechanism to be preference-compatible, it has to satisfy

$$P_i(x^*) = U_i'(x_i^*), \quad \forall i \in \mathcal{A},$$

which directly follows from (18). The point  $x^*$  is, by definition, the Nash equilibrium solution of the strategic game, where no player has an incentive to deviate from it. Under the assumptions made for player utilities, the game admits a unique Nash equilibrium solution [6]. It is important to note that, if there was no pricing term in (18), each player would try to get a large proportion of the resource resulting in a suboptimal result for everyone; a situation sometime termed as *tragedy of commons*. The designer can prevent this by a carefully selected pricing scheme [13], [14].

The global objective of the designer can be maximization of the sum of player utilities while ensuring full resource usage, i.e.  $\sum_i x_i = C$ . Hence, the designer  $\mathcal{D}$  solves the counterpart of the constrained optimization problem in (6) along with (7) and (8).

When the two criteria of preference-compatibility and efficiency (designer objectives) are combined, the pricing function  $P_i$  of a player  $i$  has to satisfy

$$P_i(x^*) = U_i'(x_i^*) = \lambda, \quad \forall i \in \mathcal{A},$$

where  $\lambda > 0$  is the unique Lagrange multiplier. From the criterion of full resource usage, it follows that

$$\sum_i x_i^* = \sum_i (U_i')^{-1}(\lambda) = C. \quad (19)$$

Define  $\lambda^*$  as the optimal solution to (19) given player utilities  $U_i$  and capacity  $C$ . Then, the optimal pricing function is:  $P_i = \lambda^* \forall i$ .

If the designer wants to compute the unit prices  $P$  directly by solving (19), it needs to ask the individual players for their utilities. However, the players have an incentive to misrepresent their utilities to gain a larger share of resources, if they are asked directly by the designer. Such a direct mechanism has two significant disadvantages. First, the designer has to

have additional schemes in place to detect potential player misbehavior (for which players have an incentive). Second, it brings another layer of communication overhead to the system. The disadvantages of such direct mechanism will be illustrated more concretely in the scope of an example in the next subsection.

Alternatively, one can design an **iterative pricing mechanism** that is based on observation of player actions  $x$  instead of asking for their word (utilities). Then, the designer deploys this iterative mechanism to compute the optimal prices  $P_i = \lambda^*$  as a solution to (19).

For example, consider the following iterative pricing mechanism

$$\lambda(n+1) = \lambda(n) + \kappa \left( \sum_i x_i - C \right), \quad (20)$$

where  $\kappa > 0$  is a small step size,  $\lambda > 0$ , and

$$x_i(n+1) = \phi x_i(n) + (1-\phi) \left( \frac{\partial U_i}{\partial x_i} \right)^{-1}(\lambda), \quad \forall i \in \mathcal{A}, \quad (21)$$

where  $0 < \phi < 1$ . Here,  $n \geq 1$  denotes the time (update) step. Note that, the players adopt a relaxed or gradient update scheme instead of best response taking into account variability of the system. The gradient update also helps with convergence.

*Example 4:*

As a special case, let the utility function of players be logarithmic and weighted by parameter  $\alpha$  such that

$$U_i(x_i) = \alpha_i \log x_i$$

for player  $i$ . Such utility functions have been utilized in the literature, for example, to model user demand in rate or congestion control on networks. The solution aligning the player and designer objectives, in other words the efficient Nash equilibrium, has the following properties:

$$\begin{aligned} P_i^* &= \frac{\alpha_i}{x_i} = \lambda; \quad x_i = \frac{\alpha_i}{\lambda} \quad \forall i \in \mathcal{A} \\ \Rightarrow \sum_i x_i &= \frac{\sum_i \alpha_i}{\lambda} = C; \quad \lambda = \frac{\sum_i \alpha_i}{C}. \end{aligned}$$

Hence, the resulting optimal pricing mechanism for all players is

$$P = \frac{\sum_i \alpha_i}{C}. \quad (22)$$

Although this solution is preference-compatible from the players' perspective and solves the global optimization problem of the designer, it is not strategy proof if the designer explicitly asks the players for their utility parameter  $\alpha$ . To see this, assume that player  $i$  has a true utility parameter  $\alpha_i$  but misrepresents it to the designer as  $\tilde{\alpha}_i = \alpha_i + \delta$  for some  $\delta \in \mathbb{R}$ . Then, the new price is  $\tilde{P} = (\sum_i \alpha_i + \delta)/C$  and player  $i$  real cost becomes

$$\tilde{J}_i(\tilde{x}_i, x_{-i}) = \tilde{P}\tilde{x}_i - \alpha_i \log(\tilde{x}_i)$$

instead of

$$J_i(x) = Px_i - \alpha_i \log(x_i).$$

Substituting  $\tilde{P}$  and computing  $\tilde{x}_i$  yields

$$\tilde{J}_i(\tilde{x}_i, x_{-i}) = \alpha_i - \alpha_i \log \left( \frac{\alpha_i}{\sum_i \alpha_i + \delta} C \right),$$

and similarly we have

$$J_i(x) = \alpha_i - \alpha_i \log \left( \frac{\alpha_i}{\sum_i \alpha_i} C \right).$$

Clearly, the player  $i$  can decrease its cost ( $\tilde{J}_i < J_i$ ) by choosing a  $\delta < 0$  despite being charged the same total price. Thus, the mechanism is not strategy-proof.

This issue is remedied by adopting the proposed iterative pricing mechanism:

$$\lambda(n+1) = \lambda(n) + \kappa \left( \sum_i x_i - C \right), \quad (23)$$

$$x_i(n+1) = \phi x_i(n) + (1-\phi) \frac{\alpha_i}{\lambda} \quad \forall i \in \mathcal{A}. \quad (24)$$

The unique (Nash) equilibrium solution of this iterative algorithm,  $(x^*, \lambda^*)$  solves the designer problem (6). Furthermore, since the players adopt here a relaxed (gradient) best response at each step and there is no explicit communication between the players and the designer, the scheme is strategy-proof. To see this, assume otherwise and let player  $i$  "misrepresent" its actions  $\tilde{x}_i = x_i + \delta$  for some  $\delta \in \mathbb{R}$ . Then, the player's instantaneous cost is  $\tilde{J}_i > J_i$  at each step of the iteration. Hence, the players have no incentive to "cheat".

The communication requirements of the algorithm (23)-(24) are minimal and suitable for a distributed implementation in a networking environment. The designer only needs to observe the total amount  $y = \sum_i x_i$  and communicate the common price  $P$  back to the players (see Figure 4 for a visualization).

Now, a basic stability analysis is provided for the following continuous-time approximation of the iterative pricing mechanism

$$\begin{aligned} \dot{\lambda} &= \frac{d\lambda}{dt} = \kappa \left( \sum_i x_i - C \right), \\ \dot{x}_i &= -\frac{\partial J_i}{\partial x_i} = \bar{\kappa}_i \left( \frac{\alpha_i}{x_i} - \lambda \right), \end{aligned}$$

where  $t$  denotes time and  $\bar{\kappa}_i > 0$  is a user-specific step size. As in the discrete-time version, the players adopt here a gradient best response algorithm. Define the Lyapunov function

$$V_L := \frac{1}{2} \left( \sum_i x_i - C \right)^2 + \frac{1}{2} \sum_i \left( \frac{\alpha_i}{x_i} - \lambda \right)^2,$$

which is nonnegative and satisfies  $V_L(x^*, \lambda^*) = 0$ . It is straightforward to show through algebraic manipulations that  $\dot{V}_L(x, \lambda) < 0$  for all  $(x, \lambda) \neq (x^*, \lambda^*)$ . Hence, the continuous-time algorithm is globally asymptotically stable [25]. This result is a strong indicator of convergence [26] of the discrete-time iterative pricing mechanism (23)-(24).

## B. Pricing Mechanisms for Non-separable Utilities

In some problem formulations, such as interference coupled systems consisting of a base station (acting as the designer)



and mobile devices as players, the players' actions are beyond the control of the base station. Let, specifically,  $x_i = h_i p_i$  denote the received power level as a product of uplink transmission power  $p_i$  and channel loss  $0 < h_i < 1$  of player  $i$ . If linear interference is assumed, then the signal-to-interference ratio (SIR) of the received signal is

$$\gamma_i = \frac{x_i}{\sum_{j \neq i} x_j + \sigma}, \quad (25)$$

as in (12).

In pricing mechanism, similar to the auction in Section III-B, each player  $i$  minimizes its respective cost

$$J_i(x) = P_i(x)x_i - \alpha_i \log(\gamma_i(x)), \quad (26)$$

which is strictly convex in  $x_i$ . Consequently, the general condition for player preference-compatibility is  $P_i = \alpha_i/x_i$ ,  $\forall i \in \mathcal{A}$ .

The global objective of the designer aims to maximize sum of utilities of players while trying to limit the total interference effect to  $C$ , motivated by e.g. limiting the aggregate interference created by the mobile devices in a wireless network. Hence, the designer  $\mathcal{D}$  solves

$$\max_x V(x) \Leftrightarrow \max_x \sum_i \alpha_i \log(\gamma_i(x)) \text{ such that } \sum_i x_i \leq C.$$

This problem differs from the one in Section III-B as it is non-convex. However, it can be convexified using the nonlinear transform  $x_i = e^{s_i}$ , and then admits a unique solution [20].

The resulting necessary and sufficient conditions for optimality are

$$\frac{\alpha_i}{x_i} - \sum_{j \neq i} \frac{\alpha_j}{I_j} = \lambda \text{ and } \sum_i x_i = C,$$

where  $I_i := \sum_{j \neq i} x_j + \sigma$  is the interference affecting player  $i$ . Hence, aligning the player and designer optimization problems leads to

$$P_i = \lambda + \sum_{j \neq i} \frac{\alpha_j}{I_j}.$$

Using the definition of  $\gamma$ , this can be rewritten as

$$P_i = \lambda + \sum_{j \neq i} P_j \gamma_j$$

or in matrix form

$$A \cdot P = \mathbf{1} \lambda,$$

where

$$A := \begin{pmatrix} 1 & -\gamma_2 & \cdots & -\gamma_N \\ -\gamma_1 & 1 & \cdots & -\gamma_N \\ \vdots & & \ddots & \vdots \\ -\gamma_1 & -\gamma_2 & \cdots & 1 \end{pmatrix}, \quad (27)$$

and  $\mathbf{1} = [1, \dots, 1]^T$ . Note that the matrix  $A$  is clearly full rank, and hence invertible.

As in Example 4, we define now an iterative pricing mechanism  $\mathcal{M}^P$  such that

$$\lambda(n+1) = \lambda(n) + \kappa_D \left( \sum_i x_i - C \right), \quad (28)$$

$$P(n+1) = (A)^{-1} \mathbf{1} \lambda(n), \quad (29)$$

and

$$x_i(n+1) = x_i(n) - \kappa_i \frac{\partial J_i}{\partial x_i} = U'_i(\gamma_i(n)) - P_i(n) \quad \forall i \in \mathcal{A}, \quad (30)$$

where the players adopt a gradient best response for convergence purposes. Here,  $\kappa_D$  and  $\kappa_i$  denote the step sizes of the designer and player  $i$ , respectively. Based on the analysis above, the mechanism is preference-compatible and efficient. Since the players have no incentive to deviate from their (gradient) best responses, it is also inherently strategy-proof as discussed in Example 4. This result is summarized in the following theorem.

**Theorem IV.1.** *The unique equilibrium outcome of the pricing mechanism  $\mathcal{M}^P$  defined by (28)-(30) is preference-compatible, strategy-proof, and efficient.*

The implementation of mechanism  $\mathcal{M}^P$  requires minimum information overhead. The designer only needs to observe the aggregate received power level  $\sum_i x_i$  and the individual SIRs,  $\gamma$ , of players both of which are already available. The player  $i$ , in return only needs to know the current price  $P_i$  and SIR  $\gamma_i$  to be able to compute the (gradient) best response (see Figure 4 for visualization). Finally, the computation of actual uplink power levels  $p$  can be computed from  $x$  using the measured channel gains.

*Example 5:*

The iterative pricing mechanism  $\mathcal{M}^P$  is illustrated with a numerical example. 10 players with the utility parameters

$$\alpha = [0.23 \ 1.33 \ 0.73 \ 0.28 \ 1.13 \ 1.65 \ 1.35 \ 2.00 \ 1.92 \ 0.12],$$

update their power levels according to (30) at each time step  $n \geq 1$  with a stepsize of  $\kappa_i = 0.05 \ \forall i$ . The designer, on the other hand, updates the Lagrangian multiplier  $\lambda$  and prices  $P$  based on (28), where  $C = 5$  and  $\kappa_D = 0.01$ . The background noise parameter in (25) is  $\sigma = 0.5$ . The convergence of the mechanism  $\mathcal{M}^P$  summarized in Algorithm 1 is depicted in Figures 5 and 6.

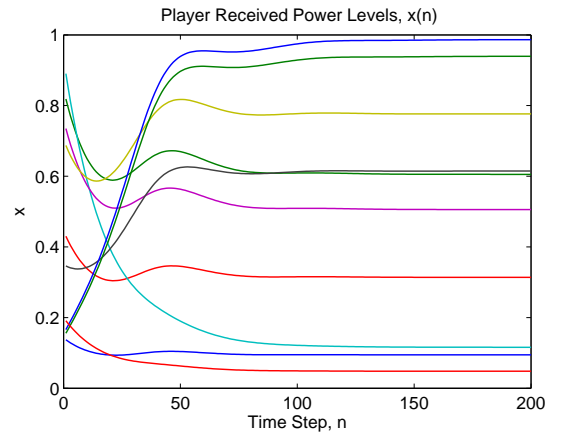


Fig. 5. The evolution of user power levels,  $x$ , which are updated by players (30) under mechanism  $\mathcal{M}^P$ .

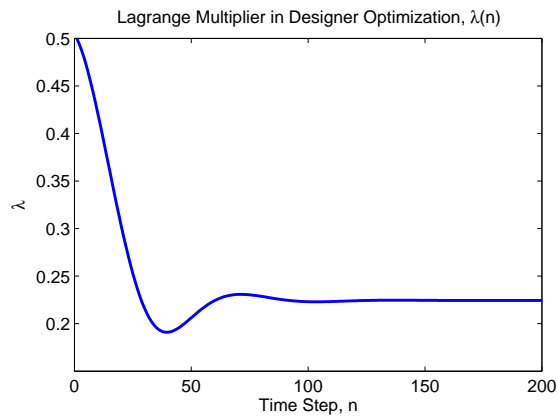


Fig. 6. The evolution of Lagrange multiplier,  $\lambda$ , used in computing player prices in (28).

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**Algorithm 1: Iterative Pricing Mechanism  $\mathcal{M}^P$** 


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**Input:** Designer (base station): Interference target  $C$  and objective  $\sum_i U_i$

**Input:** Players (users): Utilities  $U_i = \alpha_i \log(\gamma_i(x))$ ,  $\forall i$

**Result:** Power levels  $x$  and SIRs  $\gamma(x)$

- 1 Initial power levels  $x(0)$  and prices  $P_i(0)$  ;
  - 2 **repeat**
  - 3     **begin** Designer:
  - 4         Observe player power levels  $x$  ;
  - 5         Compute the matrix matrix  $A$  in (27) ;
  - 6         Update  $\lambda$  and prices  $P$  according to (30) ;
  - 7     **end**
  - 8     **begin** Players:
  - 9         **foreach** player  $i$  **do**
  - 10             Estimate marginal utility  $\partial U_i(x)/\partial x_i$  ;
  - 11             Compute power level  $x_i$  from (28) ;
  - 12         **end**
  - 13     **end**
  - 14 **until** end of iteration;
- 

## V. DISCUSSION AND LITERATURE REVIEW

There is a rich literature on Mechanism design both in the field of economics [7] and recently in engineering [8], [9], [12], [22], [27], [28]. The auction-based mechanism framework presented in Section III is based in principle on progressive second price (PSP) auctions [8], [10], [22]. The framework, on the one hand, simplifies PSP auctions by considering the users demanding as much of the resources as possible, which is a reasonable assumption in many cases since players often cannot estimate their demand accurately. On the other hand, it presents a unifying optimization framework which also allows analysis and design of games with non-separable player utilities.

The literature on pricing schemes is even richer than mechanism design one, especially in the networking community (see e.g. [4], [5] and references therein). The pricing mechanism framework in Section IV extends those results by building on [13], [14], and taking into account all of the criteria in Table I. Among other things, the presented framework

captures different types of global objectives, e.g. quality-of-service regions, information limitations, and system dynamics. The fact that an iterative pricing scheme similar to the one in [29] is required to satisfy all three criteria in Table I is an interesting result. This can be attributed to the designer having less leverage (no explicit resource allocation) in pricing mechanisms compared to auction-based ones.

There are many impossibility results in the mechanism design literature [15]–[18]. The framework presented in this paper does not actually contradict these results for in many cases analyzed one of the criteria in Table I is achieved only approximately. Similar approximations are quite common in game theory literature, e.g.  $\epsilon$ -NE. Hence, such relaxations are part of the constructive approach adopted here, and show its value.

We present next a brief survey of the literature on auctions, pricing, and mechanism design in general.

### Literature Review

*Auctions and Pricing in Games:* The book [30] provides a good overview of a variety of topics ranging from mechanism design, inefficiency of the equilibria, preference-compatibility issues and certain types of auctions. Lazar and Semret [8] have shown that a certain form of the Nash equilibrium holds when the progressive second price auction is applied by independent sellers on each link of a network with arbitrary topology.

Wu et al. [28] have proposed a repeated spectrum sharing game with cheat-proof strategies. By using the punishment-based repeated game, users get the incentive to share the spectrum in a cooperative way; and through mechanism-design-based and statistics-based approaches, user honesty is further enforced. Sengupta and Chatterjee [31] have presented an economic framework that can be used to guide the dynamic spectrum allocation process and the service pricing mechanisms that the providers can use. They have demonstrated how pricing can be used as an effective tool for providing incentives to the providers to upgrade their network resources and offer better services. Keon and Anandalingam [32] have formulated the optimal pricing problem as a nonlinear integer expected revenue optimization problem. They simultaneously solve for prices and the resource allocations necessary to provide connections with guaranteed QoS. Maille and Tuffin [33] have analyzed a multi-bid auction scheme where users compete for bandwidth at a link by submitting e.g. amount of bandwidth asked, associated unit price so that the link allocates the bandwidth and computes the charge according to the second price principle. In this case, the backbone network is overprovisioned and the access networks have a tree structure. The works [34]–[36] have discussed other interesting approaches in relation to auctions and bidding algorithms.

*Strategy Proofness and Efficiency:* The property of *strategy-proofness* is a fairly restrictive property. When it is combined with the property of *efficiency*, this often leads to special solutions. Hurwicz [16] has shown that there is no *strategy-proof, efficient* and *individually rational* mechanism in 2 user 2 resource pure exchange economy. Dasgupta et al. [17] have attempted to replace *individual rationality* in Hurwicz's result

with a weaker axiom of *non-dictatorship*. Ameliorating upon both results, Zhou [18] has established an impossibility result that there is no *strategy-proof*, *efficient* and *non-dictatorial* mechanism in 2 user  $m$  resource ( $m \geq 2$ ) pure exchange economies. He conjectures that there are no *strategy-proof*, *efficient* and *non-inversely dictatorial* mechanisms in the case of 3 or more users. In [37], Zhou's conjecture has been examined and a new class of *strategy-proof* and *efficient* mechanisms in the case of four or more users (operators) are discovered.

*Mechanism Design in Wireless Networks:* Huiping and Junde [38] have proposed a *strategy-proof* trust management system in the context of wireless ad-hoc networks. This system is preference-compatible in which nodes can honestly report trust evidence and truthfully compute and broadcast trust value of themselves and other nodes. Pal and Tardos [39] have developed a general method for turning a primal-dual algorithm into a group *strategy-proof* cost-sharing mechanism. The method was used to design approximately budget-balanced cost sharing mechanisms for two NP-complete problems: metric facility location, and single source rent-or-buy network design. Both mechanisms are competitive, group *strategy-proof* and recover a constant fraction of the cost. The works [40], [41] have presented a game theoretic framework for truthful broadcast protocol and *strategy-proof* pricing mechanism. Guanxiang et al. [42] have proposed an auction-based admission control and pricing mechanism for priority services, where higher priority services are allocated to the users who are more sensitive to delay, and each user pays a congestion fee for the external effect caused by their participation. The mechanism is proved to be *strategy-proof* and *efficient*. Wang and Li [43] have addressed the issue of user cooperation in selfish and rational wireless networks using an incentive approach. They have presented a *strategy-proof* pricing mechanism for the unicast problem and given a time optimal method to compute the payment in a centralized manner and discussed implementation of the algorithm in a distributed manner. In addition, they have presented a truthful mechanism when a node only colludes with its neighbors. Garg et al. [44], [45] have provided a tutorial on mechanism design and attempted to apply it to various concepts in engineering. Huang et al. [22], [46] have utilized SIR and power auctions to allocate resources in a wireless scenario and presented an asynchronous distributed algorithm for updating power levels and prices to characterize convergence using supermodular game theory. Wu et al. [28] have proposed a repeated spectrum sharing game with cheat-proof strategies. They have proposed specific cooperation rules based on maximum total throughput and proportional fairness criteria. Sharma and Teneketzis [47] have presented a decentralized algorithm to allocated transmission powers, such that the algorithm takes into account the externality generated to the other users, satisfies the informational constraints of the system, and overcomes the inefficiency of pricing mechanisms.

*Interference Coupling:* An axiomatic approach to interference functions has been proposed by Yates in [21] with extensions in [48], [49]. The Yates framework of *standard interference functions* is general enough to incorporate cross-layer effects and it serves as a theoretical basis for a variety

of algorithms. Certain examples include: beamforming [50], CDMA [51], base station assignment, robust design and networking [20]. The framework can be used to combine power control and adaptive receiver strategies. Certain examples, where this has been successfully achieved are as follows. In [52] it has been proposed to incorporate admission control to avoid unfavorable interference scenarios. In [53] the QoS requirements have been adapted to certain network conditions. In [54] a power control algorithm using fixed-point iterations has been proposed for a modified cost function, which permits control of convergence behavior by adjusting fixed weighting parameters.

## VI. CONCLUSIONS

An unified framework is presented for developing mechanisms such as auctions and pricing schemes, which is applicable to a fairly general class of strategic (noncooperative) games on networked systems. It has been shown that although the participating players of these mechanisms are selfish, the outcome is optimal with respect to a global criterion (e.g. maximizing a social welfare function), preference-compatible, and strategy-proof. The mechanism designer achieves these objectives by imposing rules and prices to the players. In auction-based mechanisms the designer explicitly allocates the resources based on bids of the participants in addition to setting prices. In pricing mechanism, however, global objectives are enforced by only charging the players for the resources they used. The unified framework as well as its information structures are illustrated through specific example resource allocation problems from wireless and wired networks.

The presented mechanism design framework can be extended in multiple directions. One immediate extension is multiple decision variables. A related but more challenging extension is multi-criteria decision making, where preferences are not simply expressed through scalar-valued utility or objective functions. Some of the other open research directions follow directly from relaxing the assumptions in Section II-A. Improving the robustness of the incentive mechanisms against malicious units who do not follow the rules and detection of such misbehavior is of both practical and theoretical interest. In parallel, the relaxation of the assumption on designer's honesty leads to similarly interesting questions such as how can a unit detect and respond to misbehavior (e.g. unfairness) of the designer. Additional future research directions include a more precise quantification of asymptotic approximations in the paper and analysis of networking effects between players.

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