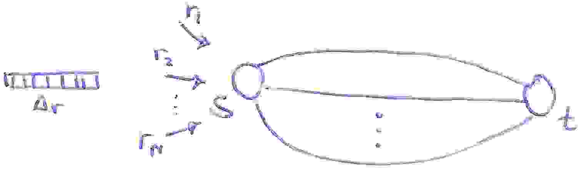
2-11-10

ΟΝΟΜΑ: ΓΙΑΝΝΟΠΟΥΛΟΣ ΜΕΝΕΛΑΟΣ

Non-atomic (splittable)



if $\ell_e(x)$ affine $\rightarrow PoA \leq \frac{4}{3}$

$s_1 \rightarrow t_1$
$s_2 \rightarrow t_2$
⋮

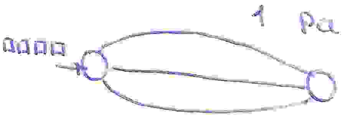
$\ell_e(x)$ $\exists$ at least one equilibrium

$\ell_e(f_e) = \ell_e(\tilde{f}_e)$ $\quad f, \tilde{f}$ NE

minimize $\sum_{e\in E} f_e' c_e(f_e)$

Best Response $\longrightarrow$ NE
(potential method)

Atomic (microscopic)



1 path

- Generally there isn't NE
- if $r_i = R$, $\exists$ equilibrium when $\ell_e(x)$ affine $PoA \leq \frac{3+\sqrt{5}}{2}$

• Lemma (Equilibrium condition)
Let f NE flow
and f^* Optimal flow
Let player $i = 1, \dots k$ use the path P_i (NE), P_i^* (Soc. Opt.)
for each edge $\ell_e(x) = a_e f_e + b_e$
then

$$C_{P_i}(f) = \sum_{e\in P_i} (a_e f_e + b_e) \leq \sum_{e\in P_i^*} (a_e(f_e + r_i) + b_e) \quad (*)$$

• Lemma (Equilibrium inequality):

$$\underbrace{C(f)}_{\text{total cost in NE}} \leq \underbrace{C(f^*)}_{\text{total cost in S.O}} + \sum_{e\in E} a_e f_e f_e^* \quad \text{where} \quad C(f) = \sum_{e\in E} f_e \ell_e(f_e)$$

Proof: $C(f) = \sum_e \underbrace{(a_e f_e + b_e)}_{\ell_e(x)} f_e = \sum_i f_p^{(i)} \cdot C_{P_i}(f) =$

$= \sum_i f_p^{(i)} \sum_{e\in P_i} (a_e f_e + b_e) \overset{(*)}{\underset{f_p^{(i)}=r_i}{\leq}} \sum_i r_i \sum_{e\in P_i^*} (a_e(f_e + r_i) + b_e) =$

$$= \sum_e \sum_{i: P_i^* \ni e} r_i (a_e (f_e + r_i) + b_e)$$

$$\underset{f_e^* \geq r_i}{\leq} \sum_e f_e^* (a_e (f_e + f_e^*) + b_e)$$

$$= \sum_e a_e f_e^{*2} + b_e f_e^* + \sum_{e \in E} a_e f_e f_e^* = \sum_e (a_e f_e^* + b_e) f_e^* + \sum_{e \in E} a_e f_e f_e^* =$$

$$= C(f^*) + \sum_{e \in E} a_e f_e f_e^* \Rightarrow C(f) \leq C(f^*) + \sum_{e \in E} a_e f_e f_e^*$$

When $l_e(x)$ affine cost functions then $PoA \leq \frac{3+\sqrt{5}}{2} \simeq 2{,}618$

Proof: Apply the Cauchy-Schwarz Inequality to the vectors $\{\sqrt{a_e}\, f_e\}_{e \in E}$ and $\{\sqrt{a_e}\, f_e^*\}_{e \in E}$ to obtain

$$\sum_{e \in E} a_e f_e f_e^* \leq \sqrt{\sum_{e \in E} a_e f_e^2} \cdot \sqrt{\sum_{e \in E} a_e (f_e^*)^2} \leq \sqrt{C(f)}\sqrt{C(f^*)}$$

$$\boxed{\text{Cauchy-Schwarz} \quad (\alpha^T \beta) \leq (\alpha^T \alpha)^{1/2} (\beta^T \beta)^{1/2}}$$

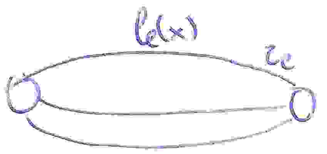
Proved that $C(f) \leq C(f^*) + \sqrt{C(f)} \cdot \sqrt{C(f^*)}$

$$\frac{C(f)}{C(f^*)} \leq 1 + \sqrt{\frac{C(f)}{C(f^*)}}$$

Solving $x \leq 1 + \sqrt{x}$ and $(x-1)^2 \leq x$

we find the result $x \leq \frac{3+\sqrt{5}}{2} \simeq 2{,}618$

Reducing PoA (non Atomic)



1) Pricing

$$\tilde{\ell}_e(x) = \ell_e(x) + \tau_e = \ell_e(x) + x\,\ell_e'(x) = \left(x \cdot \ell_e(x)\right)'$$

$\uparrow$ nonnegative tax $\quad \tau_e = f_e \cdot \ell_e'(f_e)$, $\quad \ell_e'(f_e) = \dfrac{d\ell_e(f_e)}{df_e}$

<u>Theorem</u>: (G, r, ℓ), f^* optimal flow and $\tau_e = f_e^* \ell_e'(f_e^*)$

Then f^* N.E flow for $(G, r, c+\tau)$

$$\ell_e(x) = \left(x \cdot \ell_e(x)\right)'\Big|_{f_e^*}$$

2) Capacity Augmentation

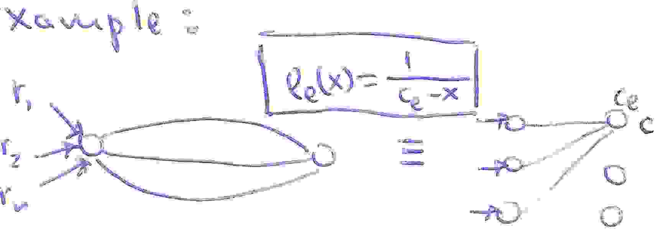
- if f NE flow for (G, r, ℓ) and f^* is feasible for $(G, 2r, \ell)$ then $C(f) \leq C(f^*)$

- Let (G, r, ℓ) be an instance and define the modified cost function $\tilde{\ell}_e(x) = \frac{1}{2}\ell_e\left(\frac{x}{2}\right)$

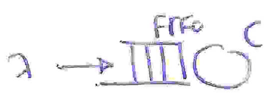
Let $\tilde{f}$ NE for $(G, r, \tilde{\ell})$ with $C(f^*)$ cost.

Then $\tilde{C}(\tilde{f}) \leq C(f^*)$

Example:



All functions are M/M/1 delay functions



$\frac{1}{C-\lambda}$ average delay per customer

The modified function is $\tilde{\ell}_e(x) = \frac{1}{2} \frac{1}{c_e - \frac{x}{2}} = \frac{1}{2c_e - x}$

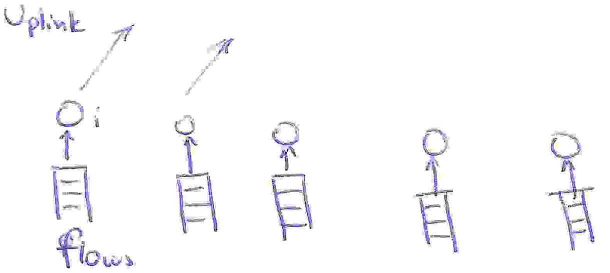
For selfish routing networks with M/M/1 delay functions: to outperform optimal routing, double the capacity of every edge. $\tilde{C}(\tilde{f}) \le C(f^*)$

<u>Example</u>: resources allocation

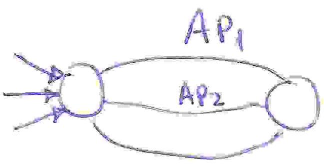
<u>Association Game</u>

IEEE 802.11 protocol

× × j × APs access points



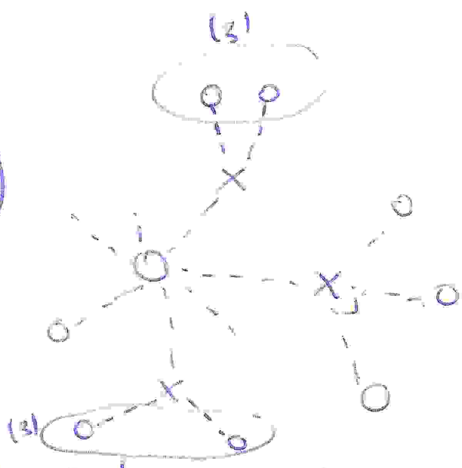
customers (they want to connect to the access point where there is little traffic)



<u>Cost factors</u>

1) <u>PHY rate</u> $i \to j$ ↑ rate, faster transmission of information

(α): $\frac{1}{R_{ij}}$



2) the rate of all 1-hop neighbours

(β) i: $\sum_{k \in N_i} \frac{1}{R_k}$

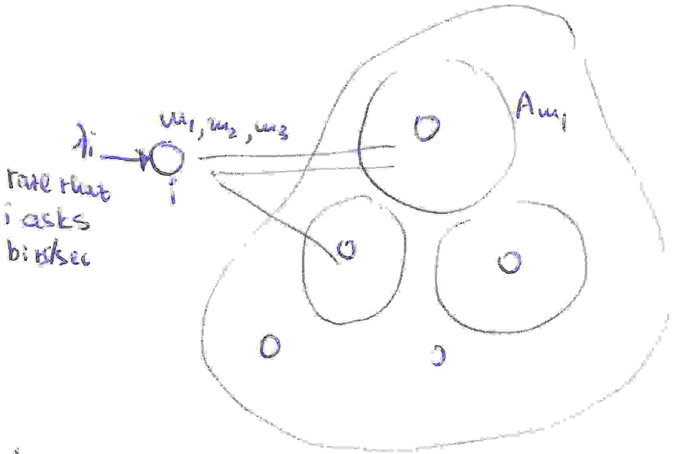
3) 2-hop neighbours (users) who speak to 1-hop neighbours access points

(γ) $\sum_{\ell \in N^i_{2hop}} \frac{1}{R_\ell}$

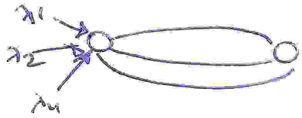
→ the total cost for connection of i with j

$C_{ij} = a_{ij} + \beta_i + \gamma_j$ $\quad C_{ij} = \frac{1}{R_{ij}} + \sum_{k \in N_i} \frac{1}{R_k} + \sum_{\ell \in N^i_{2hop}} \frac{1}{R_\ell}$

Peer to Peer networks



(atomic splittable)



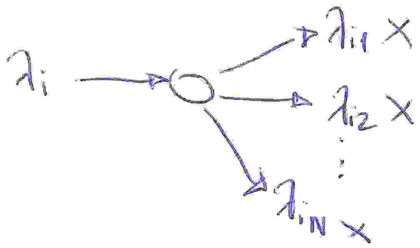
$\ell_e(x) = \frac{1}{c_e - x}$

L_i : $\frac{\text{bits}}{\text{request}}$

$\frac{\lambda_i}{L_i}$: $\frac{\text{request}}{\text{sec}}$

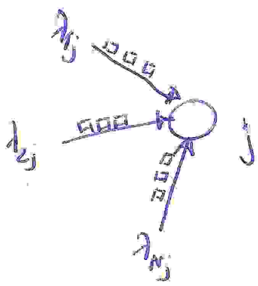
$\sum_{j \neq i} \lambda_{ij} = \lambda_i$

Client



$(\lambda_{ij} : j = 1, \ldots N \quad j \neq i)$

Server



$C_j \left(\frac{\text{bits}}{\text{sec}} \right)$

Let FIFO service

Queuing delay

Processing delay

$$\frac{1}{C_j - \Lambda_j}$$

delay / flow

$$\Lambda_j = \sum_{i \neq j} \lambda_{ij}$$

Problem:

$$D_i = \sum_{j \neq i} \frac{\lambda_{ij}}{\lambda_i} \cdot \frac{1}{C_j - \Lambda_j}$$

S.t. $\quad \sum \lambda_{ij} = \lambda_i$