The Impact of Space Division Multiplexing on Resource Allocation: A Unified Treatment of TDMA, OFDMA and CDMA

Iordanis Koutsopoulos, *Member, IEEE*, and Leandros Tassiulas, *Fellow, IEEE*

***Abstract*— Space division multiple access (SDMA) with an antenna array at the transmitter is a promising means for increasing system capacity and supporting rate-demanding services. However, the presence of an antenna array at the physical layer raises significant issues at higher layers. In this paper, we attempt to capture the impact of SDMA on access layer channel allocation, reflected on channel reuse. This impact obtains different twists in TDMA, CDMA and OFDMA due to the different nature of co-channel and cross-channel interference and the different interaction of user spatial channel characteristics with system channels, namely time slots, codes and subcarriers. We consider these access schemes in a generalized unified framework and propose heuristic algorithms for channel allocation, downlink beamforming and transmit power control so as to increase total provisioned system rate and provide QoS to users in the form of minimum rate guarantees. We study the class of greedy algorithms that rely on criteria such as induced or received interference and signal-to-interference ratio (SIR), and a class of SIR balancing algorithms. Results show superior performance for SIR balancing resource allocation and expose the performance benefits of cross-layer design.**

***Index Terms*— Beamforming, SDMA, power control, cross-layer design, resource allocation, T/C/OFDMA.**

I. INTRODUCTION

THE fundamental problem in wireless networks is to provide diverse quality of service (QoS) guarantees to users in the inherently volatile wireless medium by using limited resources. QoS at the physical layer is perceived as an acceptable signal-to-interference and noise ratio (SINR) or bit error rate (BER) at the user receiver, while QoS at the access and network layers implies provisioning of minimum rate or maximum delay guarantees. The fulfillment of QoS requirements relies on control mechanisms at different layers, such as scheduling, channel allocation, transmission power control and modulation level adaptation [2]. Existing and envisioned wireless systems use Time, Code or Orthogonal Frequency Division Multiple Access (TDMA, CDMA, OFDMA). The work in [3] is a representative one of joint treatment of channel allocation and power control in TDMA/FDMA systems. In currently employed CDMA-based 3G UMTS networks, the channels are spreading codes that are either randomly generated or designed in a deterministic manner to maximize user capacity and minimize total squared cross-correlation (TSC) [4]. Multi-code structures and spreading gain adaptation further aid transmission rate control and provisioning of diverse rates [5].

OFDMA is included in the IEEE 802.11a/g WLAN standards and is also considered in the IEEE 802.15.3 standard for wireless personal area networks (WPANs) and in the evolving IEEE 802.16x WiMAX standards for broadband wireless access. In OFDMA, the spectrum is divided into narrow-band subcarriers with overlapping spectra, which are orthogonal since they are appropriately spaced. The user bit stream is split into subsets, and each bit subset is called a subsymbol. Each subsymbol modulates a subcarrier and several subsymbols of a user are transmitted in parallel over subcarriers. OFDM transmission reduces effective symbol transmission rate and alleviates the effects of inter-symbol interference (ISI). A basic problem in OFDMA is to allocate subcarriers, transmission powers and rates to users so as to maximize information-theoretic rate. For a given subcarrier allocation this problem is solved by water-filling. For single-cell systems, the work [6] formulates this allocation problem as an integer programming one and finds a suboptimal solution for the continuous-valued problem. The same objective with a total power constraint over all users is achieved by assignment of each subcarrier to the user with the largest gain in it and subsequent power water-filling [7]. A continuous relaxation approach is applied in [8] for the dual problem of minimizing transmission power subject to minimum rate constraints. Heuristic algorithms for channel allocation, modulation and power control for a multi-cell OFDM system are presented in [9].

Adaptive antenna arrays promise to offer multiples of currently achieved data rates through Space Division Multiple Access (SDMA) [10], [11]. SDMA with a connection-oriented or connectionless access scheme allows reuse of conventional channels by spatially separable users. Within a channel, multiple beams are formed by the transmitter or receiver antenna array, with the main lobe of each beam steered to the direction of a desired user and nulls placed to directions of interferers. The objective is to separate co-channel users, that is, ensure acceptable SINRs at each user receiver. At reception, user separation can be achieved by computing one beam per user independently of others, while at transmission this issue becomes cumbersome since (i) each user beam

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The authors are with the Department of Computer and Communications Engineering, University of Thessaly, Volos, GR 38221, Greece (e-mail: {jordan, leandros}@uth.gr).

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affects interference at all other receivers and (ii) receivers are not collocated so that all user signals are jointly detected. Even if user receivers are equipped with multiple antennas or employ multi-user detection, extraction of a user signal takes place at each receiver separately. Downlink beamforming for power minimization in a single cell is studied in [12], where beamforming and power control are decoupled. In [13], the authors study beamforming for single-user OFDM transmission with multiple antennas at the transmitter and the receiver with the objective to maximize SINR. In [14], an iterative algorithm for transmit power control and receive beamforming for the uplink is proposed for a set of co-channel links, each with a minimum SINR requirement. The algorithm converges to a feasible solution if there exists one, and this solution minimizes total transmit power. In [15], the corresponding problem for the downlink is transformed to an equivalent problem for the uplink and is solved with the method of [14]. This approach cannot a priori detect infeasible instances where target SINRs cannot be reached. In [16], [17], an iterative algorithm for downlink beamforming and power control is presented, which always converges to the maximum common scaled SINR (scaled by each link SINR target).

The employment of antenna arrays introduces novel challenges at higher layers and recent works begin to address them. For an SDMA/TDMA system, heuristics for time slot assignment and user scheduling subject to deadlines are proposed in [18], [19]. In [20], subcarrier allocation, modulation control and beamforming are addressed for an OFDMA/SDMA system, where an algorithm for constructing co-channel user sets with large subcarrier rate is presented when channel reuse is allowed. Beamforming is also viewed there as an additional dimension to enhance user SINR for the case of no channel reuse. An effort to study distributed resource allocation in a multi-cell SDMA system is made in [21]. In a multi-channel system, channel allocation is coupled with physical layer beamforming and power control. Different users experience different quality in different channels, and spatial separability of users in a channel depends on beamforming, power control and user spatial channel characteristics in that channel. A given user allocation to channels constrains the choice of beamforming and power control, and vice versa. A channel allocation is efficient if it leads to beamforming and transmit power instances with good physical and access layer QoS.

In this work, we investigate the impact of SDMA on access layer channel allocation for the downlink of a single cell, with the objective to increase total achievable rate while providing user minimum rate guarantees. Our contribution to the current literature is summarized as follows: (i) we adopt a generalized framework for channel allocation, out of which TDMA, CDMA and OFDMA emerge as special cases, (ii) we incorporate downlink multi-user beamforming and power control in our approach and set up the framework for inclusion of transmission rate adaptation, (iii) we present and compare two classes of heuristic algorithms for identifying spatially separable co-channel sets of users, which can be viewed as instances of cross-layer design, since they employ physical and access layer mechanisms and strive to ensure acceptable QoS at both layers. Specifically, we focus on the class of greedy algorithms with assignment criteria that rely on induced and received interference or user SIR, as well as on the class of SIR balancing channel allocation algorithms. The goals of our study are to identify the structure of algorithms in each multiple access scheme and to demonstrate the benefits of cross-layer design in terms of achievable rate. The intense interest in adaptive antenna arrays is denoted by ongoing standardization efforts in the IEEE 802.11n standard for high throughput in conjunction with OFDMA [22]. Furthermore, multi-user beamforming is incorporated in the evolving cellular standards Cdma2000 1xEV-DO (Ultra Mobile Broadband) Revision C [23] and UMTS Long-Term Evolution (LTE) [24] for achieving high data rates through intra-cell channel reuse.

The rest of the paper is organized as follows. In section II we present the model and in section III we give the rationale of our approach and proposed algorithms. Section IV includes numerical results and section V concludes our study. A few words about notation. Vectors and matrices are set in boldface. The cardinality of set $\mathcal{X}$ is $|\mathcal{X}|$. Superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ denote transpose, complex conjugate and conjugate transpose and $\|\mathbf{u}\| = \sqrt{\sum_{i=1}^{n} |u_i|^2}$ is the ℓ_2-norm of complex vector $\mathbf{u} = (u_1, \ldots, u_n)^T$ respectively. The dominant generalized eigenvector of matrix pair $(\mathbf{A}, \mathbf{B})$, $\mathbf{u}_{\max}(\mathbf{A}, \mathbf{B})$ is the normalized eigenvector that corresponds to the largest positive eigenvalue of problem $\mathbf{Ax} = \lambda\mathbf{Bx}$. When $\mathbf{A}, \mathbf{B}$ are symmetric and positive-definite, the above is equivalent to $\mathbf{Cy} = \lambda\mathbf{y}$, with $\mathbf{C} = \mathbf{L}^{-1}\mathbf{A}(\mathbf{L}^{-1})^H$ and $\mathbf{y} = \mathbf{L}^H\mathbf{x}$, where $\mathbf{L}$ is a non-singular lower-triangular matrix from Cholesky decomposition of $\mathbf{B}$, $\mathbf{B} = \mathbf{LL}^H$.

II. System Model

We consider the downlink of a single-cell system with K users, operating in a frequency band of certain bandwidth. Depending on the access scheme (TDMA, CDMA or OFDMA), transmission occurs in channels that can be time slots, codes or subcarrier frequencies within the specified bandwidth and time frame. The base station (BS) has a uniform linear array of M antennas. Each user receiver has an omni-directional antenna. Packetized data arrive from higher layers and are decomposed into bits before transmission with an underlying slotted scheme. A fixed number of symbols, S are transmitted in a slot of duration T_s and the symbol (signaling) period is T. A user k has minimum rate requirement of r_k bits/sec over a certain time interval $(0, t)$, which is mapped to a minimum number of required channels x_k for single-rate transmission.

The block diagram of a generic SDMA transmitter is depicted in Figure 1(a). The channel allocation module allocates channels to each user and identifies co-channel user sets for each channel. Beamforming and power adaptation are then performed for each user allocated to a channel. The transmitter antenna array can form a unit-power beam vector $\mathbf{u}_{n,k} = (u_{n,k}^1, \ldots, u_{n,k}^M)^T$ of controllable orientation, determined by complex elements $\{u_{n,k}^m\}_{m=1}^M$ and it can transmit with controllable power $p_{n,k}$ to user k assigned to channel n. Since a transmit antenna array with M elements can form at most M linearly independent beam vectors to M users in a channel, at most M users can be separated in a channel. A beam is formed by a dedicated transceiver (beamformer) hardware unit, and we assume there exist at least NM such hardware units. A set of

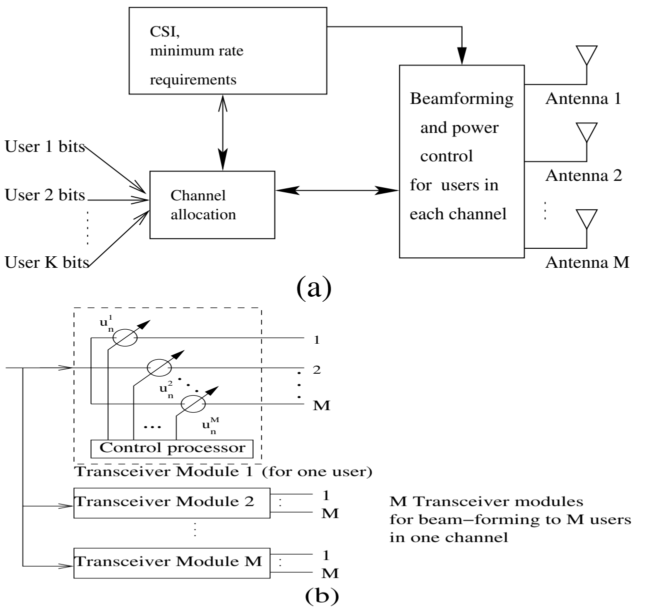

Fig. 1. (a) SDMA transmitter structure for a generic multiple access scheme. (b) M transceiver modules that form one beam for each assigned user to a channel n. These modules reside in the "Beamforming and power control" box of Figure (a).

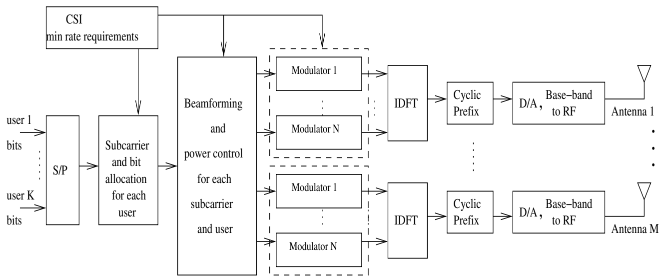

Fig. 2. Block diagram of a multi-user OFDMA/SDMA transmitter.

M transceivers is shown in Figure 1(b). User minimum rate requirements and channel state information (CSI) are inputs to the resource allocation algorithms.

Channel quality for a user remains constant in a time slot but may change between slots. The time-invariant (within a slot) channel between antenna m and user k has impulse response

$$h_k^m(t) = \sum_{\ell=1}^{L} \beta_{k,\ell}\, \delta\left(t - \tau_{k,\ell} + \tau_{k,\ell}^m\right), \tag{1}$$

where L is the number of paths, $\beta_{k,\ell}$ and $\tau_{k,\ell}$ are the complex gain and delay of the ℓ-th path of user k with respect to a reference antenna element respectively, and $\delta(\cdot)$ is the impulse function. Gains $\beta_{k,\ell}$ are complex Gaussian random variables with zero mean and variance $\sigma_{k,\ell}^2$ and delays $\tau_{k,\ell}$ are uniformly distributed in $[0, T]$. In general, different paths of a user are correlated. Close spacing between antennas is assumed, so that multi-path characteristics of a user are similar across antennas. The term $\tau_{k,\ell}^m = \frac{\Delta}{c}(m-1)\cos\theta_{k,\ell}$ captures the delay caused by the spacing between the mth antenna and the reference one, where Δ is the spacing between antennas, $\theta_{k,\ell}$ is the angle of the ℓth path of user k with respect to the antenna array and c is the electromagnetic wave propagation speed.

We now provide models for the OFDMA, TDMA and CDMA schemes. An OFDM/SDMA transmitter is shown in Figure 2. After subcarrier allocation, beamforming and power control, user bits are forwarded into M parallel modules of N modulators. A modulator modulates the corresponding subcarrier with b bits of each user assigned to that subcarrier. The complex subsymbol at the output of each modulator is formed by a QAM constellation with b bits per subsymbol. All subsymbols of each user are fed into the Inverse Discrete Fourier Transform (IDFT) module and are transformed into N time samples that make an OFDM user symbol. After cyclic prefix addition, D/A conversion and up-conversion, the continuous signals are transmitted from the M antennas.

Assuming that OFDM symbols do not overlap in time, we concentrate on one symbol. In the following, we use baseband equivalent signal models. At receiver k, after down-conversion, sampling at times $\{\frac{iT}{N}, i = 0, \ldots, N-1\}$ and DFT on time samples, the useful signal for user k at subcarrier n is $y_{n,k}^0 = \sqrt{p_{n,k}}\,(\mathbf{a}_{n,k}^H \mathbf{u}_{n,k})\, d_{n,k}$, where $d_{n,k}$ denotes a unit-power complex subsymbol. Vector

$$\mathbf{a}_{n,k} = \sum_{\ell=1}^{L} \xi_{k,\ell}(n) \mathbf{v}_n(\theta_{k,\ell}) \tag{2}$$

is called *spatial signature* of user k at subcarrier n. Factors $\xi_{k,\ell}(n) = \beta_{k,\ell}^* \exp\left(j2\pi \frac{n}{T} \tau_{k,\ell}\right)$ capture the impact of delay of path ℓ of user k on channel response at subcarrier n, and vector $\mathbf{v}_n(\theta_{k,\ell})$, whose mth component is $v_n^m(\theta_{k,\ell}) = \exp\left(-j2\pi \frac{n}{T} \tau_{k,\ell}^m\right)$, is the $M \times 1$ *antenna steering vector* at subcarrier n and direction $\theta_{k,\ell}$. Clearly $\mathbf{a}_{n,k}$ captures angular and multi-path properties of user k at subcarrier n. The expected useful received signal power is $\mathbb{E}\{|y_{n,k}^0|^2\} = p_{n,k}(\mathbf{u}_{n,k}^H \mathcal{H}_{n,k} \mathbf{u}_{n,k})$, where the $M \times M$ matrix $\mathcal{H}_{n,k}$ is

$$\mathcal{H}_{n,k} \triangleq \sum_{\ell_1=1}^{L} \sum_{\ell_2=1}^{L} \mathbb{E}\left\{\xi_{k,\ell_1}(n) \xi_{k,\ell_2}^*(n)\right\} \mathbf{v}_n(\theta_{k,\ell_1})\, \mathbf{v}_n^H(\theta_{k,\ell_2}) \tag{3}$$

and is called *spatial covariance matrix* of user k at subcarrier n. In general, rank$(\mathcal{H}_{n,k}) > 1$. If paths are uncorrelated, namely if it is $\mathbb{E}\left\{\xi_{k,\ell_1}(n) \xi_{k,\ell_2}^*(n)\right\} = 0$ for $\ell_1 \neq \ell_2$, then $\mathcal{H}_{n,k} = \sum_{\ell=1}^{L} \sigma_{k,\ell}^2 \mathbf{v}_n(\theta_{k,\ell}) \mathbf{v}_n^H(\theta_{k,\ell})$ and rank$(\mathcal{H}_{n,k}) > 1$ unless there is only a line-of-sight (LOS) path.

A note about CSI is in place here. Different forms of transmitter CSI are captured by modeling a spatial signature $\mathbf{a}$ as a complex Gaussian vector random variable with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, namely $\mathbf{a} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Perfect CSI is modeled by $\boldsymbol{\Sigma} = \mathbf{0}$. This arises in time duplexing with reasonably small channel variation rate implying low Doppler spread. The BS can learn the average vector channel for each user through uplink measurements and can use them to adapt the downlink beam. For perfect CSI, beamforming towards the signature vector is optimal in the sense of maximizing capacity [25]. For no CSI, transmission in orthogonal directions is optimal [26]. For rapid channel variations, the channel realization cannot be tracked. However, the relative geometry of propagation paths changes more slowly and this is reflected in the entries of the spatial covariance matrix. Then, CSI is modeled by knowledge of $\mathbf{\Sigma}$, and beamforming toward a direction corresponding to the largest eigenvalue of the covariance matrix is asymptotically optimal for low SNRs [25] and close to optimal in general [27]. In this work, we assume that transmitter CSI consists of estimates of spatial covariance matrices $\mathcal{H}_{n,k}$ for each user k and subcarrier n. Each matrix $\mathcal{H}_{n,k}$ is estimated by sampling received vector signal of user k in subcarrier n several times with known pilot symbols and by performing sample averaging. This presupposes that the channel process corresponding to an antenna element should be ergodic and wide-sense stationary.

The average signal-to-interference ratio (SIR) at the output of the matched filter receiver of user k at subcarrier n is

$$S_{n,k} = \frac{p_{n,k}\left(\mathbf{u}_{n,k}^H \mathcal{H}_{n,k}\mathbf{u}_{n,k}\right)}{\sum\limits_{j\in\mathcal{U}^{(n)}:j\neq k} p_{n,j}\left(\mathbf{u}_{n,j}^H \mathcal{H}_{n,k}\mathbf{u}_{n,j}\right)}, \tag{4}$$

where the denominator denotes co-channel interference and $\mathcal{U}^{(n)}$ is the set of users in subcarrier n. Our model is interference-limited in the sense that co-channel interference prevails. Apart from practical implications, this approach eliminates the need for total transmit power constraints. In the discussion above for OFDMA, we omitted time variation from $\mathbf{a}_{n,k}(t)$ and $\mathcal{H}_{n,k}(t)$ for notational simplicity.

In TDMA, time is partitioned in time slots and the entire bandwidth is used. The spatial signature and spatial covariance matrix of a user in each slot are obtained by averaging over frequency. That is, frequency selectivity is averaged out whenever frequency dependence needs to be taken into account. These quantities depend on temporal variations of multi-path characteristics in different slots. In OFDMA and TDMA, the SIR of a user does not depend on transmissions in other channels due to channel orthogonality.

In CDMA, the entire bandwidth and time frame is used. Channels are deterministic normalized codes of processing gain G that emerge from a code design or code generation method. We refer to vector $\mathbf{c}_n = (c_{n1},\ldots,c_{nG})$ as code n. A code pair (n,m) has cross-correlation $\rho_{nm} = \mathbf{c}_n^T\mathbf{c}_m$, with $\rho_{nn} = 1$. Code n is expressed as $c_n(t) = \sum_{r=1}^{G} c_{nr} q\left(t-(r-1)T_c\right)$, where $q(\cdot)$ is the chip pulse and T_c is the chip duration. The signal of user k carried by code n is $s_{n,k}(t) = \sum_i d_{n,k}(i)c_n(t-iT)$, where $\{d_{n,k}(i)\}$ is the symbol sequence. A single symbol is denoted by $s_{n,k}(t) = d_{n,k}c_n(t)$, where $d_{n,k}$ is a complex symbol formed by a linear modulation scheme with b bits per symbol. A code is associated with rate $b/(GT_c)$. A user k that uses n_k codes achieves rate $bn_k/(GT_c)$ bits/sec. The signal of user k carried by code n is multiplied by beam vector $\mathbf{u}_{n,k}$ and is allocated power $p_{n,k}$ before transmission. A code can be reused by several users if beamforming ensures user spatial separation.

The receiver of user k consists of a bank of matched filters, each of which is matched to a code used by that user. The signal at the output of the matched filter to code n is given by $y_{n,k} = \mathbf{c}_n^T\mathbf{y}_k$, where $\mathbf{y}_k = \sum_{m=1}^{N}\sum_{j\in\mathcal{U}^{(m)}}\sqrt{p_{m,j}}\mathbf{c}_{m,j}(\mathbf{a}_k^H\mathbf{u}_{m,j})d_{m,j}$ is the total received signal at the input of receiver k. The average SIR at the output of the matched filter to code n of user k is

$$S_{n,k} = \frac{p_{n,k}(\mathbf{u}_{n,k}^H\mathcal{H}_k\mathbf{u}_{n,k})}{\sum\limits_{\substack{j\in\mathcal{U}^{(n)}\\ j\neq k}} p_{n,j}(\mathbf{u}_{n,j}^H\mathcal{H}_k\mathbf{u}_{n,j}) + \sum\limits_{\substack{m=1\\ m\neq n}}^{N}\sum\limits_{j\in\mathcal{U}^{(m)}}\rho_{nm}^2 p_{m,j}(\mathbf{u}_{m,j}^H\mathcal{H}_k\mathbf{u}_{m,j})} \tag{5}$$

The two terms in the denominator are co-channel interference from other users that use code n and cross-channel interference from codes other than n respectively. At the receiver, multi-path components at different delays can be coherently combined with a RAKE structure and frequency selectivity is compensated. Spatial signatures $\mathbf{a}_k$ and spatial covariance matrices $\mathcal{H}_k$ for user k do not depend on code n.

In this work we adhere to a conventional matched filter receiver in order to place emphasis on the impact of SDMA on resource allocation for OFDMA, TDMA and CDMA. We note that beamforming and power control have also been considered in conjunction with advanced multi-user receiver structures in single-channel CDMA [28], [29]. A fixed modulation level with b bits/symbol is assumed. The minimum required SIR (in dB) for BER $\leq \epsilon$ at the receiver is threshold $\gamma = -(\ln(5\epsilon)/1.5)(2^b - 1)$ [30]. In a later section, we discuss implications of adaptive modulation on the problem, but we defer detailed study for a future work.

III. Resource Allocation in SDMA-Based Systems

A. Problem Statement

In TDMA and OFDMA, where channels are orthogonal, a set of users is *spatially separable in a channel* if there exist beamforming vectors and powers for each user i such that $\text{SIR}_i \geq \gamma$ for each user i in the channel. In a given channel, spatial separability depends on user spatial channel characteristics (that are captured by spatial covariance matrices) and on beamforming vectors and transmit powers that affect SIRs at all receivers. In TDMA, user spatial covariance matrices vary due to temporal variations of multi-path characteristics and spatial separability is addressed for each slot. In OFDMA, spatial separability depends on the particular subcarrier due to frequency selectivity of the wide-band channel, and also on the temporal variations of spatial characteristics between slots. That is, within a time slot, angular and multi-path characteristics of a user depend on the subcarrier frequency. Large channel reuse induces high total rate in a channel but also renders spatial separability difficult due to high interference.

In CDMA, user multi-path is compensated at each receiver with a RAKE path combiner after matched filtering. The spatial covariance matrix of a user is the same across all codes. The salient feature of CDMA is cross-channel interference due to code cross-correlation. A user with an assigned code receives co-channel interference from other users that use the same code and also receives cross-channel interference from other used codes that are non-orthogonal to its code. Hence, spatial separability of a user set $\mathcal{U}$ cannot be addressed separately for each code but must be considered collectively for a set of codes $\mathcal{C}$ that are used by $\mathcal{U}$. A user set $\mathcal{U}$ is spatially separable with respect to a channel (code) set $\mathcal{C}$ if there exists a beamforming vector and power for each user i assigned to a channel in $\mathcal{C}$ such that $\mathrm{SIR}_i \geq \gamma$ for each user i.

The arising problem is to perform channel allocation and user spatial separation jointly so as to increase total rate and provide QoS guarantees to users. For TDMA and OFDMA, a large co-channel set of spatially separable users needs to be identified for each channel. Spatial separability of users amounts to large angular separation (if only a LOS path exists) or to nearly-orthogonal user spatial signatures (if several paths exist), so that the joint effect of spatial covariance matrices and beams is a small induced interference. Finding the largest spatially separable co-channel user set is a hard combinatorial optimization problem. Spatial separability depends jointly on beamforming vectors and powers of all users and enumeration of all possible user assignments in a channel has exponential complexity. In CDMA, users in different channels also create cross-channel interference among themselves. Code assignment should be such that user spatial channel characteristics, code cross-correlations, beamforming vectors and powers result in a spatially separable user set. We therefore need to resort to efficient heuristic algorithms and study three of them in the sequel. The first two belong in the class of greedy heuristics and use criteria based on minimum induced or received interference and worst case SIR. We refer to them as algorithms A and B. The third one relies on the concept of SIR balancing in each channel and is referred to as algorithm C. The algorithms are presented for the general case, that is for non-orthogonal channels and channel-dependent spatial covariance matrices. TDMA, OFDMA and CDMA emerge as special cases of this general case.

B. Greedy Assignment Algorithms A and B

In order to keep reasonable complexity, we consider algorithms that involve sequential user assignment in a channel and no reassignments. User beams are controlled upon each assignment. Transmit powers are initially fixed. Power control is considered only if beamforming alone cannot ensure acceptable SIRs. Let $\mathbf{u}_{m,j}$ and $p_{m,j}$ be the beam vector and power for user $j \in \mathcal{U}^{(m)}$, where $\mathcal{U}^{(m)}$ is the co-channel user set of channel m. We start with algorithm A.

1. Beamforming vector control. A user assignment to a channel is followed by beamforming vector adjustment for existing users so that all user SIRs exceed γ. A potential insertion of user k in channel n creates co-channel interference to users in $\mathcal{U}^{(n)}$ and cross-channel interference to users in cross-correlated channels $m \neq n$. An inserted user should induce small total interference to users that are already assigned in channels, and should receive small total interference from these users. For user k and channel n we define the ratio of useful signal power from beam $\mathbf{u}_{n,k}$ over interference that is caused by beam $\mathbf{u}_{n,k}$ to users in other channels. We need the maximum value of this ratio over all beams $\mathbf{u}_{n,k}$,

$$\Psi_{n,k} = \max_{\mathbf{u}_{n,k}} \frac{\mathbf{u}_{n,k}^H \mathcal{H}_{n,k} \mathbf{u}_{n,k}}{\mathbf{u}_{n,k}^H \Big(\sum_{j \in \mathcal{U}^{(n)}} \mathcal{H}_{n,j} + \sum_{\substack{m=1 \\ m \neq n}}^{N} \sum_{i \in \mathcal{U}^{(m)}} \rho_{mn}^2 \mathcal{H}_{m,i} \Big) \mathbf{u}_{n,k}} \tag{6}$$

such that $\|\mathbf{u}_{n,k}\| = 1$. The denominator captures co-channel and cross-channel interference caused by beam $\mathbf{u}_{n,k}$ to other users. The vector $\mathbf{u}_{n,k}^*$ that maximizes the ratio in (6) is the dominant generalized eigenvector corresponding to the two matrices in the numerator and denominator of the fraction in (6) and is computed by the method outlined at the end of section I. We also compute the ratio $\Psi_{m,j}^{(n,k)}$ that quantifies the impact of user k's insertion in channel n on user $j \in \mathcal{U}^{(m)}$ and is equal to

$$\max_{\mathbf{u}_{m,j}} \frac{\mathbf{u}_{m,j}^H \mathcal{H}_{m,j} \mathbf{u}_{m,j}}{\mathbf{u}_{m,j}^H \Big(\sum_{\substack{\mu=1 \\ \mu \neq m}}^{N} \sum_{i \in \mathcal{U}^{(\mu)}} \rho_{\mu m}^2 \mathcal{H}_{\mu,i} + \sum_{\substack{i \in \mathcal{U}^{(m)} \\ i \neq j}} \mathcal{H}_{m,i} + \rho_{mn}^2 \mathcal{H}_{n,k} \Big) \mathbf{u}_{m,j}}, \tag{7}$$

such that $\|\mathbf{u}_{m,j}\| = 1$. Note that the computed beams are the ones that maximize user SIRs in a virtual uplink system. With the computed beamforming vectors, we evaluate the SIRs for all assigned users in channels.

2. Transmit power control. If SIRs for some users do not exceed γ, we employ power control (while keeping the computed beamforming vectors fixed) so that all SIRs exceed γ. For channel n, let i, j be indices of users in that channel. Let $\mathbf{U}$ be the computed ensemble of beamforming vectors for all users in channels, i.e $\mathbf{U} = \big\{\mathbf{u}_{n,k} : k \in \mathcal{U}^{(n)},\, n = 1, \ldots, N\big\}$. Define the $(\sum_{n=1}^N |\mathcal{U}^{(n)}|) \times (\sum_{n=1}^N |\mathcal{U}^{(n)}|)$ block matrix

$$\mathbf{A}(\mathbf{U}) = \begin{pmatrix} \mathbf{A}_{11}(\mathbf{U}) & \ldots & \mathbf{A}_{1N}(\mathbf{U}) \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{N1}(\mathbf{U}) & \ldots & \mathbf{A}_{NN}(\mathbf{U}) \end{pmatrix}. \tag{8}$$

The $[i,j]$-th element of the $(|\mathcal{U}^{(n)}| \times |\mathcal{U}^{(n)}|)$ matrix $\mathbf{A}_{nn}(\mathbf{U})$ in the diagonal of $\mathbf{A}(\mathbf{U})$ denotes co-channel interference caused by the beam of user j to user i in channel n,

$$\mathbf{A}_{nn}(\mathbf{U})[i,j] = \begin{cases} \mathbf{u}_{n,j}^H \mathcal{H}_{n,i} \mathbf{u}_{n,j} & \text{if } i \neq j \\ 0, & \text{if } i = j. \end{cases} \tag{9}$$

The $[i,j]$-th element of the $(|\mathcal{U}^{(n)}| \times |\mathcal{U}^{(m)}|)$ matrix $\mathbf{A}_{nm}(\mathbf{U})$, $n \neq m$ denotes the cross-channel interference caused by the beam of user $j \in \mathcal{U}^{(m)}$ to user $i \in \mathcal{U}^{(n)}$ and is given by $\mathbf{A}_{nm}(\mathbf{U})[i,j] = \rho_{nm}^2 (\mathbf{u}_{m,j}^H \mathcal{H}_{n,i} \mathbf{u}_{m,j})$, where ρ_{nm} is the cross-correlation between channels n and m. We also define the diagonal matrix $\mathbf{\Delta} = \mathrm{diag}\big\{\frac{1}{\mathbf{u}_{n,k}^H \mathcal{H}_{n,k} \mathbf{u}_{n,k}} : k = 1, \ldots, |\mathcal{U}^{(n)}|, n = 1, \ldots, N\big\}$ and the $(\sum_{n=1}^N |\mathcal{U}^{n)}|) \times 1$ vector $\mathbf{p}$ of user transmission powers in channels. Then, the condition $\mathrm{SIR}_{n,k} \geq \gamma$ for each user k in each channel n is written in matrix form as:

$$\mathbf{p} \geq \gamma \mathbf{\Delta} \mathbf{A}(\mathbf{U}) \mathbf{p}. \tag{10}$$

Matrix $\mathbf{\Delta A}(\mathbf{U})$ is non-negative definite and irreducible. From the Perron-Frobenius theorem, it has exactly one positive, real eigenvalue $\lambda^* = \max_i |\lambda_i|$, where λ_i, for $i = 1, \ldots, (\sum_n |\mathcal{U}^{(n)}|)$ are the eigenvalues of $\mathbf{\Delta A}(\mathbf{U})$. Eigenvalue λ^* has an associated eigenvector $\mathbf{p}^*$ with strictly positive entries. Furthermore, the minimum real λ for which inequality $\lambda \mathbf{p} \geq \mathbf{\Delta A}(\mathbf{U}) \mathbf{p}$ has solutions $\mathbf{p} > 0$ is $\lambda = \lambda^*$. We start by finding the maximum real positive eigenvalue λ^* of $\mathbf{\Delta A}(\mathbf{U})$. If $\lambda^* \leq 1/\gamma$, then (10) holds and the SIR level γ is called *feasible* or equivalently users are said to form a feasible set. The power vector for feasible γ is the eigenvector that corresponds to λ^*.

With the procedure above, we compute beams and powers for a specific user assignment. We need to evaluate different assignments that lead to feasible user sets and select the best one. For each possible insertion of user k in channel n that leads to a feasible user set, we define the assignment preference factor $\Phi_{n,k}$. This should be large if beams and powers yield strong useful signal for user k, low interference $I_{n,k}$ caused by user k to other users and low induced interference $I'_{n,k}$ by other users on k. We consider the largest of these two amounts of interference and define factors

$$\Phi_{n,k} = \frac{p_{n,k}(\mathbf{u}_{n,k}^{*H}\mathcal{H}_{n,k}\mathbf{u}_{n,k}^*)}{\max\left\{I_{n,k}\,,\,I'_{n,k}\right\}}\,, \tag{11}$$

where $I_{n,k}$ and $I'_{n,k}$ are given by

$$I_{n,k} = p_{n,k}\mathbf{u}_{n,k}^{*H}\Big(\sum_{j\in\mathcal{U}^{(n)}}\mathcal{H}_{n,j} + \sum_{\substack{m=1\\m\neq n}}^{N}\sum_{i\in\mathcal{U}^{(m)}}\rho_{mn}^2\mathcal{H}_{m,i}\Big)\mathbf{u}_{n,k}^* \tag{12}$$

$$I'_{n,k} = \sum_{j\in\mathcal{U}^{(n)}}p_{n,j}(\mathbf{u}_{n,j}^{*H}\mathcal{H}_{n,k}\mathbf{u}_{n,j}^*) + \sum_{\substack{m=1\\m\neq n}}^{N}\sum_{i\in\mathcal{U}^{(m)}}\rho_{mn}^2 p_{m,i}(\mathbf{u}_{m,i}^{*H}\mathcal{H}_{n,k}\mathbf{u}_{m,i}^*). \tag{13}$$

When power control is not active, these expressions do not include powers. At each step, factors $\Phi_{n,k}$ are computed for all possible insertions of users k that have not satisfied minimum rate requirements x_k and for all channels n where insertion of a user leads to a feasible user set. The assignment with maximum $\Phi_{n,k}$ is selected and the rate of user k is updated. When a user k reaches x_k, it is not considered for further assignments until all users reach minimum rate requirements. A channel is not further considered if M users are already assigned to it. The algorithm terminates if no further user insertion in any channel leads to feasible user set.

The greedy approach of least incremental interference in algorithm A aims at inserting as many users as possible. Greedy algorithm B relies on maximizing the minimum SIR of users: a user assignment in a channel is performed if it maximizes the minimum SIR of users in the system over all possible assignments. That is, algorithm B also captures the impact of an assignment on other users, so that SIRs that are closer to γ are maximized, future assignments are facilitated and ultimately the number of users with SIRs above γ is increased. The assignment factors for algorithm B are $\Phi_{n,k} = \min\left\{S_{n,k}, \min_m \min_{j\in\mathcal{U}^{(m)}} S_{m,j}\right\}$.

A large time interval of several time frames is considered, over which users need to achieve their minimum rates. In TDMA, channels are orthogonal time slots and spatial covariance matrices of users change due to temporal variations of multi-path channel characteristics between slots. Only interference from co-channel users exists. Each slot is considered separately and users are assigned sequentially based on factors $\Phi_{n,k}$. In OFDMA, channels are orthogonal subcarriers and a user k has different spatial covariance matrices $\mathcal{H}_{n,k}$ in each subcarrier n and slot. Subcarriers are filled with users in each slot of the time frame. In CDMA, channels are non-orthogonal codes due to nonzero pairwise code cross-correlations. Spatial covariance matrices do not depend on codes and change only due to multi-path temporal variations. In each frame, codes are allocated to users until the algorithm terminates, and the procedure repeats in the next frame.

C. SIR Balancing Assignment Algorithm C

In algorithms A and B, beamforming and power control were decoupled. Algorithm C is based on SIR balancing and attempts to provide maximum common user SIR by employing joint beamforming and power control.

1) Single-channel algorithm: Consider channel n with user set $\mathcal{U}^{(n)}$. Let $\mathbf{p}_n$ and $\mathbf{U}_n = \{\mathbf{u}_{n,k} : k \in \mathcal{U}^{(n)}\}$ be the transmit power vector and ensemble of beamforming vectors for users in $\mathcal{U}^{(n)}$. Consider interference matrix $\mathbf{A}_{nn}(\mathbf{U})$ in (9) and call it $\mathbf{B}(\mathbf{U}_n)$. Define the diagonal matrix $\mathbf{\Delta}_n = \mathrm{diag}\big\{\frac{1}{\mathbf{u}_{n,i}^H\mathcal{H}_{n,i}\mathbf{u}_{n,i}} : i \in \mathcal{U}^{(n)}\big\}$. The condition $S_{n,k} \geq \gamma$ for users is channel n with beamforming vectors $\mathbf{U}_n$ and power vector $\mathbf{p}_n$ in the *downlink* is written in matrix form as [16], [17]:

$$\mathbf{p}_n \geq \gamma\mathbf{\Delta}_n\mathbf{B}(\mathbf{U}_n)\mathbf{p}_n\,. \tag{14}$$

Matrix $\mathbf{\Delta}_n\mathbf{B}(\mathbf{U}_n)$ has the same properties as $\mathbf{\Delta}\mathbf{A}(\mathbf{U})$ with respect to existence of a positive eigenvalue and an eigenvector $\mathbf{p}_n$ with positive components. With the same reasoning as before, the maximum common SIR is given by

$$\gamma_c^* = \frac{1}{\min\limits_{\mathbf{U}_n}\lambda^*(\mathbf{\Delta}_n\mathbf{B}(\mathbf{U}_n))}. \tag{15}$$

For the corresponding problem in the *uplink*, the SIR requirements with ensemble of beamforming vectors $\tilde{\mathbf{U}}_n$ and power vector $\tilde{\mathbf{p}}_n$ are expressed as $\tilde{\mathbf{p}}_n \geq \gamma\mathbf{\Delta}_n\mathbf{B}^T(\tilde{\mathbf{U}}_n)\tilde{\mathbf{p}}_n$ and the maximum possible common SIR $\tilde{\gamma}_c^*$ is

$$\tilde{\gamma}_c^* = \frac{1}{\min\limits_{\tilde{\mathbf{U}}_n}\lambda^*(\mathbf{\Delta}_n\mathbf{B}^T(\tilde{\mathbf{U}}_n))}. \tag{16}$$

The following properties for the relationship between the downlink and uplink problems hold [16], [17]:

Property 1: For given set of beamforming vectors $\mathbf{U}_n$, it is $\lambda^*(\mathbf{\Delta}_n\mathbf{B}(\mathbf{U}_n)) = \lambda^*(\mathbf{\Delta}_n\mathbf{B}^T(\mathbf{U}_n))$.

Property 2: The downlink and uplink problems have the same maximum achievable common SIR, namely $\gamma_c^* = \tilde{\gamma}_c^*$.

Property 3: The beamforming vectors that solve downlink and uplink problems (15),(16) are the same, i.e. $\mathbf{U}_n^* = \tilde{\mathbf{U}}_n^*$.

Property 4: In algorithm I below, the sequence of eigenvalues $\lambda^{*(t)}$ is monotonically decreasing with iteration t and converges to a minimum eigenvalue that is related to the maximum common SIR through (15) and (16).

As a side note, the properties above also hold in the presence of noise, with equal noise power level at all receivers [17]. The steps of Algorithm I are:

- **STEP 1**: Set $t = 0$. Start with arbitrary beamforming vectors $\mathbf{U}_n^{(0)}$.
- **STEP 2**: $t \leftarrow t + 1$. For given $\mathbf{U}_n^{(t)}$, solve the uplink eigenproblem $\mathbf{\Delta}_n\mathbf{B}^T(\mathbf{U}_n^{(t)})\mathbf{p}_n^{(t)} = \lambda^{*(t)}\mathbf{p}_n^{(t)}$.

- **STEP 3**: For the computed $\mathbf{p}_n^{(t)}$, solve a set of *decoupled* generalized eigen-problems:

$$\mathbf{u}_{n,k}^{(t)} = \arg\max_{\mathbf{u}_{n,k}} \frac{\mathbf{u}_{n,k}^H \mathcal{H}_{n,k} \mathbf{u}_{n,k}}{\mathbf{u}_{n,k}^H \mathcal{R}_{n,k}(\mathbf{p}_n^{(t)}) \mathbf{u}_{n,k}^H} \tag{17}$$

subject to $\|\mathbf{u}_{n,k}\| = 1$ for $k \in \mathcal{U}^{(n)}$, where matrix $\mathcal{R}_{n,k}(\mathbf{p}_n^{(t)}) = \sum_{j \in \mathcal{U}^{(n)}, j \neq k} p_{n,j}^{(t)} \mathcal{H}_{n,j}$.
- **STEP 4**: With the computed $\mathbf{U}_n^{(t)}$, go to Step 2. Continue until $|\lambda^{*(t+1)} - \lambda^{*(t)}| < \epsilon$, with $\epsilon > 0$ a small constant.

In Step 3, the quantity to be maximized is the uplink SIR of user k. Beamforming vectors $\mathbf{U}_n^*$ at the end of the algorithm are the desired downlink beams. If $\lambda_0 = \lambda^*(\mathbf{\Delta}_n \mathbf{B}^T(\mathbf{U}_n^*))$ is the eigenvalue at the end of the algorithm, the downlink power vector is the eigenvector of $\mathbf{\Delta}_n \mathbf{B}(\mathbf{U}_n^*)$ corresponding to λ_0. If $1/\lambda_0 \geq \gamma$, SIR γ is achievable for all co-channel users.

2) Description of Algorithm C: Algorithm C is based on the observation that a system with K users in N non-orthogonal channels is equivalent to a single-channel system of interfering users and can be described by block matrix $\mathbf{A}(\mathbf{U})$ in (8). A system where users achieve common SIR γ_c in the downlink is described by equations $\mathbf{p} = \gamma_c \mathbf{\Delta} \mathbf{A}(\mathbf{U}) \mathbf{p}$ and the matrix in Step 3 of algorithm I is

$$\mathcal{R}_{n,k}(\mathbf{p}^{(t)}) = \sum_{\substack{j \in \mathcal{U}^{(n)} \\ j \neq k}} p_{n,j}^{(t)} \mathcal{H}_{n,j} + \sum_{\substack{m=1 \\ m \neq n}}^{N} \sum_{j \in \mathcal{U}^{(m)}} \rho_{nm}^2 p_{m,j}^{(t)} \mathcal{H}_{m,j}. \tag{18}$$

Fix an assignment of users to channels and let γ_c^* be the maximum common SIR of users as the outcome of algorithm I. For each user $k \in \mathcal{U}^{(n)}$, $n = 1, \ldots, N$ let $\gamma_{c,n}(k)$ be the maximum common SIR of remaining users when k is *removed* from channel n. Again $\gamma_{c,n}(k)$ is found by Algorithm I with an appropriately modified matrix $\mathbf{A}(\mathbf{U})$. Initially, all users are assigned in all channels. At each step, a user is removed from a channel, such that the highest common SIR is incurred for remaining users. The procedure continues until the desired common SIR γ is reached. The goal is to remove few users until common SIR γ is reached, so as to achieve high total rate. The steps of algorithm C are as follows:

- **STEP 0** : Start by assigning all K users in each channel n, $n = 1, \ldots, N$.
- **STEP 1** : Run algorithm I and find the maximum common SIR γ_c^* for the system.
- **STEP 2** : If $\gamma_c^* \geq \gamma$, the algorithm is terminated. Otherwise go to Step 3.
- **STEP 3** : For each $k \in \mathcal{U}^{(n)}$, $n = 1, \ldots, N$ compute $\gamma_{c,n}(k)$ with Algorithm I. Select pair (n^*, k^*) with the maximum $\gamma_{c,n}(k)$ and remove user k^* from channel n^*.
- **STEP 4** : Update user rates. If a user k reaches minimum rate requirements x_k, do not consider it for removal. Set $\gamma_{c,n^*}(k^*) = \gamma_c^*$. Go to Step 2.

Again users need to achieve some minimum rates over a time interval. In CDMA, Algorithm C runs in each frame. In TDMA and OFDMA, where matrices $\mathbf{A}_{ij}(\mathbf{U}) = \mathbf{0}$ for $i \neq j$, a separate problem (15) is solved for each channel n. In TDMA, each slot is considered separately and the single-channel version of algorithm C is applied. In OFDMA, in each slot we start by assigning all users in each subcarrier and run algorithm I for each subcarrier to get a vector of common channel SIRs, $\boldsymbol{\gamma}_c = (\gamma_{c,1}, \ldots, \gamma_{c,N})$, where $\gamma_{c,n}$ is the common SIR for users in subcarrier n. If $\gamma_{c,n} \geq \gamma$ for all $n = 1, \ldots, N$, the algorithm terminates. Otherwise users must be removed from subcarriers n with $\gamma_{c,n} < \gamma$. For each user k in such a subcarrier n, let $\gamma_{c,n}(k)$ be the common SIR in n after k is removed. At each step we remove the user k from subcarrier n so that $\gamma_{c,n}(k)$ is maximum. Next, user rates are updated. When a user k reaches x_k, it is not considered in later iterations. If $\gamma_{c,n} \geq \gamma$ for a subcarrier n at some stage of the algorithm, no more users are removed from n.

D. Optimal solution for K=2 users and N=1 channel

For $K = 2$ co-channel users and $M \geq 2$, let $\mathcal{H}_i$, $\mathbf{u}_i$ and p_i be the spatial covariance matrix, beam and power of user i. We start with beams $\{\mathbf{u}_i^{(0)}\}$. In the first iteration of algorithm I, we have $\lambda^{*(1)}$ as a function of $\mathcal{H}_i$ and $\{\mathbf{u}_i^{(0)}\}$ and we get power ratio $\mu^{(1)} = p_2/p_1$ in Step 2. In Step 3, we find beams $\mathbf{u}_1 = \mathbf{u}_{\max}(\mathcal{H}_1, \mathcal{H}_2)$ and $\mathbf{u}_2 = \mathbf{u}_{\max}(\mathcal{H}_2, \mathcal{H}_1)$. In second iteration, we have $\lambda^{*(2)} = \left[\lambda_{\max}(\mathcal{H}_1, \mathcal{H}_2)\lambda_{\min}(\mathcal{H}_1, \mathcal{H}_2)\right]^{1/2}$ and ratio $\mu^{(2)} = \left[\lambda_{\max}(\mathcal{H}_1, \mathcal{H}_2)/\lambda_{\min}(\mathcal{H}_1, \mathcal{H}_2)\right]^{1/2}$, where $\lambda_{\max}(\mathcal{H}_1, \mathcal{H}_2)$, $\lambda_{\min}(\mathcal{H}_1, \mathcal{H}_2)$ are the maximum and minimum generalized eigenvalues of $(\mathcal{H}_1, \mathcal{H}_2)$. These do not change in later iterations. Thus, the maximum common SIR is $\gamma_c^* = 1/\lambda^{*(2)}$ with $\mathbf{u}_1, \mathbf{u}_2$ and power ratio p_2/p_1 above.

E. Adaptive Modulation

When adaptive modulation is employed, the number of bits of a user is selected from a L_0-element set $\mathcal{M}$ of available QAM or QPSK modulation levels with different number of bits per symbol, $\{b_i\}_{i=1}^{L_0}$. In OFDMA, different number of bits of a user can modulate a subcarrier depending on subcarrier quality. In TDMA, the number of bits $b_i S$ conveyed to a user in a slot is adaptable, while in CDMA the rate b_i/GT_c carried by a code is controlled.

Consider $m \leq M$ co-channel users in channel n. Let $\mathbf{b} = (b_1, b_2, \ldots, b_m)^T$ be the user modulation level vector and $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_m)^T$ the corresponding threshold vector, with $\gamma_i = -(\ln(5\epsilon)/1.5)(2^{b_i} - 1)$. Co-channel user set $\mathcal{U}^{(n)}$ is *spatially separable* with respect to modulation vector $\mathbf{b}$ or equivalently $\mathbf{b}$ is *achievable* if $\mathbf{p} \geq \tilde{\mathbf{\Delta}}_n \mathbf{B}(\mathbf{U}_n)\mathbf{p}$, where $\tilde{\mathbf{\Delta}}_n = \text{diag}\left\{\frac{\gamma_i}{\mathbf{u}_{n,i}^H \mathcal{H}_{n,i} \mathbf{u}_{n,i}} : i \in \mathcal{U}^{(n)}\right\}$ and $\mathbf{p}$ is the corresponding transmit power vector. If the maximum positive eigenvalue of matrix $\tilde{\mathbf{\Delta}}_n \mathbf{B}(\mathbf{U}_n)$ satisfies $\lambda^{**} \leq 1$, the modulation vector $\mathbf{b}$ is achievable and the power vector that achieves $\mathbf{b}$ is the eigenvector that corresponds to λ^{**}. High modulation levels for users in a channel imply more transmitted bits per user in a channel but do not favor large channel reuse since they are vulnerable to interference. On the other hand, low modulation levels can sustain more interference and thus more crowded co-channel sets but transmit few bits per user. Clearly, these two aspects of impact of modulation level on channel rate are conflicting. The objective of a resource allocation algorithm is to identify co-channel user sets of maximum rate. The design of such algorithms is left for future consideration.

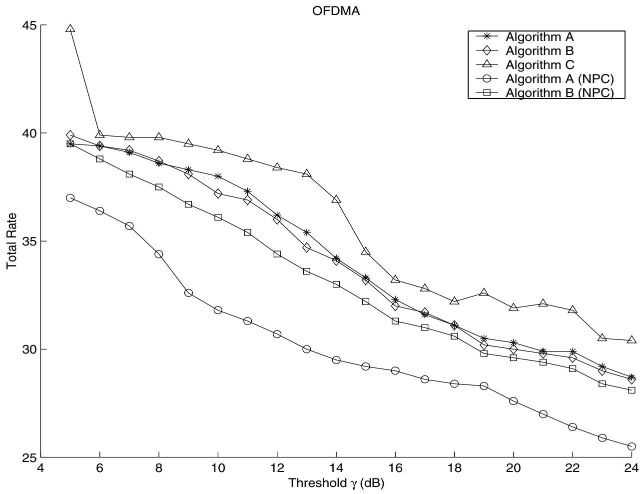

Fig. 3. Total achievable system rate vs. SIR threshold for OFDMA and M=4 antennas.

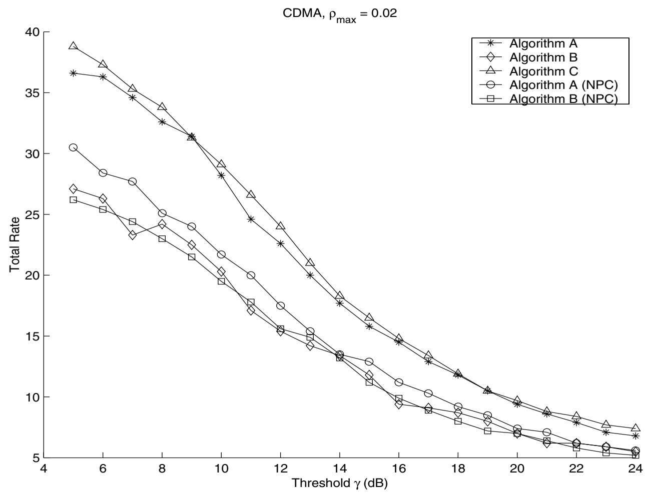

Fig. 5. Total achievable system rate vs. SIR threshold for CDMA with low code cross-correlation and M=4 antennas.

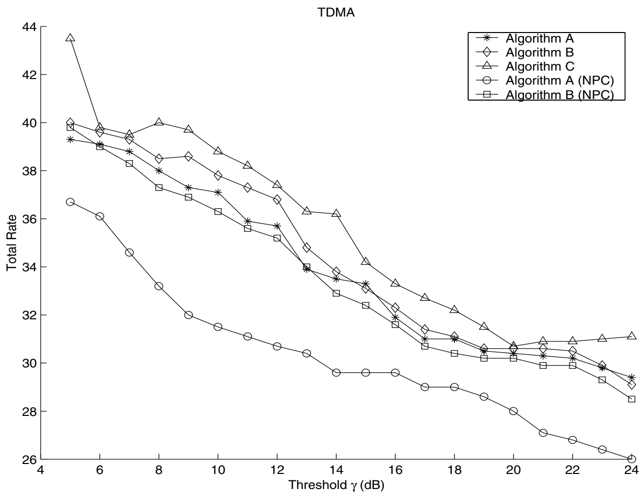

Fig. 4. Total achievable system rate vs. SIR threshold for TDMA and M=4 antennas.

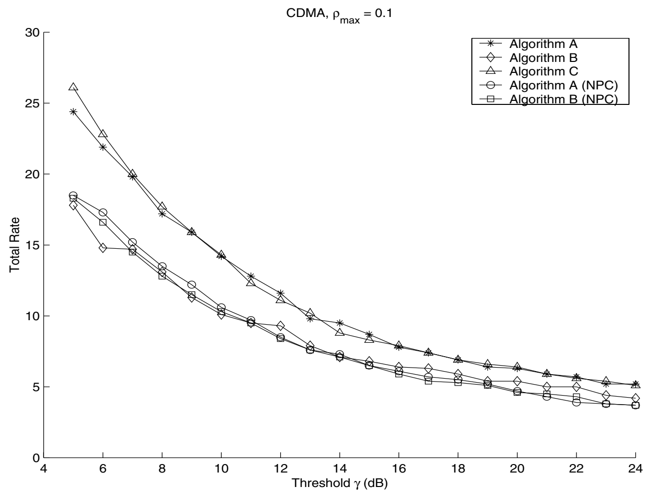

Fig. 6. Total achievable system rate vs. SIR threshold for CDMA with high code cross-correlation and M=4 antennas.

IV. Numerical Results

We consider a base station (BS) with $K = 10$ users, $N = 10$ channels, and an antenna array with $M = 4$ or 8 elements and $\Delta = \lambda/2$. Minimum rate requirements in terms of minimum number of required channels are $x_k = 3$ for each user k. Received power decays with distance d from the BS as d^{-4}. For a link between an antenna element and a user, multi-path is modeled by 2 paths with angles θ_1, θ_2, where θ_1 is uniformly distributed in $[0, 2\pi]$ and θ_2 deviates from θ_1 by a random quantity, uniform in $[0, 0.1\pi]$. The complex gain of each path is a normal random variable with standard deviation $\sigma = 6\,\mathrm{dB}$ that accounts for shadowing. Path gains of different users are uncorrelated. Our goal is to compare the performance of proposed algorithms for TDMA, OFDMA or CDMA. We also assess the benefit of power control in algorithms A, B and thus we present results with and without power control (NPC). The performance metrics are (i) total user rate in terms of user channels and (ii) total residual rate, namely additional rate so that users reach minimum rate requirements. For CDMA, we assume that code cross-correlation is uniformly distributed in $[0, \rho^2_{\max}]$ and consider low and high cross-correlation scenarios with $\rho^2_{\max} = 0.02$ and 0.1 respectively. Results are averaged over several experiments with different channel conditions. The observed fluctuations in the plots are mostly due to minimum rate requirements of users.

In Figure 3, the total rate is depicted as a function of SIR threshold γ for OFDMA. A high value of γ implies stringent BER requirement. Algorithm C achieves the best performance while algorithms A and B perform almost the same. For moderate values of γ (10-15dB), rate improvements of 20-25% are achieved with power control for algorithm A, while the corresponding benefit for algorithm B is only 5-10%. The performance of algorithm B-NPC is close to that of A with power control. This suggests that method B-NPC can be adopted in situations where low complexity is needed. For larger values of γ (e.g. $\gamma > 17$dB), three of the four alternatives of algorithms A and B lead to similar performance. Similar conclusions are derived for TDMA in Figure 4. However, the performance difference between algorithm C and

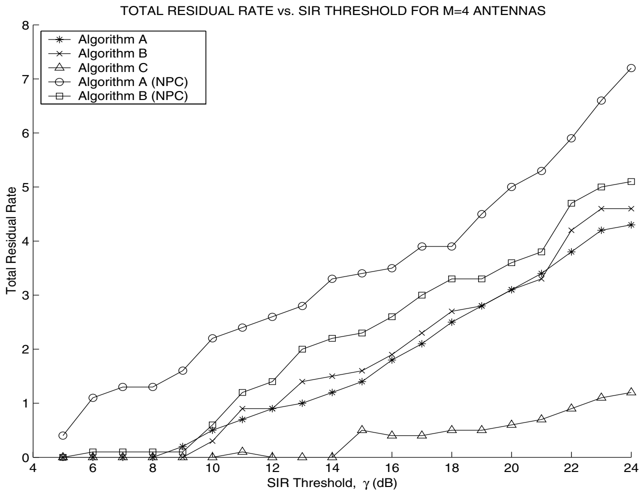

Fig. 7. Total residual rate vs. SIR threshold for OFDMA and M=4 antennas.

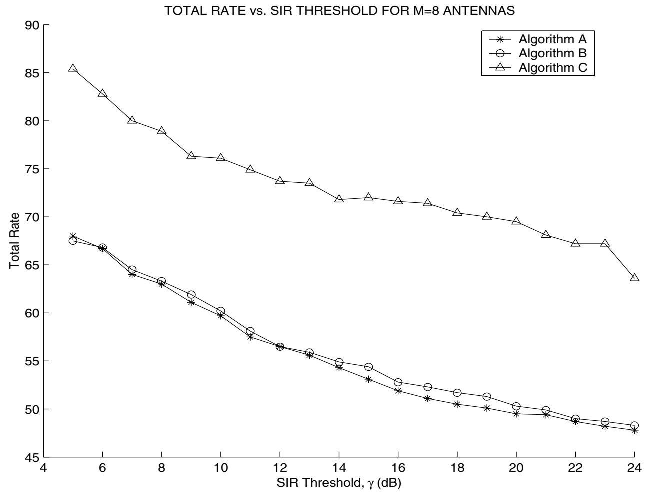

Fig. 8. Total achievable system rate vs. SIR threshold for OFDMA and M=8 antennas.

other techniques is smaller than in OFDMA.

The case of CDMA is depicted in figures 5 and 6. For low code cross-correlation, algorithm C yields the highest total rate and algorithm A leads to similar performance. Algorithm A-NPC performs better than B regardless of the use of power control in B, although it has the lowest rate in TDMA and OFDMA. As code cross-correlation increases, algorithms A and C achieve similar rates and the performance gap between these and other algorithms decreases. In Figure 7 we show performance in terms of total residual rate for OFDMA, which again verifies the superiority of algorithm C. Minimum rate requirements of users are fulfilled for $\gamma \leq 14$dB and a small portion of user requirements remains unsatisfied for larger γ. For $M = 8$ antennas (Figure 8), algorithms A and B yield only $30 - 35\%$ more rate than algorithm C with $M = 4$, while C achieves double rate for $M = 8$ than with $M = 4$. This justifies the claim that performance depends jointly on physical and access layer adaptation methods. The superiority of algorithm C over A and B is more evident in OFDMA and TDMA and is marginal in CDMA with non-orthogonal channels. This is mostly due to the joint adaptation of beams and powers which achieves SIR balancing, but also due to the machinery of allocation, which starts from an all-users-to-all-channels initial condition and proceeds by iteratively removing users with the SIR balancing criterion. While this scheme has similar effect with the one of least-interference greedy incremental user insertion for a virtual channel (encountered in CDMA), its performance is better when channels are orthogonal and separate, since then the greedy algorithm is applied in two dimensions, namely user and channel selection.

V. Discussion

The impact of SDMA on resource allocation should lead to efficient channel reuse in the presence of interference and high total rate. We adhered to a unified approach for TDMA, OFDMA and CDMA and presented algorithms for joint channel allocation, beamforming and power control that capture properties of a good solution. We observed that SIR balancing with joint beamforming and power control leads to very good performance, superior to that of greedy heuristics, especially in cases of orthogonal channels. Our conclusions about superiority of SIR balancing over greedy least interference channel assignment are in line with existing findings for systems with no SDMA [3], [31].

The resource allocation problem and associated algorithms obtain an interesting twist if multi-rate transmission is employed with adaptive modulation in TDMA and OFDMA and also by spreading gain adaptation in CDMA. Then, user spatial separability will also depend on modulation levels as discussed in section III. Another interesting issue is to devise distributed algorithms for multi-cell systems, which will be executed independently by each BS as best responses to actions of other BSs, so that BS rate is maximized. Finally, an important extension in multi-cell systems is BS assignment for load balancing and interference mitigation.

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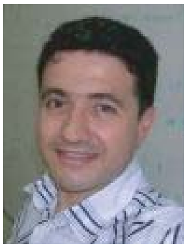
Iordanis Koutsopoulos (S '99, M '03) is a Lecturer at the Department of Computer and Telecommunications Engineering, University of Thessaly, Greece. He was born in 1974 in Athens, Greece. He obtained the Diploma in Electrical and Computer Engineering from the National Technical University of Athens, Athens, Greece in 1997 and the M.S and Ph.D degrees in Electrical and Computer Engineering from the University of Maryland, College Park (UMCP) in 1999 and 2002 respectively. From 1997 to 2002 he was a Fulbright Fellow and a Research Assistant with the Institute for Systems Research (ISR) of UMCP. He has held internship positions with Hughes Network Systems, Germantown, MD, Hughes Research Laboratories LLC, Malibu, CA, and Aperto Networks Inc., Milpitas, CA, in 1998, 1999 and 2000 respectively. For the summer period of 2005 he was a visiting scholar with University of Washington, Seattle, WA. For the period 2005-2007 he was awarded a Marie Curie International Reintegration Grant (IRG).

His research interests are in the field of networking with emphasis on wireless networks, cross-layer design, sensor networks, smart antennas and wireless network security.



Leandros Tassiulas (S'89, M'91, SM/06, F07) obtained the Diploma in Electrical Engineering from the Aristotelian University of Thessaloniki, Thessaloniki, Greece in 1987, and the M.S. and Ph.D. degrees in Electrical Engineering from the University of Maryland, College Park in 1989 and 1991 respectively.

He is Professor in the Dept. of Computer and Telecommunications Engineering, University of Thessaly, since 2002. He has held positions as Assistant Professor at Polytechnic University New York (1991-95), Assistant and Associate Professor University of Maryland College Park (1995-2001) and Professor University of Ioannina Greece (1999-2001). His research interests are in the field of computer and communication networks with emphasis on fundamental mathematical models, architectures and protocols of wireless systems, sensor networks, high-speed internet and satellite communications. Dr. Tassiulas is a Fellow of IEEE. He received a National Science Foundation (NSF) Research Initiation Award in 1992, an NSF CAREER Award in 1995, an Office of Naval Research, Young Investigator Award in 1997 and a Bodosaki Foundation award in 1999. He also received the INFOCOM 1994 best paper award and the INFOCOM 2007 achievement award.