A Distributed Resource Allocation based on Noncooperative Game and Distributed Pricing for Multi-cell OFDMA Systems

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Hojoong Kwon and Byeong Gi Lee

Abstract

In this paper, we present a distributed resource allocation algorithm for multi-cell OFDMA systems by adopting a game theoretic approach. We first define a utility function that represents both the reward of the weighted sum of the data rates and the cost of power consumption in a cell by using the concept of *power price*. Then we model the resource allocation problem as a noncooperative game. We prove that there exists a Nash equilibrium for the game and investigate the uniqueness of the equilibrium. Based on the game, we devise a *distributed resource allocation* (DRA) algorithm that finds the Nash equilibrium point by arranging each base station to adjust its own resource allocation strategy iteratively without coordination among the base stations. The DRA algorithm turns out to converge fast to the equilibrium within a small number of iterations. In addition, in order to improve the efficiency of the Nash equilibrium, we propose a *load-balancing based distributed pricing* (LBDP) algorithm that induces cooperation among the base stations implicitly by controlling the power price of each base station according to the load distribution. We demonstrate through simulation results that the combination of the DRA and LBDP algorithms, called DRA-LBDP, exhibits a performance that is close to a centralized algorithm.

Index Terms

OFDMA, distributed algorithms, noncooperative game, pricing, co-channel interference

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I. Introduction

As *orthogonal frequency division multiple access* (OFDMA) has emerged as one of the most promising multiple access techniques for high data rate transmission over wireless channels, resource allocation in OFDMA systems has become an important research topic in wireless communications. In particular, resource allocation for multi-cell OFDMA systems is a very important subject as it is directly applicable to practical use. In multi-cell environment, one of the major issues for research is how to maximize the performance by controlling the co-channel interference among the neighboring cells. The algorithms that have been developed to date mostly handle this issue in a centralized manner or with limited cooperation among the neighboring cells [1]–[3]. However, such approaches induce signaling overhead and also require efforts for cell planning.

In multi-cell environment, therefore, distributed operation is a preferred approach for the management of co-channel interference. For distributed operation, the base station in each cell should be capable of operating without the information about other cells. In this situation, game theory can render a useful and powerful tool for efficient resource management. The game theory was previously applied for solving power control problems in communication systems [4] such as the uplink power control in *code division multiple access* (CDMA) systems [5, 6] and the power control in multi-cell *orthogonal-frequency-division-multiplexing* (OFDM) systems [7]. In those applications, the power control problems were modelled as noncooperative games and their distributed solutions were sought for.

In this paper, we present a noncooperative game for the downlink resource allocation in multi-cell OFDMA systems. We adopt the concept of *power price* to define a utility function that represents both the reward of the weighted sum of the data rates and the cost of power consumption in a cell. Then we model the problem of maximizing the overall utility under the maximum power constraint as a noncooperative resource allocation game. In this game, a base station in each cell individually controls the assignment of sub-channels to the users and the allocation of power to each sub-channel in order to maximize its own utility. The game model we take in this paper has some unique features that are different from those in the previous works [5]–[7]: First, a *player*, which used to be each user, is now each base station. In addition, a *strategy* chosen by each player, which used to deal with only power control, now involves both the sub-channel assignment and the power allocation. As is the case in [7], the strategy becomes multi-dimensional for the OFDMA systems since they have multiple sub-channels. For the proposed game, we investigate the existence and uniqueness of the Nash equilibrium point. We construct a distributed resource allocation algorithm from the game.

We also present a distributed pricing algorithm that can improve the efficiency of a Nash equilibrium. The power price in the above utility function represents the cost imposed on each base station for the co-channel interference generated by itself as well as its power consumption. So, we use a pricing mechanism that can incite cooperation among base stations implicitly: We charge a high price to the base stations in lightly loaded cells such that they do not increase the transmission power unnecessarily and thereby suppress the co-channel interference to other base stations in highly loaded cells. Such "social" behaviors of the base stations can lead to load balancing effects and improved network performance. We arrange each base station to control the power price based only on the knowledge of the measured load in its own cell. By doing so, it becomes possible to apply the pricing algorithm without requiring additional signaling among the base stations. We demonstrate through computer simulations that the pricing mechanism enables the relevant distributed resource allocation algorithm to perform close to a centralized algorithm performing near-optimal.

The rest part of the paper is organized as follows: We first describe the system model under consideration in Section II. Then, in Section III, we present the noncooperative resource allocation game, investigate the existence of the Nash equilibrium for the game, propose the distributed resource allocation algorithm that converges to the Nash equilibrium, and examine the uniqueness of the Nash equilibrium. In Section VI, we present the distributed pricing algorithm that can improve the overall network performance at the Nash equilibrium. Finally, in Section V, we present simulation results.

II. System Model

We consider a downlink resource allocation in an OFDMA system with N cells serving K users. We assume all the cells in the network share the same frequency band, and the total bandwidth B is divided into M sub-channels. We denote by $\mathbf{p}^n = (p_1^n, \cdots, p_M^n)$ the *transmission power vector* of base station n, with p_m^n denoting the transmission power at sub-channel m. In addition, we denote by $\mathbf{P} = [\mathbf{p}^1 \ \mathbf{p}^2 \ \cdots \ \mathbf{p}^N]$ the *network power vector*, a concatenation of the transmission power vectors of the N base stations in the network. We assume that the total transmission power (i.e., $\sum_{m=1}^{M} p_m^n$) of each base station is constrained to be less than $P_{\max}$, and each sub-channel can be assigned to only one user. We define by $\mathbf{A}^n = [a_{mk}^n]_{M \times K}$ the *assignment matrix*, where a_{mk}^n is 1 if sub-channel m is assigned to user k and 0 otherwise.

Let G_{mk}^n denote the channel gain between user k and base station n in sub-channel m. Then the *signal-to-noise-and-interference-ratio* (SINR) of user k in cell n for the given network power vector $\mathbf{P}$ can be expressed by

$$\gamma_{mk}^n(\mathbf{P}) = \frac{G_{mk}^n p_m^n}{\sum_{l=1, l \neq n}^{N} G_{mk}^l p_m^l + \sigma^2}, \tag{1}$$

where σ^2 is the noise power, and the achievable data rate of user k is given by

$$R_{mk}^n(\mathbf{P}) = \frac{B}{M} \log_2(1 + \frac{\gamma_{mk}^n(\mathbf{P})}{\Gamma}), \tag{2}$$

where $\Gamma = -\ln(5\text{BER})/1.5$ for the *bit error rate* requirement BER [8].

The total data rate of user k, R_k, depends on the power allocation in other cells as well as the sub-channel assignment and the power allocation in the corresponding cell. Specifically, R_k

is determined by $\mathbf{P}$ and $\mathbf{A}^n$ as follows:

$$R_k(\mathbf{P}, \mathbf{A}^n) = \sum_{m=1}^{M} a_{mk}^n R_{mk}^n(\mathbf{P}). \tag{3}$$

The weighted sum of the data rates of all the users in cell n is given by $\sum_{k \in U_n} \mu_k R_k$, where U_n denotes the set of users in cell n and μ_k is the user-dependent weighting factor. This weighted sum data rate increases if the transmission power of the base station increases but it causes an increase of co-channel interference in the neighboring cells too. Consequently, the transmission power increase leads to conflicting interests among multiple base stations. In the subsequent sections, we are going to solve this conflict problem in a distributed manner by adopting a game theoretic approach.

III. Distributed Resource Allocation Algorithm

A. *Noncooperative Resource Allocation Game*

We first define the *utility function* (or *payoff function*) and construct a game in which each base station individually controls the resource allocation to maximize its own utility. The weighted sum of the data rates can be considered as the reward obtained by consuming the power resource and the cost is the total transmission power, $\sum_{m=1}^{M} p_m^n$. So the utility function is defined to be the reward less the cost. Specifically, the utility function of base station n is given by

$$u_n(\mathbf{P}, \mathbf{A}^n) = \sum_{k \in U_n} \mu_k R_k(\mathbf{P}, \mathbf{A}^n) - c \sum_{m=1}^{M} p_m^n, \tag{4}$$

where c is the price per unit power, or *power price*, having the unit bps/W.[1] The power price represents the price that each base station should pay not only for its power consumption but also for the co-channel interference generated by itself. Therefore, the utility function based on the power price renders a measure to implicitly induce cooperation among base stations, that is, to encourage each base station to maximize the weighted sum of the data rates while

[1] In the next section, we will use different power price for each base station and introduce a pricing algorithm.

minimizing the co-channel interference to other cells.[2] For the utility function, we may use an alternative form of notation $u_n(\mathbf{p}^n, \mathbf{P}^{-n}, \mathbf{A}^n)$ in which $\mathbf{p}^n$ is separated out of $\mathbf{P}$. Note that $\mathbf{P}^{-n}$ denotes the reduced network power vector that remains after separating out $\mathbf{p}^n$, which indicates the interference power.

Let $G = [\mathcal{N}, \{\mathbb{P}^n \times \mathbb{A}^n\}, \{u_n\}]$ denote a *noncooperative resource allocation game* (NRAG) where $\mathcal{N} = \{1, 2, \cdots, N\}$ is the index set of the base stations and $\mathbb{P}^n \times \mathbb{A}^n$ is the strategy space of each base station, defined by $\mathbb{P}^n = \{\mathbf{p}^n \mid 0 \le \sum_{m=1}^{M} p_m^n \le P_{\max}\}$ and $\mathbb{A}^n = \{\mathbf{A}^n \mid a_{mk}^n = 0 \text{ or } 1 \text{ for all } m,\ k,\ \text{and } \sum_{k \in U_n} a_{mk}^n = 1\}$, respectively. Each base station tries to determine the transmission power vector $\mathbf{p}^n$ and the assignment matrix $\mathbf{A}^n$ such that $\mathbf{p}^n \in \mathbb{P}^n$ and $\mathbf{A}^n \in \mathbb{A}^n$.

In the game, each base station maximizes its own utility, regardless of other base stations in a distributed fashion. Formally, the game can be expressed as

$$\text{NRAG:} \max_{\mathbf{p}^n \in \mathbb{P}^n, \mathbf{A}^n \in \mathbb{A}^n} u_n(\mathbf{p}^n, \mathbf{P}^{-n}, \mathbf{A}^n), \text{ for all } n \in \mathcal{N}. \tag{5}$$

This indicates that the base station n optimizes its own transmission power vector $\mathbf{p}^n$ and the assignment matrix $\mathbf{A}^n$ for the given interference from all the other cells.

We first consider the problem of optimizing the sub-channel assignment for a given network power vector $\mathbf{P}_o$, which is given by

$$\max_{\mathbf{A}^n \in \mathbb{A}^n} \sum_{k \in U_n} \mu_k R_k(\mathbf{P}_o, \mathbf{A}^n). \tag{6}$$

Note that the cost term is suppressed as it depends only on the transmission power vector. By Eq. (3), we get

$$\sum_{k \in U_n} \mu_k R_k(\mathbf{P}_o, \mathbf{A}^n) = \sum_{k \in U_n} \sum_{m=1}^{M} a_{mk}^n \mu_k R_{mk}^n(\mathbf{P}_o). \tag{7}$$

Since each a_m^n has only two values, 0 or 1, we can obtain the optimal solution by using a greedy approach. Specifically, the assignment matrix that assigns each sub-channel to the user

[2] Note that in previous game models for the uplink power control in CDMA systems, the transmission power is involved in the utility function in order to capture the trade-off between throughput and energy consumption [5, 6].

who yields the maximum weighted data rate is the optimal solution of the problem in (6). That is, for each m,

$$a^{*n}_{mk}(\mathbf{P}_o) = \begin{cases} 1, \text{ if } k = \arg\max_k \mu_k R^n_{mk}(\mathbf{P}_o), \\ 0, \text{ otherwise.} \end{cases} \tag{8}$$

According to Eq. (8), we can determine the optimal sub-channel assignment matrix $\mathbf{A}^{*n}(\mathbf{P})$ once the network power vector $\mathbf{P}$ is determined. Therefore, the resource allocation problem turns into a power allocation problem and, consequently, the NRAG reduces to a *noncooperative power allocation game* (NPAG) $G' = [\mathcal{N}, \{\mathbb{P}^n\}, \{u_n\}]$ as follows:

$$\begin{aligned} \text{NPAG: } & \max_{\mathbf{p}^n \in \mathbb{P}^n} u_n(\mathbf{p}^n, \mathbf{P}^{-n}, \mathbf{A}^{*n}(\mathbf{P})) \\ & = \sum_{m=1}^{M} \max_k \left[\mu_k R^n_{mk}(\mathbf{P})\right] - c p^n_m, \text{ for all } n \in \mathcal{N}. \end{aligned} \tag{9}$$

We call a network power vector $\mathbf{P}$ a *Nash equilibrium* of the NPAG if for every $n \in \mathcal{N}$, $u_n(\mathbf{p}^n, \mathbf{P}^{-n}, \mathbf{A}^{*n}(\mathbf{P})) \geq u_n(\mathbf{q}^n, \mathbf{P}^{-n}, \mathbf{A}^{*n}(\mathbf{Q}))$ for all $\mathbf{q}^n \in \mathbb{P}^n$ and $\mathbf{Q} \equiv [\mathbf{p}^1 \cdots \mathbf{p}^{n-1}\, \mathbf{q}^n\, \mathbf{p}^{n+1} \cdots \mathbf{p}^N]$. In other words, at a Nash equilibrium, once the power vectors of other base stations are given, no base station can improve its utility level by changing its own transmission power vector unilaterally. Such a Nash equilibrium exists in the NPAG, as is proved in Appendix A. More formally,

Theorem 1: A Nash equilibrium exists in the NPAG.

B. Design of Distributed Resource Allocation Algorithm

The transmission power vector of a base station that maximizes the utility function in the strategy space is called the *best response* to the transmission power vectors chosen by the other base stations. Let $\mathbf{r}_n(\mathbf{P}^{-n})$ denote the best response of base station n to a given interference power vector $\mathbf{P}^{-n}$. In formal expression, the best response is the correspondence $\mathbf{r}_n : \mathbf{P}^{-n} \rightarrow \mathbf{p}^n$. Unfortunately, the max function in the utility function is not differentiable, so it is hard to determine the best response by solving the problem in (9) analytically. To get around the difficulty, we consider an iterative algorithm as follows:

Proposition 1: For a given sub-channel assignment matrix ${\mathbf{A}_o}^n$, let ${k^*}_m^n$ denote the index of the user with $a_{omk}^n = 1$. Then, in the NPAG, the best response of base station n is given by

$$[\mathbf{r}_n(\mathbf{P}^{-n})]_m = \left[\frac{B\mu_{{k^*}_m^n}}{(c+\lambda^{*n})M\ln 2} - \frac{\Gamma(\sum_{l=1,l\neq n}^{N} G_{m{k^*}_m^n}^l p_m^l + \sigma^2)}{G_{m{k^*}_m^n}^n} \right]^+, \tag{10}$$

$$\lambda^{*n}(\sum_{m=1}^{M}[\mathbf{r}_n(\mathbf{P}^{-n})]_m - P_{max}) = 0,\ \lambda^{*n} \geq 0, \tag{11}$$

where $[\mathbf{r}_n(\mathbf{P}^{-n})]_m$ denotes the m-th power of the best response (i.e., the power of sub-channel m) and λ is the Lagrangian multiplier for the maximum power constraint.[3]

Proof: See Appendix B. ■

The proposition indicates that the best response of base station n is the water-filling power allocation with the water level determined either by the power price or by the maximum transmission power. This means that each base station avoids using the sub-channel having high interference and allocates more power to the sub-channel having low interference instead. This contrasts to the power control in CDMA systems in which the transmission power of each user increases as the interference from other users increases [5, 6]. The reason is that an OFDMA system has multiple sub-channels, while a CDMA system has a single channel.

From Proposition 1, we can obtain the optimal transmission power vector for a given sub-channel assignment matrix. This enables us to devise a *distributed resource allocation* (DRA) algorithm that determines the sub-channel assignment and power allocation iteratively as follows:

i) Initially, each base station distributes the total power equally to each sub-channel.
ii) Each user measures the SINR of all the sub-channels for the given transmission power vectors of the other base stations in the previous iteration.
iii) Each user feeds back the measured values to the base station associated with it.
iv) Each base station performs sub-channel assignment according to (8).
v) Each base station performs power allocation according to (10) and (11).

[3]Note that $[x]^+ = x$ if $x \geq 0$ and 0 otherwise.

vi) We iterate the steps ii) $\sim$ v) until the resource allocation converges to an equilibrium.

In the above DRA algorithm, we update both the interference from other cells and the sub-channel assignment and perform the water-filling power allocation process through iterations.[4] Note that at each iteration each base station can maximize its own utility function using only local information received from the users, that is, the SINR in each sub-channel. So a base station does not need the information about the transmission power levels that the other base stations use. In such a way, the DRA algorithm can operate in a distributed manner without requiring any signaling among the base stations.

It is obvious that if the DRA algorithm converges, it will converge to a Nash equilibrium point. As will become clear in Section V, the algorithm converges indeed and the number of steps required for the convergence of the algorithm is small.

C. Numbers of Nash Equilibrium Points

Now we consider whether the Nash equilibrium is unique or not. In order to gain some insight into the uniqueness issue, we consider the simple case of two cells with one user located in each cell. From Proposition 1, a network power vector $\mathbf{P}$ is a Nash equilibrium if and only if the water-filling condition (10) and (11) is satisfied for all the cells simultaneously. We assume that the noise power σ^2 is much smaller than the co-channel interference.[5] For the case of the two cells, the necessary and sufficient condition can be written as follows: For any sub-channel m,

$$p_m^1 = \left[w_1 - \frac{\Gamma G_{m1}^2 p_m^2}{G_{m1}^1} \right]^+, \tag{12}$$

$$p_m^2 = \left[w_2 - \frac{\Gamma G_{m2}^1 p_m^1}{G_{m2}^2} \right]^+, \tag{13}$$

[4]The proposed iterative water-filling power allocation among multiple base stations is similar to the iterative water-filling approach that was proposed for the power control in *digital subscriber line* (DSL) systems [9].

[5]This assumption is reasonable in the interference-limited systems.

where $w_i = \frac{B\mu_i}{(c+\lambda^{*i})M\ln 2}$, $i = 1,2$ and the users are labled by the same index as the corresponding cell. Depending on the set of the channel gains G^i_{mj} for $i,j = 1,2$, the NPAG may have one or more equilibrium points as follows: As one can easily prove, there exists a unique Nash equilibrium if there is at least one user for which $\frac{G^i_{mi}}{G^j_{mi}} > \frac{w_j}{\Gamma w_i}$ for $i,j = 1,2$, $i \neq j$. At the Nash equilibrium, each base station may allocate a positive transmission power to that particular user. On the other hand, it can also be easily proved that if $\frac{G^1_{m1}}{G^2_{m1}} < \frac{w_2}{\Gamma w_1}$ and $\frac{G^2_{m2}}{G^1_{m2}} < \frac{w_1}{\Gamma w_2}$, there exist three Nash equilibria.[6] Fig. 1 illustrates the regions of unique and three Nash equilibria for the case $w_1 = 2$, $w_2 = 1$, and $\Gamma = 1$, with the notation (x_1, x_2) indicating that the transmission power of base station i ($i = 1,2$) is lager than zero if $x_i = 1$ and zero otherwise.

In the multi-user case with multiple users existing in a single cell, the second case with three equilibria occurs with a low probability. In this multi-user case, a sub-channel is supposed to be assigned to the user who yields the highest weighted data rate. If all the users have the same weighting factor μ_k, i.e, $\mu_k = \mu$ for all k, for a given network power vector, then we get

$$\arg\max_k \mu_k \log_2\left(1 + \frac{G^i_{mk} p^n_m}{\Gamma G^j_{mk} p^j_m}\right) = \arg\max_k \frac{G^i_{mk}}{G^j_{mk}}. \tag{14}$$

So the optimal solution of the problem in (6) leads to assigning each sub-channel to the user having the maximum ratio $\frac{G^i_{mk}}{G^j_{mk}}$, denoted by ${k^*}^i_m$. If we assume that $\frac{G^i_{mk}}{G^j_{mk}}$ is an independent random variable for all k, then we get

$$Pr\left[\frac{G^i_{m{k^*}^i_m}}{G^j_{m{k^*}^i_m}} < w\right] = \prod_{k\in U_i} F_k(w), \tag{15}$$

where $F_k(x)$ is the CDF of $\frac{G^i_{mk}}{G^j_{mk}}$, i.e., the probability of $\frac{G^i_{mk}}{G^j_{mk}} < w$. Therefore, the probability of

[6]Note that we except the case of $\frac{G^i_{mi}}{G^j_{mi}} = \frac{w_j}{\Gamma w_i}$ for $i,j = 1,2$, $i \neq j$. If the conditional cdf of G^i_{mi}, $F_{G^i_{mi}}(x|G^j_{mi} = g)$, is a continuous function, we have $\Pr\left(\frac{G^i_{mi}}{G^j_{mi}} = w\right) = 0$ for a constant w. As an exceptional case, let us consider the case that G^i_{mi} and G^j_{mi} are fully correlated, i.e., $G^i_{mi} = rG^j_{mi}$ for a constant r. In this case, $F_{G^i_{mi}}(x|G^j_{mi} = g) = \delta(x - rg)$, so we get $\Pr\left(\frac{G^i_{mi}}{G^j_{mi}} = r\right) = 1$. However, such an exceptional case does not exist because the channel gain is a random variable, determined by the path loss, the multi-path fading and the shadowing, and the multi-path fadings from different base station are mutually independent in general.

$\frac{G^i_{mk^*i_m}}{G^j_{mk^*i_m}} < w$ for $i \neq j$ and a constant $w > 0$ becomes smaller as the number of users increases. Even for the case where each user has a different weighting factor, it is likely that there is a user who has a high SINR as well as a high weighting factor at any time when the number of users is large. Therefore, we may conclude that the Nash equilibrium is likely to be unique with a high probability in multi-user systems. This argument may be applied to the case with more than two cells in a similar way.

IV. Load-Balancing based Distributed Pricing Algorithm

In the above, we defined the utility function by adopting the concept of power price and designed the DRA algorithm based on it. However, the Nash equilibrium achieved by the distributed algorithm could yet be less efficient than the resource allocation obtained through centralized optimization. We can expect that the DRA algorithm would become more efficient if the power price of each base station could be controlled adequately. So, in this section, we investigate how to achieve this by introducing a distributed pricing algorithm.

In general, users are distributed non-uniformly over the network and each user requires different data rate for different service applications. Even when the same data rate is to be provided to different users, the required transmission powers are different depending on their channel conditions and interference levels. As such, the load is not uniform over the network and the amount of power resource needed to support the load vary among cells. In such non-uniform load distribution environment, it is desirable to employ load balancing approach [3] to improve the overall network performance. So we take an approach that adapts the price of each base station according to load distribution, thereby yielding a *load-balancing based distributed pricing* (LBDP) algorithm.

The basic idea of the LBDP algorithm is as follows: In the DRA algorithm, the price is designed to control the degree of willingness of a base station to reduce the co-channel interference to the neighboring cells while sacrificing the data rate of the users within its own cell. So it is desirable to impose a higher price on the base station having lighter load. Then, in a lightly-loaded cell, the corresponding base station will try to use a low price, setting the transmission power to a low level which is just good enough to support the given light load. This mechanism prevents base stations from increasing transmission power unnecessarily, consequently decreasing the co-channel interference to other cells as well. On the other hand, in a heavily-loaded cell, the corresponding base station will try to use a high price, increasing the transmission power to a high level adequate to support the given heavy load. Apparently it will cause an increase of co-channel interference, but it can be mitigated if the neighboring cells having light load use some extra power. It is possible to implement this pricing scheme in a distributed way by arranging each base station to control the price only based on its own load information. Similar to the DRA algorithm, we can achieve desirable price values through iterative adjustment of price by each base station.[7]

Specifically, let $c^n(i)$ denote the price of base station n in the i-th iteration, L^n the load of base station n, and $f_u(x)$ and $f_l(x)$ non-decreasing and non-increasing functions of x, respectively. Then the LBDP algorithm operates in such a way that each base station updates the price iteratively in the form

$$c^n(i+1) = \begin{cases} c^n(i)(1 - f_u(L^n)) \ , & \text{if } L^n \text{ is high,} \\ c^n(t)(1 + f_l(L^n)) \ , & \text{if } L^n \text{ is low,} \\ c^n(t) \ , & \text{otherwise.} \end{cases} \tag{16}$$

In the following, we discuss how to define the load and the price update functions depending on the traffic model, for the two special models: dynamic arrival model and infinite backlog model.

A. Distributed Pricing under Dynamic Arrival Traffic

We first consider a dynamic arrival model in which data packets for the users arrive randomly at each time instant. We consider a frame and super-frame structure in the time domain, with the

[7]When the load of all the neighboring cells is high all the cells will continue increasing the transmission power up to the maximum level. In this case, it is necessary to force some cells to decrease the transmission power such that the other cells can support the given high load. However this requires a cental controller that enforces cooperation among the cells. In this paper, we assume that such a case hardly happens, so we choose to adopt the distributed operation.

time divided into periodic frames and with each super-frame composed of N_f frames. Assuming that the channel condition of each user is fixed during each frame and varies between frames, we perform the DRA algorithm on frame basis. On the other hand, taking into account that the traffic characteristics vary slowly, we perform the LBDP algorithm on super-frame basis.

Each base station has multiple queues, with each queue associated with a different user. In the beginning of each frame, data packets arrive according to a random process and are stored in one of the queues according to their destinations. We denote by $A_k(t)$ the number of bits that arrive for user k at the beginning of frame t and denote by $R_k(t)$ the data rate allocated to user k during frame t. Then the number of bits transmitted from the queue is given by $T_f R_k(t)$ for the frame duration T_f. We denote by $W_k(t)$ the size of the queue of user k at the beginning of the frame t. Then the evolution of the queue size takes the expression

$$W_k(t+1) = \min\{\max\{W_k(t) - T_f R_k(t), 0\} + A_k(t+1), \overline{W}_k\}, \tag{17}$$

where $\overline{W}_k$ is the maximum size of queue k. Incoming packets are to be dropped when the queue is fully occupied but the allocated data rate will not be fully utilized if the number of packets to transmit is not large enough.

In order to maximize the throughput, it is necessary to minimize the amount of data dropped from the queue. It was well known that a resource allocation algorithm that maximizes the queue-size-weighted sum of rates is throughput optimal, that is, the queue is kept stable if the arrival rate lies within the capacity region [10]–[13]. The optimality of such resource allocation algorithm has been proved in various contexts. So we adopt the algorithm in our system as follows: In the utility function, we set the weighting factor of user k in cell n to its queue size normalized by the average value, i.e.,

$$\mu_k(t) = \frac{W_k(t)}{\sum_{k \in U_n} W_k(t)/|U_n|}, \tag{18}$$

where $|U_n|$ denotes the number of elements in set U_n. Then each base station determines the sub-channel assignment and the power allocation to maximize the queue-size-weighted sum of rate by using the DRA algorithm.

When employing the above policy, the load of a cell can be represented by the queue sizes of the users associated with the corresponding base station. So we define the load of a cell as the average queue size of all the users within the cell. We measure the load by comparing the average queue size W_{avg} ($\equiv \frac{1}{|U_n|}\sum_{k\in U_n} W_k$) with the lower and upper thresholds, Q_l and Q_u: The load is light if $0 \le W_{avg} < Q_l$ and heavy if $W_{avg} > Q_u$. Based on the measure load, we define the price update function as follows:

$$c^n(i+1) = \begin{cases} c^n(i)\left(1-\delta_u \frac{W^n_{avg}-Q_u}{Q_u}\right) & \text{, if } W_{avg} > Q_u, \\ c^n(i)\left(1+\delta_l \frac{Q_l-W^n_{avg}}{Q_l}\right) & \text{, if } W_{avg} < Q_l, \\ c^n(i)\ , & \text{otherwise,} \end{cases} \tag{19}$$

for two predetermined step sizes δ_u and δ_l. Each base station updates the price in the end of each super-frame. Note that each base station uses only the load information of its own cell for the price adaptation. Therefore, the resulting DRA algorithm, combined with the LBDP algorithm, still operates in distributed manner.

B. *Distributed Pricing under Infinite Backlog Traffic*

There are a large number of resource allocation algorithms available to date for the case with infinite-backlog of packets in queue. Among them, we adopt the most popular *proportional fairness* (PF) algorithm [14]. The PF algorithm achieves proportional fair resource allocation by selecting the user who have the highest ratio of the current data rate to the average throughput. So we can implement the proportional fair criterion by setting the weighting factors of the users to the reciprocals of the average throughputs. Specifically, the average throughput of user k, $T_k(t)$, is updated such that

$$T_k(t+1) = (1-\frac{1}{t_c})T_k(t) + \frac{1}{t_c}R_k(t), \tag{20}$$

where t_c is a constant for exponentially weighted low-pass filter. We set the weighting factor of user k in cell n to

$$\mu_k(t) = \frac{\sum_{k\in U_n} T_k(t)/|U_n|}{T_k(t)}. \tag{21}$$

In the infinite backlog model, the load is not well defined due to the absence of any quality of service requirement. So we adopt the concept of token as follows: We generate a token virtually to each user at each frame. The arrival rate of the token is the same for all users. The generated tokens are stored in a token queue and are removed by the amount of the data rate allocated to the corresponding user. Then the dynamics of the token queue is given by an expression similar to Eq. (17). So we can determine the price of each base station using the same equation as in (19) with the queue size replaced with the token queue size.

V. Simulation Results

We conducted computer simulations over the network composing of 19 cells. We set the values of the involved parameters as follows: the number of sub-channels, M, 21; bandwidth of each sub-channel, 0.1 MHz; cell radius, 1km; the number of users per cell, 15; the maximum transmission power, P_{max}, 43dBm; and the path loss exponent, 3.76.

We considered a hexagonal 3-sectorized cell structure. For performance comparisons, we considered a simple algorithm that fixes the *frequency reuse factor* (FRF). The fixed FRF algorithm with FRF = 1 is designed to distribute the total power equally among all the sub-channels and that with FRF 3 is designed to distribute the total power equally among one third of the sub-channels. In the FRF = 3 case, we arranged the three sectors in a single cell to use different groups of sub-channels. In addition, we considered a centralized algorithm that can achieve near-optimal performance. Since the problem of finding the optimal solution is NP-complete, we designed a *centralized resource allocation* (CRA) algorithm that has a tractable complexity as it adopts the *pseudo-cell structure*, which is defined in [3] and repeated in Fig. 2.[8] Fig. 3 shows the pseudo-code of the CRA algorithm. Note that we use the sector index s instead of the cell index n.

[8]The CRA algorithm determines both the FRF values of the sub-channels and the sub-channel assignment for each pseudo-cell by utilizing the channel information of all users in the corresponding pseudo-cell. The CRA algorithm takes a greedy approach to maximize the weighted sum of the data rates. Since pseudo-cells are coupled with each other due to the co-channel interference, we iteratively determined the resource allocation of each pseudo-cell after updating the power allocation of other pseudo-cells.

A. Distributed Resource Allocation Algorithm

We first investigated the convergence of the DRA algorithm to the Nash equilibrium point. We considered the two cases: (i) when $\mu_k = 1$ for all k, and (ii) when μ_k's are uniformly distributed from 1 to 4. We set the power price to 0.3 and 1.5 Mbps/W for the two cases, respectively. Initially, we made the maximum transmission power equally distributed to each sub-channel. Fig 4. depicts the resulting utility per cell for the two cases with respect to the number of iterations. The utility in the figure is normalized by the value at the equilibrium point. We observe that for the both cases, the DRA algorithm converges to the equilibrium within about 3 iterations. This indicates that the required number of iterations for the convergence is small. We also observe that the DRA algorithm can improve the utility significantly when compared with the equal-power allocation algorithm.

Then we examined the performance of the DRA algorithm in comparison with the algorithms using fixed-FRF. We considered simply one pseudo-cell where we located one user in each sector on the line connecting the corresponding base station to the center of the three base stations. Fig. 5 depicts the resulting total data rate with respect to the distance between the base station and the user. As expected, the FRF = 1 case outperforms the FRF = 3 case when the users are located near the base station and the FRF = 3 case outperforms the FRF = 1 case when they approach the cell boundary. We observe that the DRA algorithm can adapt the power allocation efficiently to the user distribution. In addition, the DRA algorithm outperforms the both fixed power allocation cases by adopting water-filling power allocation across the sub-channels.

B. Load-Balancing based Distributed Pricing Algorithm

We examined the performance of the combination of the DRA algorithm and the LBDP algorithm (namely, DRA-LBDP). We set the sizes of the frame and the super-frame to 5 msec and 100 frames, respectively. For the dynamic arrival model, we considered CBR traffic that generates a packet with the size of 125 bytes at each frame. We set the queue size of each user to 50 packets, and the two thresholds, Q_l and Q_h, to 5 and 15 packets, respectively. For the infinite backlog model, we set the generation rate of token to be the same as the packet arrival rate of the dynamic arrival model. We set δ_u and δ_l to 1.6 and 0.8, respectively.[9]

We first investigated the convergence of the LBDP algorithm under the dynamic arrival traffic model. We considered a non-uniform load distribution scenario where for each pseudo-cell, 22, 10 and 10 users were located in sector 1, 2 and 3, respectively. Figs. 6 and 7 depict the average drop probability and the average total transmission power over each super-frame with respect the number of price update. We observe that without the LBDP algorithm, some packets are dropped from the queue in the the heavily-loaded sector (i.e., sector 1) while no drop occurs in the lightly-loaded sectors (i.e., sectors 2 and 3). However, as the power update proceeds in each sector, sector 1 increases the total transmission power up to the maximum power and sectors 2 and 3 decrease the total transmission power. Accordingly, the drop probabilities in three sectors get balanced at zero. This shows that the power price in the LBDP algorithm converges to a point where the resources utilized by the three sectors are balanced according to their respective loads.

We then compared the DRA-LBDP algorithm with the CRA and the fixed-FRF algorithms. We randomly located 700 users over the network. Fig. 8 depicts the resulting drop probability of the various algorithms with respect to the arrival rate. We observe that the DRA-LBDP and the CRA algorithms outperforms the fixed-FRF algorithm, with the DRA-LBDP algorithm performing close to the CRA algorithm. The DRA-LBDP algorithm performs even better when the traffic load is low. This happens because the DRA-LBDP algorithm controls the transmission power in continuous level while the CRA algorithm controls only the frequency reuse factor, that is, controls the transmission power in discrete level.

We also evaluated the performance of the DRA-LBDP algorithm in different load conditions.

[9]We set the initial power cost of each cell as follows: Assuming that the total transmission power is inversely proportional to the power price, during the initial three super-frames before performing the LBDP algorithm, we updated the power price by $p^n(i+1) = p^n(i)P_{avg}(i)/P_0$, where $P_{avg}(i)$ is the average total transmission power over super-frame i, such that the total transmission power becomes as large as $P_0 = 30$dBm.

We divided all the sectors into two groups — heavily-loaded sector having 22 users and lightly-loaded sector having 10 users. Fig. 9 depicts the drop probability of the various algorithms with respect to the percentage of the heavily-loaded sectors. We observe that the difference between the DRA-LBDP algorithm and the CRA algorithm is small regardless of the load distribution. This indicates that the LBDP algorithm can control the power price effectively according to load environment.

Lastly, we examined the performance of the DRA-LBDP algorithm under infinite backlog model. We randomly located 700 users over the network. In order to compare the various algorithms in terms of fairness among the users, we examine the throughput of the lowest 5% of users, who usually are located at the cell edge and are affected severely by the co-channel interference. As shown in Table. I, the DRA-LBDP improves the cell-edge performance 2.15 times over the FRF 1 case. At the same time, the DRA-LBDP algorithm even increases the average throughput by 6% over the FRF =1 case.[10] We also observe that the DRA-LBDP algorithm exhibits a performance comparable to the CRA algorithm. It shows slightly higher fairness and slightly less cell throughput when compared with the CRA algorithm.

VI. Conclusions

In this paper, we have presented a new distributed resource allocation algorithm for multi-cell OFDMA systems relying on a game theoretic approach. We have presented a noncooperative game in which each base station tries to maximize the system performance while minimizing the co-channel interference. Based on the game, we have devised a practical algorithm called DRA that requires no coordination among the base stations. As the cost of this distributed operation, the DRA algorithm requires some iterative calculations of the sub-channel assignment matrix and the transmission power vector. However, the required number of iterations turned out to be very small. Due to the iterative processing, it may not be applicable in the environment where the channel condition varies very fast. In addition, we have proved that there exists a Nash

[10] The average throughput gain results from the water-filling power allocation.

equilibrium point for the noncooperative game and it is highly probable that the equilibrium is unique. This confirms that the DRA algorithm will exhibit good performance without falling into undesirable resource allocation.

For an improved performance of the DRA algorithm, we have introduced the concept of power price that helps to avoid the co-channel interference among the cells and the distributed pricing algorithm called LBDP that can improve the efficiency of the Nash equilibrium achieved by the DRA algorithm. We considered two traffic models — dynamic arrival model and infinite backlog model — and designed the LBDP algorithm suitable for each model by applying the same principle. It is possible to adapt the LBDP algorithm to other traffic models in a similar way. Simulation results revealed that the combination of the DRA and DLBP algorithms, or DRA-LBDP algorithm, performs close to the centralized resource allocation algorithm performing near-optimal in various load conditions.

To the best of our knowledge, the DRA-LBDP algorithm is the first fully distributed algorithm for the resource allocation in multi-cell OFDMA systems. The DRA-DLBP algorithm requires no exchange of information among the cells. Each base station utilizes only the SINR information measured by and fed back from the constituent users, and also determines the power price based on its own load autonomously.

The DRA-LBDP algorithm may be particularly useful in the environment with irregular cell structure. When the shapes and the locations of the cells are irregular, cell planning may not be easy, so a distributed algorithm that does not require inter-cell resource allocation controllers is very much demanding. We also expect that the algorithm may be very useful in the fourth generation (4G) communication systems. The 4G systems are expected to be established based on all-IP network architecture which can make the systems more scalable and cost-effective. In this case, the 4G systems need to employ a distributed management architecture instead of the hierarchical management architecture of the past and the current systems. Therefore, the distributed resource allocation algorithm may possibly render one of the most essential techniques for the 4G systems.

APPENDIX

A. Proof of Theorem 1

In [15], it is established that a Nash equilibrium exists in game $G = [\mathcal{N}, \{\mathbb{P}^n\}, \{u_n\}]$ if, for all $n \in \mathcal{N}$,

i) $\mathbb{P}^n$ is a nonempty, convex, and compact subset of some Euclidean space $\Re^{NM}$, and

ii) $u_n(\mathbf{P})$ is continuous in $\mathbf{P}$ and quasi-concave in $\mathbf{p}^n$.

The strategy space $\mathbb{P}^n$ of each base station is defined by a set of power vectors in which all the power values are between zero and the maximum power. Thus it is clear that the first condition is satisfied.

The utility function $u_n(\mathbf{P})$ is obviously a continuous function of $\mathbf{P}$. Let $\bar{R}^n_m(p^n_m, \mathbf{P}^{-n}) \equiv \max_k \mu_k R^n_{mk}(\mathbf{P})$. Then each $\bar{R}^n_m$ is a monotonically increasing function of p^n_m. So, for any $\alpha > 0$, the sub-level set $S^n_m \equiv \{x|\bar{R}^n_m(x) \geq \alpha\}$ is given by $\{x|x \geq x_\alpha\}$ for $x_\alpha \equiv (\bar{R}^n_m)^{-1}(\alpha)$. Since S^n_m is a convex set, $\bar{R}^n_m$ is a quasi-concave function of p^n_m. Then the utility function can be written as a sum of the functions that are quasi-concave in the corresponding p^n_m. Therefore we can prove that the utility function is also quasi-concave in $\mathbf{p}^n$.

B. Proof of Proposition 1

As mentioned in the proof of Theorem 1, the strategy set is convex. For the given sub-channel assignment matrix, the utility is a concave function of $\mathbf{p}^n$ and hence the problem is a convex optimization problem. Therefore we can apply the Karush-Kuhn-Tucker (KKT) condition [16] to get the solution in (10) and (11). We omit the details.

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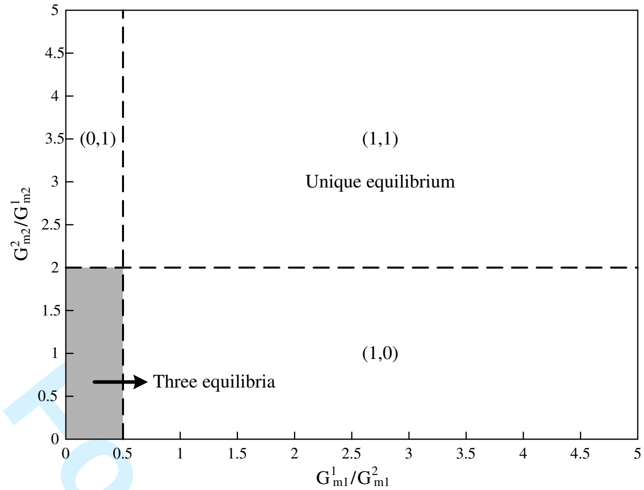

Fig. 1. Illustration of the regions of unique and three equilibria for the case $w_1 = 2$, $w_2 = 1$, and $\Gamma = 1$ (Note that (x_1, x_2) indicates that the transmission power of base station i $(i = 1, 2)$ is lager than zero if $x_i = 1$ and zero otherwise.).

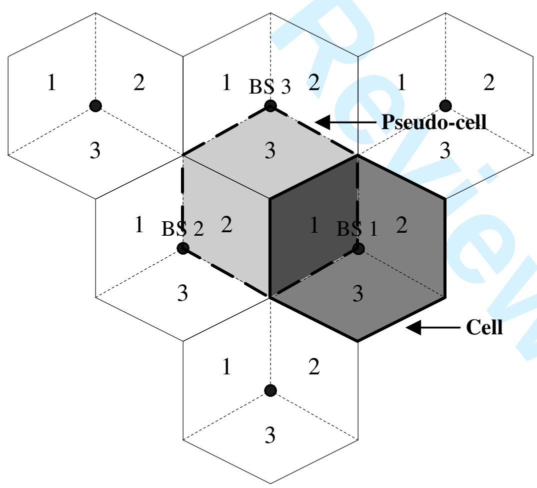

Fig. 2. Definition of “pseudo-cell” in hexagonal 3-sectorized cellular system.

1. Each cell assumes that the transmission powers of BSs in all the other pseudo-cells are zero;
2. **WHILE** (Resource Allocation is not converged)
3. **FOR** all cells
3. **FOR** all sub-channel $m = 1, 2, \cdots, M$
4. **FOR** all sector $s = 1, 2, 3$
5. // FRF 1
6. Set $\mathbf{P}$ in a way that all sectors in the pseudo-cell are active;
7. $\psi_m^s(1) = \max_{\mathbf{A}^s} \sum_{k \in U_s} \mu_k R_{mk}^s(\mathbf{P}, \mathbf{A}^s)$;
8. // FRF 3
9. Set $\mathbf{P}$ in a way that only sector s in the pseudo-cell is active;
10. $\psi_m^s(3) = \max_{\mathbf{A}^s} \sum_{k \in U_s} \mu_k R_{mk}^s(\mathbf{P}, \mathbf{A}^s)$;
11. **IF** $(\sum_s \psi_m^s(1) > \max_s \psi_m^s(3))$ **THEN**
12. Set FRF = 1;
13. **ELSE**
14. Set FRF = 3;
15. Each cell updates the transmission powers of BSs in all the other pseudo-cells;

Fig. 3. Pseudo-code of the CRA algorithm.

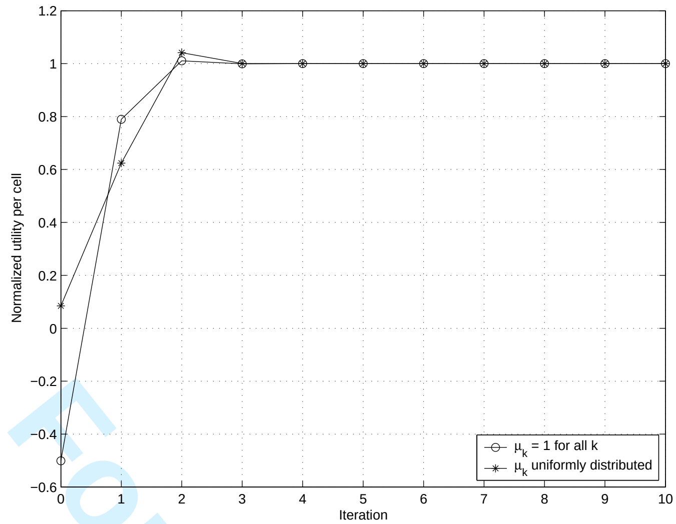

Fig. 4. Normalized utility per cell with respect to the number of iterations. (Note that the DRA algorithm converges to Nash equilibrium in 3 iterations for the both cases of μ_k.)

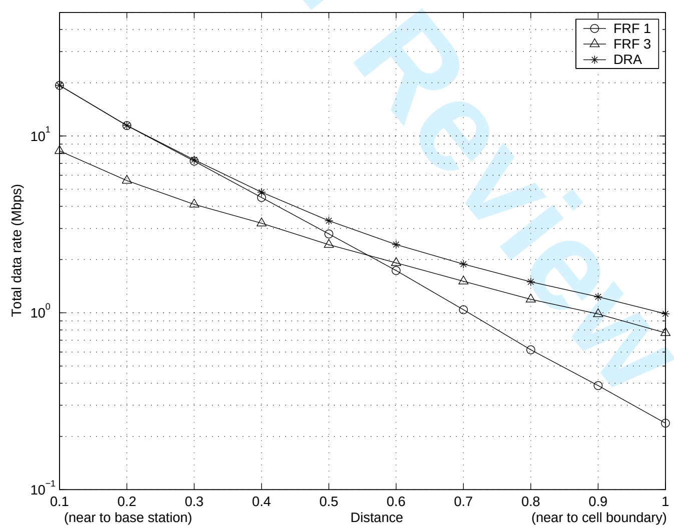

Fig. 5. Total data rate with respect to the distance between the base station and the user.

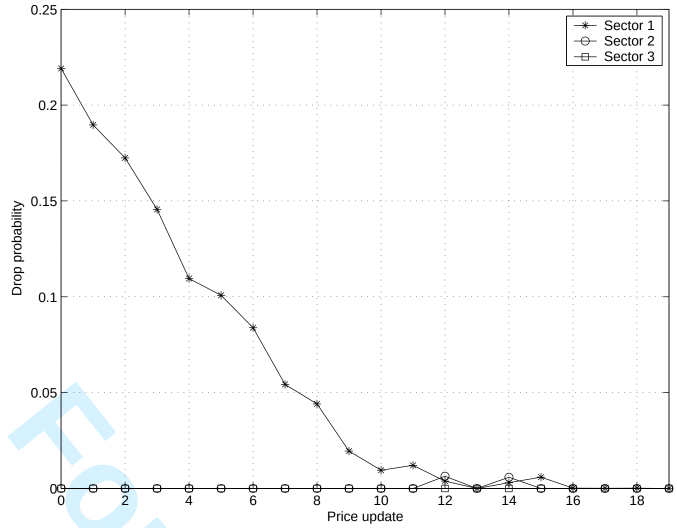

Fig. 6. Average drop probability of each sector with respect to the number of price update. (Note that the LBDP algorithm adjusts drop probabilities to zero.)

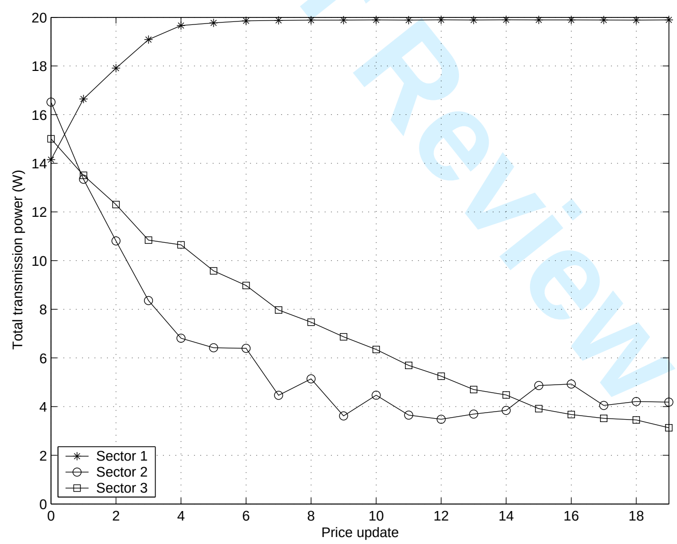

Fig. 7. Average total transmission power of each sector with respect to the number of price update.

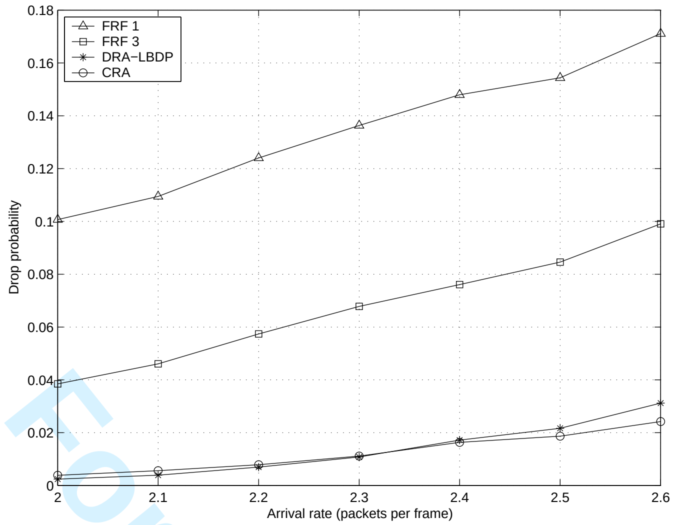

Fig. 8. Drop probability of the various algorithms with respect to the arrival rate.

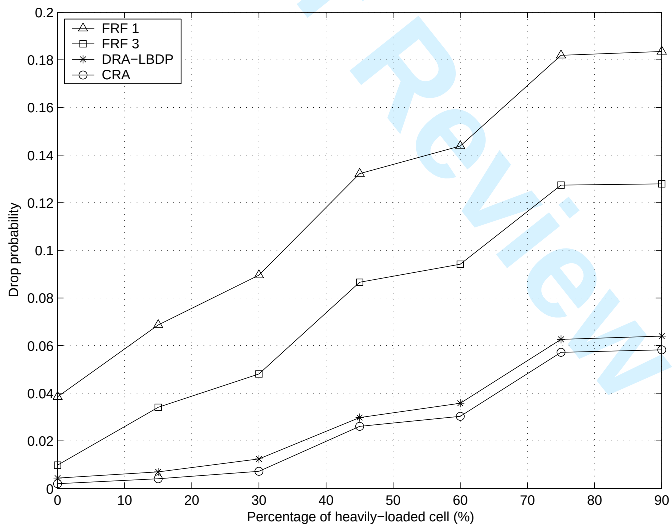

Fig. 9. Drop probability of the various algorithms with respect to the percentage of heavily-loaded cells.

TABLE I

NORMALIZED AVERAGE CELL THROUGHPUT AND THE LOWEST 5 % USER THROUGHPUT

	CRA	DRA-LBDP	FRF 1	FRF 3
$\eta/\eta_{\text{FRF 1}}$	1.08	1.06	1.00	0.64
$T(5\%)/T_{\text{FRF 1}}(5\%)$	1.92	2.15	1.00	1.33