Distributed Power and Admission Control for Time Varying Wireless Networks

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***Abstract*— This paper presents new distributed power and admission control algorithms for ad-hoc wireless networks in random channel environments. Previous work in this area has focused on distributed control for ad-hoc networks with fixed channels. We show that the algorithms resulting from such formulations do not accurately capture the dynamics of a time-varying channel. The performance of the network in terms of power consumption and generated interference, can be severely degraded when power and admission control algorithms that are designed for deterministic channels are applied to random channels. In particular, some well-known optimality results for deterministic channels no longer hold. In order to address these problems we propose a new criterion for power optimality in ad-hoc wireless networks. We then show that the optimal power allocation for this new criterion can be found through an appropriate stochastic approximation algorithm. We also present a modified version of this algorithm for tracking non-stationary equilibria, which allows us to perform admission control. Ultimately, the iterations of the stochastic approximation algorithms can be decoupled to form fully distributed on-line power and admission control algorithms for ad-hoc wireless networks with time-varying channels.**

I. Introduction

Adaptive control of transmission power in wireless networks allows devices to setup and maintain wireless links with minimum power while satisfying constraints on quality of service (QoS). The benefits of power minimization are not just increased battery life. Effective interference mitigation can also increase overall network capacity by allowing higher frequency reuse.

Typically, power control and interference mitigation techniques are designed for wireless networks with cellular architectures. The benefit of such an architecture is that one can assume a centralized controller has knowledge of the channel states for all users in the system. In this paper we consider a fundamentally different architecture where there is no centralized controller to distribute power control commands or channel information. Hence, the model we consider here is that of an ad-hoc wireless network with purely distributed control (we will clarify the details of this definition in the next section).

Some of the earliest work on decentralized power control for wireless networks was published by Foschini and Miljanic [3] in 1993. Their proposed control algorithm (now well known in the wireless community as simply the Foschini-Miljanic algorithm) provides for distributed on-line power control of ad-hoc networks with user-specific SIR requirements. Furthermore, their algorithm yields the minimum transmitter powers that satisfy the SIR requirements. This seminal work spawned a number of further publications [1], [9], [10] by various authors that extended the original algorithm to account for additional issues. Of particular interest for this paper is the work in [1] that provides a detailed analysis of active link protection and admission control for the underlying Foschini-Miljanic algorithm.

The original algorithm proposed by Foschini and Miljanic (and the extensions cited above) requires that the channel gains between nodes in the ad-hoc network are constants. In some settings this assumption is reasonable as the time scale for adaptation is much faster than the time scale of the channel variability (e.g. stationary users, slowly-varying channels, and so forth). This focus of the work presented in this paper is power adaptation and admission control in an environment where the adaptation and channel variability time scales are similar (e.g. when the network users are mobile).

We consider the same distributed power and admission control problems in [3] and [1], but we permit the links between network nodes to be time-varying stochastic processes. Within this setting we evaluate the performance of the original Foschini-Miljanic algorithm and show that it does not continue to satisfy the minimum power optimality conditions (optimality in this case is in terms of expected transmitter powers). Moreover, we also show that the SIR targets of the Foschini-Miljanic algorithm change dramatically in a random channel environment. In order to address these shortcomings we propose a new criteria for power optimality in wireless ad-hoc networks. We then show that a power allocation that satisfies our new optimality criteria can be solved by stochastic approximation. In order to address the admission control problem we propose a modified version of the stochastic approximation algorithm for tracking non-stationary network equilibria (i.e. users entering and leaving the system). In both algorithms, the resulting stochastic approximation iterations yield fully distributed on-line power control algorithms that converge to the optimal power allocation for an ad-hoc network in a random channel environment.

The rest of this paper is organized as follows. In the next section we present a brief review of the formulation and results of [3] and [1]. In Section 3 we evaluate the performance of the Foschini-Miljanic algorithm in a random channel environment. In Section 4 we propose a new criteria for power optimality in wireless ad-hoc networks. Section 5 contains our proposed distributed stochastic approximation algorithm for power control. In Section 6 we address the problem of distributed admission control in a random channel environment. Numerical results are presented in Section 7 and we then conclude with a discussion of future research.

II. A Review of The Foschini-Miljanic Algorithm

In [3] the authors formulate the wireless network as a collection of radio links with each link corresponding to a transmitter and an intended receiver. Each transmitter is assumed to have a *fixed* channel gain to its intended receiver as well as fixed gains to all other receivers in the network. The quality of each link is determined by the signal to interference ratio (SIR) at the intended receiver. In a network with N interfering links we denote the SIR for the ith user as

$$R_i = \frac{G_{ii}P_i}{\eta_i + \sum_{i\neq j} G_{ij}P_j}, \quad (1)$$

where $G_{ij} > 0$ is the power gain from the transmitter of the jth link to the receiver of the ith link, P_i is the power of the ith transmitter, and η_i is the thermal noise power at the ith receiver.

Each link is assumed to have a minimum SIR requirement $\gamma_i > 0$ that represents the ith user's quality of service (QoS) requirements. This constraint can be represented in matrix form as

$$(\mathbf{I} - \mathbf{F})\mathbf{P} \geq \mathbf{u} \text{ with } \mathbf{P} > 0, \quad (2)$$

where $\mathbf{P} = (P_1, P_2, \ldots, P_n)^T$ is the column vector of transmitter powers,

$$\mathbf{u} = \left(\frac{\gamma_1\eta_1}{G_{11}}, \frac{\gamma_2\eta_2}{G_2 2}, \ldots, \frac{\gamma_N\eta_N}{G_N N}\right)^T, \quad (3)$$

is the column vector of noise powers scaled by the SIR constraints and channel gain, and $\mathbf{F}$ is a matrix with

$$F_{ij} = \begin{cases} 0, & \text{if } i = j \\ \frac{\gamma_i G_{ij}}{G_{ii}}, & \text{if } i \neq j \end{cases} \quad (4)$$

with $i, j \in \{1, 2, \ldots, N\}$.

A. Key Results for the Deterministic Channel

The matrix $\mathbf{F}$ has non-negative elements and, by assumption, is irreducible (i.e. we do not have multiple disjoint networks). Let ρ_F be the Perron-Frobenius eigenvalue of $\mathbf{F}$. Then from the Perron-Frobenius theorem and standard matrix theory [8] we have the following equivalent statements

1) $\rho_F < 1$
2) There exists a vector $\mathbf{P} > 0$ (i.e. $P_i > 0$ for all i) such that $(\mathbf{I} - \mathbf{F})\mathbf{P} \geq \mathbf{u}$
3) $(\mathbf{I} - \mathbf{F})^{-1}$ exists and is positive componentwise.

Furthermore, if any of the above conditions holds we also have that $\mathbf{P}^* = (\mathbf{I} - \mathbf{F})^{-1}\mathbf{u}$ is the Pareto optimal solution to (2). That is, if $\mathbf{P}$ is any other solution to (2) then $\mathbf{P} \geq \mathbf{P}^*$ componentwise. Hence, if the SIR requirements for all users can be met simultaneously the best power allocation is $\mathbf{P}^*$, so as to minimize power consumption.

In [3] the authors show that the following iterative power control algorithm converges to $\mathbf{P}^*$ when $\rho_F < 1$, and diverges to infinity otherwise

$$\mathbf{P}(k+1) = \mathbf{F}\mathbf{P}(k) + \mathbf{u}, \quad (5)$$

for $k \in \{1, 2, 3, \ldots\}$. Furthermore, the above iterative algorithm can be simplified into the following distributed version. Let

$$P_i(k+1) = \frac{\gamma_i}{R_i(k)} P_i(k), \quad (6)$$

for each link $i \in \{1, 2, \ldots, N\}$. Hence, each link increases power when its SIR is below its target and decreases power when its SIR exceeds its target. It is easy to show that (5) and (6) are pathwise equivalent and hence the distributed version of the power control algorithm also converges to $\mathbf{P}^*$.

B. Link Protection and Admission Control for the Deterministic Channel

In [1] the authors extend the above power control algorithm to include active link protection and distributed admission control. Active link protection provides a QoS "buffer" to active links that protects them from new users powering up into the system. Distributed admission control permits new users to make local decisions regarding the stability of the network – thereby permitting a local admission control algorithm.

Let $\mathcal{L}$ denote the set of links in the network. A link $i \in \mathcal{L}$ is considered active at time k if $R_i(k) \geq \gamma_i$ and inactive if $R_i(k) < \gamma_i$. Let $\mathcal{A}_k$ and $\mathcal{B}_k$ denote the sets of active and inactive links, respectively. Let $\delta = 1+\epsilon$ for some $\epsilon > 0$ denote the control parameter for active link protection. The new power control algorithm operates according to the following iteration:

$$P_i(k+1) = \begin{cases} \frac{\delta\gamma_i}{R_i(k)} P_i(k), & \text{if } i \in \mathcal{A}_k \\ \delta P_i(k) = \delta^{(k+1)} P_i(0), & \text{if } i \in \mathcal{B}_k \end{cases}, \quad (7)$$

where $P_i(0)$ is the initial power of the ith transmitter.

Under this scheme the active links update their power according to (6) but each user is aiming for an enhanced target of $\delta\gamma_i$. The inactive users increase their power gradually as they try to gain entry into the system. The QoS buffer and the gradual rate of power increase by new users allows the active links to maintain their required levels of QoS. In [1] the authors show that this scheme possesses a number of important properties:

1) once a link enters the active set $\mathcal{A}_k$ it will remain active,
2) if all the QoS requirements for the set of links $\mathcal{L}$ are feasible then eventually all links will become active in finite time,
3) if some of the links in $\mathcal{B}_k$ are infeasible then all of the powers in the system will explode to infinity geometrically fast,
4) as the powers tend to infinity the SIR's of each active user will converge to γ_i, the SIR's of each inactive user will saturate at some level less than γ_i.

In [1] the authors show that the above properties can be exploited to construct distributed admission control algorithms. In distributed admission control a mobile user attempts to enter the network and, at some point, must decide if the network can remain stable and satisfy the new user's QoS requirements. If so, the user enters the network and joins the collection of active links, otherwise the user leaves the system. The three general classes of admission control algorithms proposed in [1] can be roughly characterized as:

1) time based drop out, which exploits above property two,
2) maximum power based drop out, which exploits property three,
3) saturation based drop out, which exploits property four.

Due to space considerations we will not discuss the particulars of each algorithm in detail. We will refer to these general concepts in Section 6 when we discuss admission control in a random channel environment.

III. The Foschini-Miljanic Algorithm in a Random Channel Environment

We now consider the performance of (5) and (6) when the channel gains G_{ij} are allowed to vary with time. Let $\mathbf{G} = (\mathbf{G}(k) : k \geq 0)$ be a stationary ergodic sequence of random channel gain matrices. The elements of $G_{ij}(k)$ denote the channel gain from the jth transmitter to the ith receiver at time k. The sequence $\mathbf{G}$ takes values in a discrete or continuous set of N x N non-negative and irreducible matrices. Given this definition we create another sequence of random matrices $\mathbf{F} = (\mathbf{F}(\mathbf{k}) : k \geq 0)$ with elements

$$F_{ij}(k) = \begin{cases} 0, & \text{if } i = j \\ \frac{\gamma_i G_{ij}(k)}{G_{ii}(k)}, & \text{if } i \neq j \end{cases}, \tag{8}$$

and the random vector sequence $\mathbf{u} = (\mathbf{u}(\mathbf{k}) : k \geq 0)$ where

$$\mathbf{u}(\mathbf{k}) = \left(\frac{\gamma_1 \eta_1}{G_{11}(k)}, \frac{\gamma_2 \eta_2}{G_{22}(k)}, \ldots, \frac{\gamma_N \eta_N}{G_{NN}(k)} \right)^T. \tag{9}$$

For the sake of simplicity we assume the thermal noise power terms η_i remain time-invariant.

We now evaluate the properties of the "random version" of (5), which we define as

$$\mathbf{P}(\mathbf{k+1}) = \mathbf{F}(\mathbf{k})\mathbf{P}(\mathbf{k}) + \mathbf{u}(\mathbf{k}). \tag{10}$$

The distributed version of this algorithm is identical to (6)

$$P_i(k+1) = \frac{\gamma_i}{R_i(k)} P_i(k), \tag{11}$$

with $R_i(k)$ now given by

$$R_i(k) = \frac{G_{ii}(k)P_i(k)}{\eta_i + \sum_{i \neq j} G_{ij}(k)P_j(k)}. \tag{12}$$

Clearly the power vector $\mathbf{P}(\mathbf{k})$ will not converge to some deterministic constant as it did in (5). Rather, in a random channel environment a statement regarding stability requires the power vector to converge in distribution to a well defined random variable. In (5) the key convergence condition is to require that the Perron-Frobenius eigenvalue $\rho_F < 1$. Since $\mathbf{F}$ is now a random matrix process the key convergence condition is that the Lyapunov exponent $\lambda_F < 0$, where λ_F is defined as

$$\lambda_F = \lim_{k \to \infty} \frac{1}{k} \log ||\mathbf{F}(\mathbf{1})\mathbf{F}(\mathbf{2}) \cdots \mathbf{F}(\mathbf{k})||. \tag{13}$$

See [6] for more details on Lyapunov exponents and the convergence properties of products of random matrices. We have the following lemma

Lemma 1: If the transmitter powers are updated according to (10) or (11), $\lambda_F < 0$, and

$$\mathrm{E}\left[\log(1 + ||\mathbf{u}(\mathbf{k})||)\right] < \infty \tag{14}$$

then the power vector $\mathbf{P}(\mathbf{k}) \Rightarrow \mathbf{P}(\infty)$. If $\lambda_F > 0$ then $\mathbf{P}(\mathbf{k}) \to \infty$ a.s.

Proof: See [4].

Since the powers (and hence the SIRs) of all users are now random variables, clearly we cannot meet the original QoS constraint that required $R_i \geq \gamma_i$ for $i \in \{1, 2, \ldots, N\}$ with probability one. In a random environment an appropriate first step is to evaluate the expected value of R_i or the expected value of functions of R_i. For the power control algorithm (10) in a random channel environment we have

Lemma 2: If the transmitter powers are updated according to (10) or (11) and $\lambda_F < 0$ then

$$\lim_{k \to \infty} E[\log R_i(k)] = \log \gamma_i \text{ for all } i \in \{1, 2, \ldots, N\} \tag{15}$$

Proof: See [4].

Notice that in a random channel environment the "target QoS levels" have changed. Rather than aim for $R_i = \gamma_i$, the random version of the power update algorithm aims for $\mathrm{E}[\log R_i] = \log \gamma_i$. (Note that $\log E[R_i] \geq E[\log R_i]$.) Moreover, we no longer have a definition of optimality for this scenario.

IV. A Reformulation of Power Optimality

When designing an adaptive power control algorithm for a random channel environment we can develop an algorithm that predicts channel behavior based on past observations and then uses those predictions to update the transmitter powers. The resulting sequence of transmitter powers will then be a random process that potentially satisfies some QoS constraint. In this paper we will use a different approach for a number of reasons. The most significant reason is that our design objective is a simple, robust, and *distributed* algorithm for controlling the transmitter powers in a random channel environment. Any power update algorithm that uses channel prediction will require each transmitter to learn something about the channels between other transmitters and receivers. This violates our constraints on the simplicity and distributed nature of the algorithm. Hence we assume the appropriate power control algorithm should aim for an optimal fixed power allocation.

In order to facilitate our construction of a power control algorithm we develop a slightly modified version of the original QoS definition used by (5). Consider the original SIR requirement

$$R_i(k) = \frac{G_{ii}(k)P_i(k)}{\eta_i + \sum_{i \neq j} G_{ij}(k)P_j(k)} \geq \gamma_i, \tag{16}$$

and re-write it as

$$G_{ii}(k)P_i(k) - \gamma_i \eta_i - \gamma_i \sum_{i \neq j} G_{ij}(k)P_j(k) \geq 0. \tag{17}$$

Further suppose we want this constraint to hold in expected value. If we re-write the set of constraints for all $i \in \{1, 2, \ldots, N\}$ in matrix form and assume a fixed power allocation $\bar{\mathbf{P}}$, we have

$$(\mathbf{I} - \bar{\mathbf{F}})\bar{\mathbf{P}} - \bar{\mathbf{u}} \geq 0 \tag{18}$$

where

$$\bar{F}_{ij} = \begin{cases} 0, & \text{for } i = j \\ \gamma_i \frac{\mathrm{E}[G_{ij}]}{\mathrm{E}[G_{ii}]} & \text{for } i \neq j \end{cases}, \quad (19)$$

and $\bar{\mathbf{u}} = \mathrm{E}[\mathbf{u}]$. If $\rho_{\bar{F}} < 1$ then from the arguments presented in [3] we know there exists a Pareto optimal vector $\bar{\mathbf{P}}^* = (I - \bar{\mathbf{F}})^{-1}\bar{\mathbf{u}}$, such that for any other vector $\mathbf{P}$ that satisfies (18) we have $\mathbf{P} \geq \bar{\mathbf{P}}^*$ componentwise.

We now have two critical questions to answer. First, how do we design an algorithm that converges to the optimal solution of (18)? Second, will this algorithm be *distributed*? The answer is not obvious. Notice that a distributed algorithm must iteratively solve an expected value matrix equation without observations of all the (random) elements of the matrix process $\mathbf{F}$. This issue is addressed in the next section.

V. A Distributed Pareto-Optimal Algorithm in the Random Channel Environment

Our goal is to develop a distributed power control algorithm that converges to $\bar{\mathbf{P}}^*$. Recall that in the original algorithm (6) for deterministic channels, each receiver provided feedback of its SIR to the appropriate transmitter and this was all that was required for the distributed algorithm to converge. As we will see in this section, in the random channel environment we require slightly more information at each transmitter. In addition to the SIR at the intended receiver we assume each transmitter also has knowledge of the channel gain to the receiver. That is, the ith transmitter has knowledge of $R_i(k)$ and $G_{ii}(k)$ when selecting $P_i(k+1)$.

As a first step towards a distributed algorithm we first consider a centralized solution of

$$(\mathbf{I} - \bar{\mathbf{F}})\bar{\mathbf{P}}^* - \bar{\mathbf{u}} = 0, \quad (20)$$

(a centralized solution assumes knowledge of the $\mathbf{F(k)}$'s). If we write

$$g(\bar{\mathbf{P}}) = (I - \bar{\mathbf{F}})\bar{\mathbf{P}} - \bar{\mathbf{u}} \quad (21)$$

then we can view the solution $\bar{\mathbf{P}}^*$ of (20) as the zero of the function $g(\bar{\mathbf{P}})$. Of course, the centralized controller of the network might not have access to $\bar{\mathbf{F}}$ and $\bar{\mathbf{u}}$, just random observations of the matrix sequence $\mathbf{F}$ and vector sequence $\mathbf{u}$. Hence, finding a solution to our optimality equation is equivalent to estimating the zero of the function $g(\bar{\mathbf{P}})$ when we only have access to noisy estimates of $g(\bar{\mathbf{P}})$. Therefore, one possible iterative estimation procedure is a version of the Robbins-Monro stochastic approximation algorithm [7]. We will define our centralized power control algorithm as follows. Let $\hat{\mathbf{P}}(\mathbf{k})$ be our estimate for the solution to (20) at time k, then

$$\hat{\mathbf{P}}(\mathbf{k+1}) = \hat{\mathbf{P}}(\mathbf{k}) - a_k g\left(\hat{\mathbf{P}}(\mathbf{k})\right) + a_k \epsilon_k, \quad (22)$$

where a_k is the algorithm step-size satisfying $a_n \to 0$, $\sum_{n=1}^{k} a_n \to \infty$, and $\sum_{n=1}^{k} {a_n}^2 < \infty$. The error term is

$$\epsilon_k = \left(\bar{\mathbf{F}} - \mathbf{F(k)})\hat{\mathbf{P}}(\mathbf{k}) + (\bar{\mathbf{u}} - \mathbf{u(k)}\right). \quad (23)$$

In [4] we show, under appropriate regularity conditions on the channel process $\mathbf{G}$, that $\hat{\mathbf{P}}(\mathbf{k}) \to \bar{\mathbf{P}}^*$ as $k \to \infty$ if the QoS conditions (17) are feasible. Furthermore, if $\rho_{\bar{F}} > 1$ then $\hat{\mathbf{P}}(\mathbf{k}) \to \infty$.

It is interesting to note that the convergence conditions for this algorithm do not require that all possible values of $\mathbf{F}$ have $\rho_F < 1$. Recall that in the deterministic channel case $\rho_F > 1$ corresponded to an unstable system with powers increasing to infinity. In the random channel case it is possible for the wireless network to operate in an unstable environment for some fraction of time and still remain stable on average.

Finally, we can construct a distributed version of the full-information stochastic approximation algorithm (22). Let

$$\hat{P}_i(k+1) = \hat{P}_i(k) - a_n \left(G_{ii}(k)\hat{P}_i(k) - \frac{\gamma_i G_{ii}(k)\hat{P}_i(k)}{R_i(k)} \right), \quad (24)$$

and we have

Theorem 1: The components of $\hat{\mathbf{P}}(\mathbf{k})$ from the full-information algorithm (22) are equivalent (on each sample path) to the individual transmitter powers determined by the distributed algorithm (24). Hence, all of the convergence results and properties of the full information algorithm also apply to the distributed algorithm.

Proof: The proof follows immediately from element by element analysis of $\hat{\mathbf{P}}(\mathbf{k})$.

VI. Admission Control for the Random Channel Environment

In any stochastic approximation problem, appropriate selection of the step size sequence a_k is critical in order to ensure robust performance. The three conditions $a_n \to 0$, $\sum_{n=1}^{k} a_n \to \infty$, and $\sum_{n=1}^{k} {a_n}^2 < \infty$, provide a sequence that is (initially) large enough to move our estimate $\hat{\mathbf{P}}$ close to equilibrium and (eventually) small enough to remove noise from the estimate. While these properties of the step size ensure a.s. convergence of $\hat{\mathbf{P}}(\mathbf{k}) \to \mathbf{P}^*$, we are faced with a significant complication – the algorithm is non-stationary.

This non-stationarity makes it quite difficult to construct an admissions control algorithm from a stochastic approximation algorithm. For example, suppose we set $a_k = 1/k$ and further suppose that a new user attempts to enter the system at time $k = 10^4$. At this point, all of the current active users will have settled close to the equilibria $\mathbf{P}^*$ and will not be able to respond quickly to the interference from the new user since their step size is quite small. Moreover, if we enforce some form of active link protection in this scenario then the new user will have to wait an arbitrarily long time to become active. In general, the problem of admission control is fundamentally different from the problem of finding a fixed equilibrium. In a network where users are allowed to enter and leave, the equilibrium $\mathbf{P}^*$ becomes a time-varying process. Hence, our goal in admissions control is to develop an algorithm that tracks the equilibrium process $\mathbf{P}^*$ while maintaining our required QoS (17).

To this end, we propose a modified version of our stochastic approximation algorithm that uses a *fixed* step size rather than a decreasing step size. As in Section III, suppose we have two sets of links: the active links $\mathcal{A}_k$, and the inactive links $\mathcal{B}_k$ that are attempting to gain access to the system. Let

$$\hat{P}_i(k+1) = \hat{P}_i(k) - a\left(G_{ii}(k)\hat{P}_i(k) - \frac{\delta\gamma_i G_{ii}(k)\hat{P}_i(k)}{R_i(k)}\right), \quad \text{if } i \in \mathcal{A}_k \tag{25}$$

$$\hat{P}_i(k+1) = \max\left[a\delta^{(k+1-T_i)}P_0, \hat{P}_i(k) - a\left(G_{ii}(k)\hat{P}_i(k) - \frac{\delta\gamma_i G_{ii}(k)\hat{P}_i(k)}{R_i(k)}\right)\right], \quad \text{if} i \in \mathcal{B}_k \tag{26}$$

where $0 < a < 1$ is the fixed step size, P_0 is the initial power of a user when it attempts to enter the system and T_i is that time at which the new user begins transmission. In this scheme, the active links adjust their powers according to a fixed step size stochastic approximation algorithm. The inactive links perform the same algorithm steps but their maximum power is capped at $a\delta^{(k+1)-T_i}P_0$ to protect the QoS of the active links.

We first compare the performance of the fixed step size algorithm to that of the decreasing step size algorithm (assuming no users enter or leave the system). Notice that the fixed step size algorithm provides a geometric weighting of recent estimates with the weight tending to zero for the oldest estimates. Further note that for a fixed step size we do not satisfy two of the conditions that guarantee a.s. convergence, namely $a_n \to 0$ and $\sum_{n=1}^{k} {a_n}^2 < \infty$. Hence, the fixed step algorithm only provides weak convergence to a random variable rather than a.s. convergence. Specifically, in [4] we prove that if the QoS requirements for all users are feasible then under (25) we have $\hat{\mathbf{P}}(\mathbf{k}) \Rightarrow \tilde{\mathbf{P}}$, where $\tilde{\mathbf{P}}$ is a stationary random variable that satisfies $\mathrm{E}[||\mathbf{P}^* - \tilde{\mathbf{P}}||] = O(a)$. Therefore, under a fixed step size algorithm we will never converge to the optimal fixed power levels although we will come close in expectation.

The obvious question is "why should we prefer this fixed step size to the decreasing step size?" The answer lies in the ability of the fixed step size algorithm to track a time-varying equilibrium. In [4] we prove that when a new user attempts to enter the system, (25) and (26) converge geometrically fast to the new power equilibrium, provided the QoS requirements for all users are feasible. This rate of convergence is in stark contrast to that of the standard stochastic approximation algorithm which can take an arbitrarily long time to converge to the new equilibrium due to its decreasing step size (see the Numerical Results section for an example). Moreover, this fixed step size algorithm permits active link protection and admission control. In [4] we prove, under appropriate regularity conditions on $\mathbf{G}$, that the QoS requirements for the active links are protected under (25) and (26). That is, as a new user attempts to enter the system the expected value bounds in (17) are maintained.

We also show that the admission control algorithms proposed in [1] and reviewed in Section 2 can be modified to fit into this stochastic approximation framework. Specifically, the algorithms proposed in [1] provide deterministic bounds on the time required for a new user to either enter the system or determine that the system is unstable and leave. For the random channel case discussed in this paper we can at best provide probabilistic bounds. For example, the time based dropout algorithm will permit a new user to enter the system if, after a certain amount of time, the user *estimates* that its QoS requirements are being met. Therefore, in the deterministic channel environment a user knows w.p. 1 if its QoS requirements are met, whereas in a random channel environment the user must base its admission control decision on a sample average of its QoS requirements. Similar formulations exist for the maximum power dropout and saturation dropout rules in a random channel environment. We reserve the detailed proofs of these methods for [4] due to space constraints. The significant admission control issue stemming from these results is that in a random channel environment a new user can make an *incorrect* admission control decision. Specifically, a new user cannot make a "probability one" admission decision unless it is allowed to observe the network for an infinite amount of time. Therefore, since we want users to join the network quickly, once a user joins the set of active links $\mathcal{A}_k$ it will be required to continue monitoring its QoS requirements and voluntarily drop out of the network if it determines that the system is unstable.

Finally, it should be noted that the appropriate choice of a is not obvious. In most publications on fixed step size stochastic approximation algorithms a is chosen by heuristic methods. The tradeoffs that must be evaluated are the rate at which the algorithm responds to changes in the equilibrium point $\mathbf{P}^*$ (corresponding to large a) and minimization of the impact of noise (corresponding to small a). We investigate this issue further in [4] where we provide some heuristic as well as rigorous methods for determining a.

VII. Numerical Analysis

Consider an ad-hoc network consisting of four mobile devices. Further assume that the fourth device does not attempt to enter the network until time $k = 2000$. In the first numerical example we will assume every link in the system is an independent exponential random variable where the expected value of the gain matrix

$$\mathrm{E}[G] = \begin{pmatrix} 1 & .0375 & .02 & .03 \\ .0375 & 1 & .04 & .04 \\ .02 & .04 & 1 & .05 \\ .03 & .04 & .05 & 1 \end{pmatrix}. \tag{27}$$

We assume that $\gamma_i = 5$ and $\eta_i = 1$ for each transmitter. For this setup we have $\rho_{\bar{F}} = .55$ with all four users active, so we should expect the power control algorithms to be fairly stable. The plot in Figure 1 shows the power of the first transmitter for the same channel sample path using (11), (24), and (25) as the power control algorithms. Note that the power axis is on a *logarithmic scale* . Clearly the stochastic approximation algorithms provide better power stability and consumption than (11). Note that the fixed step size algorithm does not converge to a constant and therefore does achieve the optimal equilibrium point.

Fig 1: Transmit Powers for the Standard Foschini−Miljanic Algorithm and the Two Proposed Stochastic Approximation Algorithms

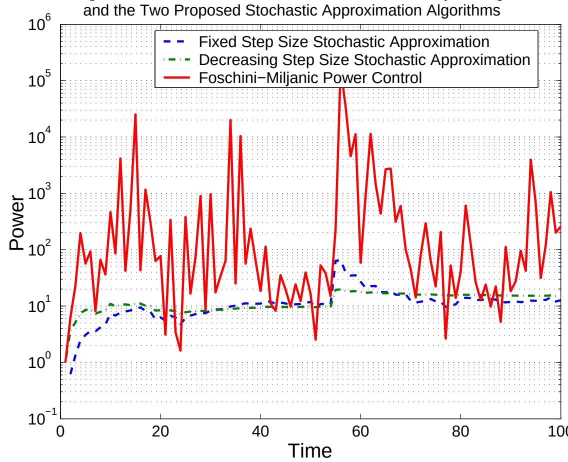


Fig 2: Power Comparison of the Foschini−Miljanic Algorithm and the Two Stochastic Approximation Algorithms in a Channel with Substantial Memory

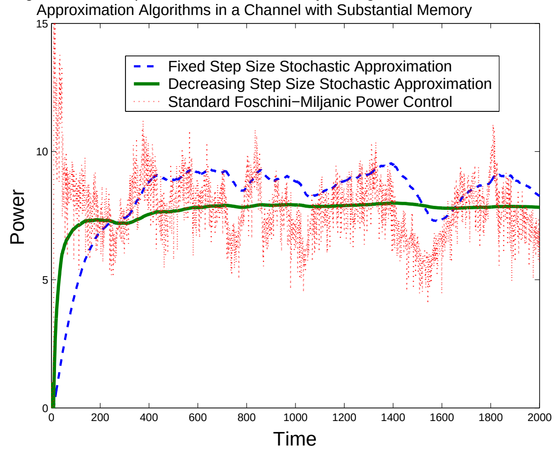
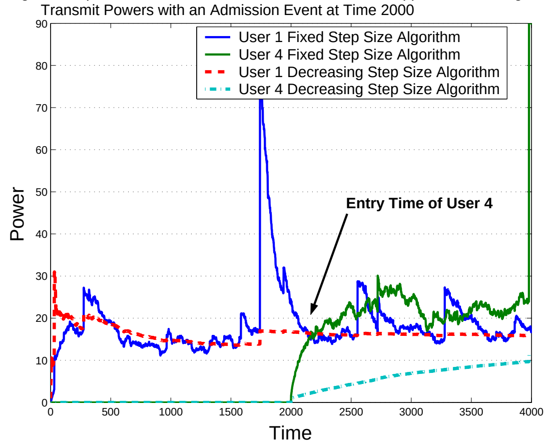


Fig. 3 Comparision of the Stand and Modified Stochastic Approximations Algorithms: Transmit Powers with an Admission Event at Time 2000



Admittedly, independent exponential fading is probably the worst case channel for (11). Suppose we now allow the channel to have memory. We will model a channel gain matrix with memory of M timeslots as

$$\mathbf{G}(k+M) = \frac{1}{M}\mathbf{G_{iid}} + \frac{1}{M}\sum_{i=1}^{M-1}\mathbf{G(k+i)}, \qquad (28)$$

where $\mathbf{G_{iid}}$ is a matrix with independent exponential elements with mean (27). This type of memory model is highly favorable for (11) as the amount of "randomness" in each iteration of the channel decreases with $1/M$. The plot in Figure 2 shows the power comparison for a channel with $M = 20$, which is a substantial amount of memory. While (11) performs much better in this case, the stochastic approximation algorithms are still preferred.

Now consider the impact of the fourth user powering up into the system at time $k = 2000$. Figure 3 plots the power of users 1 and 4 operating under the fixed step size and decreasing step size stochastic approximation algorithms (with link protection). Although the fixed step size algorithm does not provide a.s. convergence to the optimal equilibrium, it does provide *substantially faster* access times for new users. This trade-off will likely be acceptable for non-stationary systems with frequent entries and exits of users.

VIII. Conclusion

We have presented an evaluation of the Foschini-Miljanic power control algorithm in a random channel environment. The analysis shows that their proposed algorithm does not meet its intended QoS requirements nor does it perform well in terms of power consumption. In order to address these issues we proposed a new criteria for power optimality in wireless ad-hoc networks. We then showed that the optimal power allocation could be discovered through a stochastic approximation algorithm. Moreover, the structure of this stochastic approximation algorithm yielded an optimal fully distributed on-line algorithm for controlling transmitter powers in an ad-hoc network. In addition, we presented a fixed step size version of the optimal stochastic approximation algorithm that provides only weak convergence rather than a.s. convergence to the power equilibrium. However, the fixed step size algorithm provides much better response to time-varying power equilibria (i.e. users entering and exiting the system). In [4] we will present the proofs withheld from this short paper. We will also provide detailed numerical analysis of the active link protection and admission control protocols proposed in Section 4.

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