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Asymptotic analysis of fair scheduling in the OFDM systems

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Abstraction: Orthogonal Frequency Division multiplexing (OFDM) is one of the promising techniques in the next generation high-speed data rate wireless networks for its ability to combat the intersymbol interference(ISI), and fair scheduling algorithms are used to utilize the independence of the fading statistics of different user to improve the spectral efficiency. In this paper, we try to analysis the performance of fair scheduling algorithms in the OFDM systems and choose the appropriate subband size to tradeoff between the throughput and the channel feedback. We show that asymptotic scheduling gain of fair scheduling algorithms in the OFDM system increases as the number of user increases and decreases with the average received SNR. We also present the distribution of the averaged received signal-to-noise-ratio (SNR) in a subband, give a formula to evaluate the scheduling gain approximately, and point out that the maximal eigenvalue is a good indicator to choose the appropriate subband size. It is shown that the appropriate subband number is 16 for channel A or 32 for channel B.

Keywords: Orthogonal Frequency Division multiplexing (OFDM), multiuser diversity,fair scheduling, subband

I. INTRODUCTION

Orthogonal Frequency Division multiplexing (OFDM) is one of the promising techniques in the next generation high-speed data rate wireless networks for it is one of the most effective schemes to mitigate the multipath delay spread in the wireless communications by dividing the entire channel into many parallel narrowband subcharriers to increase the symbol duration[1]. It has a high spectral efficiency for the overlapping between the adjacent subcarriers. This is very important for wireless communications.

Motivated by the information theoretic results[2][3], another approach to increase the throughput of multiuser systems is to use multiuser diversity to take advantage of the independence of the fading statistics of different user. Traditionally, channel fading is viewed as a source of unreliability that has to be mitigated. In the context of multiuser diversity, however, fading can be instead considered a source of randomization that can be exploited. In a multiuser diversity system, a packet scheduler uses the instantaneous knowledge of the signal-to-noise (SNR) of each user to allocate resources to the users based on channel quality. Overall systems throughput is maximized by allocating at any time the common channel resource to the user that can best exploit it .It can also be thought of as a form of selection diversity. Multiuser diversity is appropriate for high-speed data transmission since data packets are more tolerant to scheduling delays than constant bit rate services such as voice. A proportional fair scheduling scheme called High data rate (HDR) has been used in the CDMA2000 systems to improve the downlink performance[4][5][6]. The scheduling algorithm tries to transmit data to a mobile terminal when its instantaneous data rate nears its peak rate.

In this paper we study the improvement in the average throughput due to multiuser diversity in the OFDM systems and select the appropriate scheduling parameter subband size in the multipath fading channels. For all users have to feedback the subcarrier gain information to the basestation in every slot, the channel feedback would consume a large amount of the uplink bandwidth if not preprocessed. In the CDMA2000 standard, nine data rate levels are used to reduce the feedback. In the OFDM systems, feedback can be reduced further if we consider the correlation between the adjacent subcarriers and temporal correlation of multipath channels between the consequent OFDM symbols. So a frame structure is introduced to facilitate the resource allocation in the OFDM systems. Firstly the time axis is divided into slots that consist of several OFDM symbols, and then subcarriers in a slot are partitioned into groups termed subband. Every subband is composed of some adjacent subcarriers and is disjoint with each other. The subband size is the number of subcarriers in a group and needs to be carefully selected in the OFDM systems to tradeoff between the channel feedback and the system throughput. We assume that the channels are not changed during a slot. The scheduling gain is defined to be the ratio of the spectral efficiency of a scheduler to that of the round robin scheduler (fixed time division multiplexing). We show that scheduling gain increases as the number of user increases and decreases with the average received signal-to-noise-ratio(SNR). We also present the distribution of the averaged received SNR in a subband, give a formula to evaluate the scheduling gain approximately when subband is used, and point out the maximal eigenvalue is a good indicator to select the appropriate subband size. It is shown that the appropriate subband size. It is shown that the appropriate number of subbands is 16 for channel A or 32 for channel B when considering the tradeoff between the feedback and the throughput.

The rest of this paper is organized as follows. In section II the system model is introduced and the HDR scheduling algorithm is visited and its asymptotic performance is presented in section III. In section IV, we study the performance of the scheduling algorithm in the OFDM systems and point out that appropriate subband number is 16 or 32 when considering the tradeoff between the channel feedback and throughput. The conclusion is finally given in section

II. SYSTEM MODEL

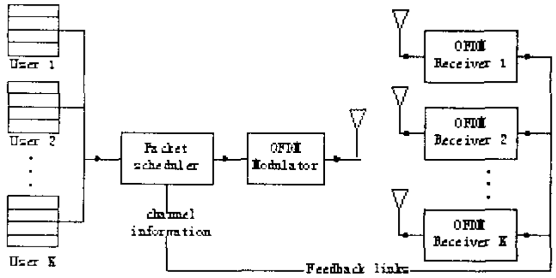

Fig.1 fair scheduler in the multiuser OFDM system

A baseband model for the downlink of an OFDM system is illustrated in Fig.1. There is a single basestation(BS) communicating with K users. A queue of packets is stored at the BS for each of the K users. We assume that the channel information is transmitted back without error to the BS just before the scheduler is to choose users for the next transmission time. Based on feedback of the channel state from each user, the BS chooses the users to which it will send the packets at the next slot. We assume there are N subcarriers in the OFDM systems and the subband size is M, i.e. a subband contains M subcarriers and the number of subbands is N/M.

We assume a slot consists of L OFDM symbols and channels are not changed during a slot but varied from slot to slot. We assume that the statistics of multipath channels of all users are same and independent. The baseband block multipath fading channel model of user k at time t is described by .

$$h_k(t,\tau)=\sum_{l=0}^{L-1}h_{k,l}(t)\delta(\tau-\tau_{k,l}) \quad (1)$$

where $h_{k,l}(t)$, $\tau_{k,l}$ represent respectively the gain and delay of the l th path and $h_{k,l}(t)$ is a zero-mean complex Guassian random variable with variance $E_{k,l}$. The correlation function of $h_{k,l}(t)$ is given by

$$R_{k,l}(\tau)=E[h_{k,l}(t)h_{k,l}^{*}(t+\tau)]=E_{k,l}J_0(2\pi f_d\tau)$$

where f_d is the maximal doppler rate, $J_0(\cdot)$ is the modified first class zero-order bessel function. For simplicity we assume $E_{k,l}=E_{j,l}=E_l$ and $\tau_{k,l}=\tau_{j,l}=\tau_l$.

At the receiver end, the mobile terminal demodulates the received OFDM symbols by using Fast Fourier transformation (FFT) and predicts the channel at the next slot from the history of channel information and feedbacks them to the scheduler without delay. In this paper, we assume that the scheduler has the perfect channel information of each user. Let T_S denote the duration of an OFDM symbol. The channel estimate at the subcarrier i of user k at time t is given by

$$H_k(t,i)=\sum_{l=0}^{L-1}h_{k,l}(t)e^{-j2\pi\tau_l i/T_S} \quad (2)$$

where $1\le k\le K$, $1\le i\le N$. The correlation between subcarriers i and m is given by

$$R(i,m)=\sum_{l=0}^{L-1}E_l e^{-j2\pi\tau_l|m-i|/T_S} \quad (3)$$

where $1\le i\le N, 1\le m\le N$.

Known from (2) and (3), $H_k(t,i)$ is a zero-mean normal random variable with variance $\sigma^2=\sum_{l=0}^{l}E_l$, and all the subcarriers have the same distribution, and there are strong correlation between the adjacent subcarriers. So the best performance in the OFDM system can be reached by scheduling every subcarrrier independently for users. Dividing a slot into subband incurs some loss in the system performance. In the following section we analysis the gain achieved by scheduling every subcarrier independently.

III. ASYMPTOTIC ANALYSIS OF THE PROPORTIONAL FAIR SCHEDULING

The scheduling metric of user k at time n at a subcarrier is given by

$$\frac{D_k(n)}{R_k(n)} \quad (4)$$

where $D_k(n)$ is the instantaneous data rate that user k at the subcarrier, $R_k(n)$ is the average data rate in a time interval of length T_C at the subcarrier. The scheduling rule is

$$j=\arg\max_k\frac{D_k(n)}{R_k(n)} \quad (5)$$

that is the scheduler selects the user at the subcarrier to be serviced who has the maximal metric. It has been proved the scheduler is proportional fair, and all the users share almost the same number of time slots in a long range.

When T_C is large enough, $R_k(n)$ is almost stationary and the scheduler rule becomes[4]

$$k=\arg\max_k\frac{D_k(n)}{R_k} \quad (6)$$

R_k is determined by the channel statistic of user k and used to normalize the channel data rate $D_k(n)$. The scheduler selects the user at the subcarrier who has the maximal normalized instantaneous data rate. Since the channel statistics of all users are same and independent, the asymptotic scheduling rule is equivalent to

$$k=\arg\max_k C_k(n) \quad (7)$$

for there is a one-to-one mapping between $D_k(n)$ and $C_k(n)$, where $C_k(n)$ is the signal-to-noise ratio (SNR) of the channel of user k at time n. The scheduler selects the user which has the maximal SNR to send at the next slot time.

We assume the noise at the subcarrier of every user is the additive white gaussion noise with variance 1. For the raylegh fading channel, the probability density function (pdf) of the SNR is given by

$$f(r)=\frac{1}{\sigma^2}e^{-\frac{r}{\sigma^2}} \tag{8}$$

where σ^2 is the average received SNR.
The probability of that user k *is selected to be serviced* is given by

$$\Pr(r_k > r_j, for \quad j \neq k | r_k\} = \prod_{j=1, j\neq i}^{K} \int_0^{r_k} f(x)dx$$

$$=\left(1-e^{-\frac{r}{\sigma^2}}\right)^{K-1} \tag{9}$$

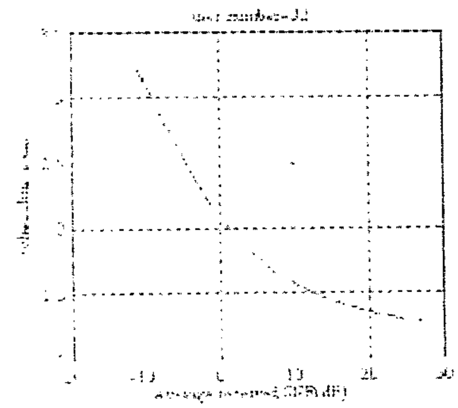

Fig.2. Scheduling gain as a function of the average received SNR

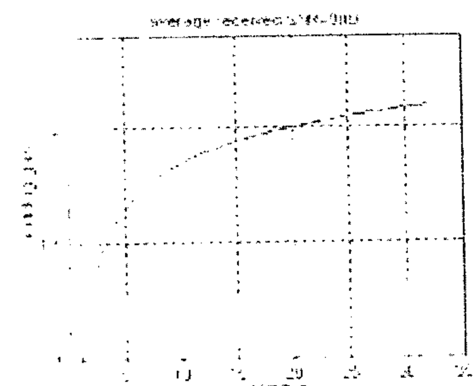

Fig.3 Scheduling gain as a function of user number

The scheduling gain is defined to be the ratio of throughput of the proportional fair scheduling algorithm to the round-robin (fixed time division multiplexing) scheduling algorithm. In the OFDM systems, the scheduling gain at the subcarrirer is given by

$$G=\Gamma_S / \Gamma_{RR} \tag{10}$$

where Γ_S is the throughput of the fair scheduling system and is given by

$$\Gamma_S = \int_0^{\infty} \log(1+r) f(r) dr \prod_{j=1, j\neq i}^{K} \int_0^{r} f(x)dx \tag{11}$$

$$= \int_0^{\infty} \log(1+r)\frac{1}{\sigma^2}e^{-\frac{r}{\sigma^2}}\left(1-e^{-r/\sigma^2}\right)^{(K-1)} dr \tag{12}$$

and Γ_{RR} is the throughput of the round-robin system and is given by

$$\Gamma_{RR} = \frac{1}{K}\int_0^{\infty} \log_2(1+r)\frac{1}{\sigma^2}e^{-\frac{r}{\sigma^2}} dr \tag{13}$$

The scheduling gain is plotted in Fig.2 and Fig.3.It is shown that the scheduling gain increases with the number of users and decreases with the average received signal strength. The scheduling gain is smaller at higher SNR because of the nonlinear function $\log$. At high SNR, it needs to increase the SNR much more to get the data rate double compared with the case at low SNR. In a word, the increasing speed of the scheduling gain decreases with the user number, and the decreasing speed of the scheduling gain also decays with the SNR.

IV. THE APPROPRIATE SUBBAND SIZE AND DISCUSSIONS

In the above section, we consider the ideal situation where the subband size is 1. In this section, we study the effect of the subband size M on the system throughput. In a subband, the SNR of the subcarriers in a subband are not same but there is strong correlation among the subcarriers. In the following analysis, we only consider the first subband because of the symmetry among the subbands.
We still assume the noise at every subcarrier is the independent additive white Gaussian noise with variance 1 and the transmitted power at every subcarrier is 1. The average received SNR S at the first subband is given by

$$S=\frac{1}{M}\sum_{i=0}^{M-1}\left|H_{j,i}\right|^2 \tag{14}$$

where $H_{j,i}$ is the channel gain which is given in (2). Substituting (2) into (14), the SNR S is given by

$$S=\frac{1}{M}\sum_{i=0}^{M-1}\left\{\left(\sum_{l=0}^{L-1}h_{j,l}e^{-j2\pi\tau_l i/T_S}\right)\left(\sum_{l=0}^{L-1}h_{j,l}e^{-j2\pi\tau_l i/T_S}\right)^*\right\} \tag{15}$$

Define

$$\boldsymbol{h}=\begin{bmatrix}h_{j,0} & h_{j,1} & \cdots & \cdots & h_{j,L-1}\end{bmatrix}\in C^{1\times L}$$

$$\boldsymbol{e}_i=\left[e^{-j2\pi\tau_0 i/T_S}, e^{-j2\pi\tau_1 i/T_S}, \cdots, e^{-j2\pi\tau_{L-1} i/T_S}\right]^T \in C^{L\times 1}$$

Thus we have

$$\left(\sum_{l=0}^{L-1} h_{j,l} e^{-j2\pi\tau_l i/T_s}\right) = \boldsymbol{he}_i \tag{16}$$

Substituting (16) into (14), S can be written as

$$S = \frac{1}{M}\sum_{i=0}^{M-1} \boldsymbol{he}_i \boldsymbol{e}_i^H \boldsymbol{h}^H = \boldsymbol{h}\left(\sum_{i=0}^{M-1} \boldsymbol{e}_i \boldsymbol{e}_i^H\right)\boldsymbol{h}^H$$

$$= \boldsymbol{hAh}^H \tag{17}$$

where $\boldsymbol{A} = \sum_{i=0}^{M-1} \boldsymbol{e}_i \boldsymbol{e}_i^H$.

Define

$$\boldsymbol{\eta} = \left[\frac{h_{j,0}}{\sqrt{E_0}} \quad \frac{h_{j,1}}{\sqrt{E_1}} \quad \cdots \quad \cdots \quad \frac{h_{j,L-1}}{\sqrt{E_{L-1}}}\right] \tag{18}$$

$$\boldsymbol{E} = diag\left[\sqrt{E_0} \quad \sqrt{E_1} \quad \cdots \quad \cdots \quad \sqrt{E_{L-1}}\right] \tag{19}$$

Substituting (18) and (19) into (17), S is given by

$$\boldsymbol{S} = \frac{1}{M}\boldsymbol{\eta E A E}^H \boldsymbol{\eta}^H = \boldsymbol{\eta B \eta}^H \tag{20}$$

where $\boldsymbol{B} = \frac{1}{M}\boldsymbol{E}^H \boldsymbol{AE}$ and is a Hermite Matrix. Thus, there exists a unitary matrix $\boldsymbol{U} \in C^{L\times L}$ to diagonalize matrix $\boldsymbol{B}$, $\boldsymbol{B} = \boldsymbol{U\Lambda U}^H$, where $\Lambda = diag(\lambda_0, \cdots, \lambda_{L-1})$ and the eigenvalues are real. We assume that $\lambda_0 \geq \cdots \geq \lambda_{L-1} \geq 0$. Let m denote the number of nonzero eigenvalues of matrix $\boldsymbol{B}$. Substituting the diagonal form into (20), we get

$$\boldsymbol{S} = \boldsymbol{\eta U \Lambda U}^H \boldsymbol{\eta}^H = \boldsymbol{\omega \Lambda \omega}^H = \sum_{i=0}^{m-1} \lambda_i |\omega_i|^2 \tag{21}$$

where $\boldsymbol{\omega} = \boldsymbol{\eta U}, \omega_i$ is the ith element of vector $\boldsymbol{\omega}$. For the unitary property of matrix $\boldsymbol{U}$, it is easy to know that $\boldsymbol{\omega}$ is circular symmetric complex Gaussian random vector with distribution $CN(0, \boldsymbol{I})$. $|\omega_i|^2$, $0 \leq i < L-1$, is a sequence of independent and identical distributed(i.i.d) exponential random variables. λ_k is the average power corresponding to ω_i and $\sum_{i=0}^{L-1}\lambda_i = \sum_{i=0}^{L-1} E_i$ if we note that the average power of every subcarrier is equal to $\sum_{i=0}^{L-1} E_i$. The ratio $\lambda_0 / \sum_{i=0}^{L-1}\lambda_i$ is also not changed when only the average received SNR increases for the structure of the eigenvalues is only related with the path delay spread profile and path power profile of the multipath fading channels. When $M = 1$, there is only one nonzero eigenvalue λ_0. As M increase, the number of nonzero eigenvalues also gradually increases.

The characteristic function of an exponential random variable with parameter λ is given by

$$\Phi(t) = \frac{j/\lambda}{t - j/\lambda}$$

Therefore, the characteristic function of S is given by

$$\Phi_S(t) = \prod_{l=0}^{m-1} \frac{j/\lambda_l}{t - j/\lambda_l} = \sum_{l=0}^{m-1} \frac{A_l}{t - j/\lambda_l} \tag{22}$$

where

$$A_i = (j/\lambda_i)\left(\prod_{l=0, l\neq i}^{m-1} \frac{j/\lambda_l}{t - j/\lambda_l}\right)\Bigg|_{t=j/\lambda_i}$$

The probability density function of S is given by

$$p_S(s) = \int_{-\infty}^{+\infty} \prod_{l=0}^{m-1} \frac{j/\lambda_l}{t - j/\lambda_l} \exp(-jst)dt, s \geq 0 \tag{23}$$

Substituting (22) into (23), we have

$$p_H(s) = \sum_{l=0}^{m-1} \frac{A_l}{j} \exp(-s/\lambda_l) \tag{24}$$

When $\lambda_i < \lambda_0, i \neq 1$, especially when $\lambda_i << \lambda_1$, the main dominant item in (24) is the iterm determined by λ_0. In other word, when $M = 1$, there is only one nonzero eigenvalue and S has an negative exponential distribution function with parameter λ_0; as M increases, λ_0 decreases and the power is dissipated on the more items in (24) and the distribution of S deviates further from the exponential distribution. This deviation means increasing the subband size incurs some loss of the scheduling gain for random variable S becomes less and less random, while less randomness means less scheduling gain for multiuser diversity is achieved through utilizing the randomness in the channel.

Define

$$\phi = \lambda_0 / \sum_{i=0}^{L-1} \lambda_i \tag{25}$$

as the measure of the deviation. As we will see in the following section, it can be used to determine the appropriate subband size.

The cumulative distribution function (CDF) of S is given by

$$\Pr(s) = \Pr(S < s) = \sum_{l=0}^{m-1} \frac{A_l \lambda_l}{j}(1 - \exp(-s/\lambda_l)) \tag{26}$$

According to the scheduling rule in (7), the asymptotic throughput of a fair scheduling system of a user is given by

$$\Gamma_S = \int_0^{\infty} \log_2(1+s) p(s) (\Pr(s))^{K-1} ds \tag{27}$$

The throughput of the round robin scheduling system of a user is given by

$$\Gamma_{RR} = \frac{1}{K}\int_0^{\infty} \log_2(1+s) p(s) ds \tag{28}$$

Substituting (24), (25) into (27), (28), we have

$$\Gamma_{RR} = \frac{1}{K}\int_0^{\infty} \log_2(1+s)\left[\sum_{l=0}^{m-1} \frac{A_l}{j} \exp(-s/\lambda_l)\right] ds \tag{29}$$

$$\Gamma_S = \int_0^r \log_2(1+s)\left[\sum_{l=0}^{m-1}\frac{A_l}{j}\exp(-s/\lambda_l)\right]\left[\sum_{l=0}^{m-1}\frac{A_l\lambda_l}{j}(1-\exp(-s/\lambda_l))\right]^{K-1} ds \tag{30}$$

TABLE.I M-1225E OUTDOOR CHANNEL MODELS

Tap	Channel A		Channel B		Doppler
	Relative delay (ns)	Average power (dB)	Relative delay (ns)	Average power (dB)	spectrum
1	0	0.0	0	−2.5	Classic
2	310	−1.0	300	0	Classic
3	710	−9.0	8 900	−12.8	Classic
4	1 090	−10.0	12 900	−10.0	Classic
5	1 730	−15.0	17 100	−25.2	Classic
6	2 510	−20.0	20 000	−16.0	Classic

Substituting (29) and (30) into (10), the scheduling gain can be evaluated through the numerical approach. The reason that we use (29) not while not (11) is that the scheduling gain in (29) is an also good measure to the subband-size-induced deviation from the exponential distribution. The results are presented in the next section.

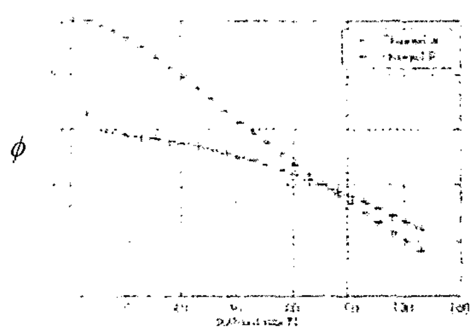

Fig.4 the maximal eigenvalue versus the subband size

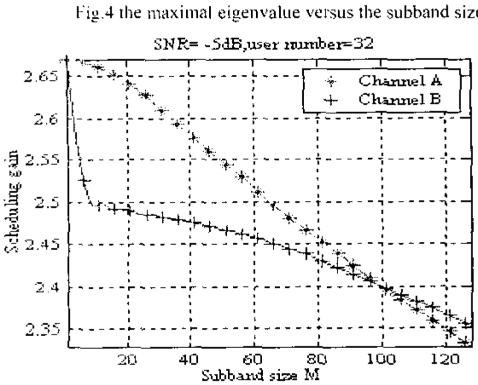

Fig.5 scheduling gain versus subband size at SNR= -5dB

V. SIMULATION AND RESULTS

In this section, the multipath fading channel models illustrated in table.1 are used in our numeric evaluations and system-level simulations. We assume N =1024, L =6. The sampling duration is assumed to be 100 ns and an OFDM symbol consists of N samples.

In Fig.4, we study the influence of subband size M on the maximal eigenvalue λ_0. It is clear that ϕ in (25) is a decreasing function of subband size M, has different characteristics in different channel models. When the subband size is small, the slope of channel B is much sharper compared with the channel A for the delay spread of channel B is much larger than channel A. In other word, the distribution of S in channel B deviates from the exponential distribution more quickly. In order to fit this condition, we have to choose a smaller subband size for channel B. To limit the loss less than 10%, the subband size for channel A is about 64 while the appropriate subband size of channel B is 32 if we constraint that the subband size must be the power of 2. The similar results can also be achieved from the viewpoint of coherent bandwidth.

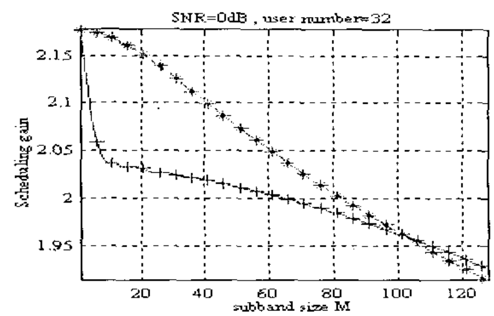

Fig.6 scheduling gain versus subband size at SNR= 0dB

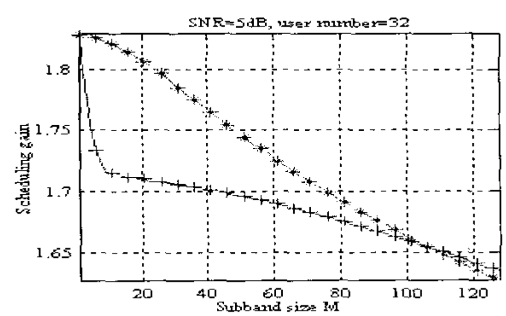

Fig.7 scheduling gain versus subband size at SNR= 5dB

Figs.5-7 show the performance of the scheduling gain versus the subband size M with the different averaged received SNR, -5 dB, 0 dB, and 5 dB. It is surprising to find that although the absolute values of the scheduling gains are different, the normalized scheduling gains at three SNRs are almost same if the same channel model and subband size are used, as illustrated in Fig.8. Here the normalization is to scale the scheduling gains in a figure to the maximal scheduling gain. The reason is that the scheduling gain can be seen as a measure to the deviation from the exponential distribution. It is easy to see that the curves have similar shapes as in Fig.4. Therefore, these two measures: the maximal eigenvalue and the scheduling gain, is equivalent. So the normalize scheduling gain can also be used to select the appropriate subband size. It is clear that the scheduling gain decays with the subband size and the decreasing speed of the scheduling gain in channel B is much faster than in channel A. The reason is that the coherent bandwidth of channel A is much larger than channel B. From the results above, the appropriate subband size is 64 for channel A and 32 for channel B when considering that the accurateness of the above analysis method decreases with the subband size.

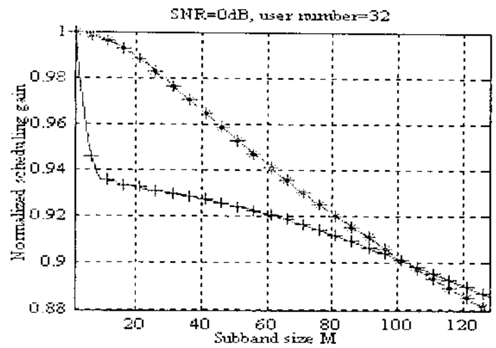

Fig.8 Normalized scheduling gain versus subband size at SNR=0dB

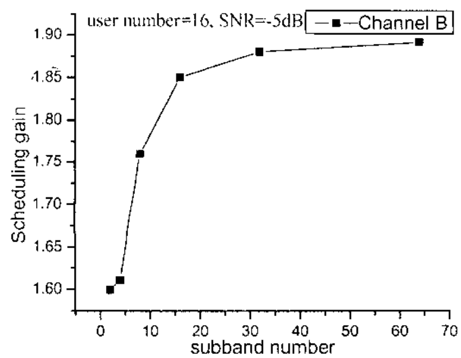

Fig.9 scheduling gain of the HDR scheduling algorithm versus the subband number

In a word, the appropriate subband size is 32 for channel B and 64 for channel B. In other word, the appropriate subband number is 32 or 16. This conclusion is confirmed through the simulation shown in the Fig.9 under the condition that $T_C = 1000$, average received SNR is -5dB, the user number is 16, channel B model is used and the HDR scheduling algorithm is used. It is clear that thescheduling gain is almost saturated when the subband number is 32 and there is some loss due to HDR scheduling algorithm compared with the asymptotic performance.

VI. CONCLUSION

In this paper, we show that the asymptotic scheduling gain of fair scheduling algorithms in the OFDM system increases as the number of user increases and decreases with the average received SNR. We also present the distribution of the averaged received SNR in a subband, give a formula to evaluate the scheduling gain approximately when subband is used, and point out the maximal eigenvalue is a good indicator to select the appropriate subband size. It is shown that the appropriate number of subbands is 16 for channel A or 32 for channel B when considering the tradeoff between the channel feedback and the throughput.

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